2nd Joint CCA-MASDOC Student Conference

CCA
Cambridge Centre for Analysis

MASDOC
Mathematics and Statistics
Centre for Doctoral Training

EPSRC
Engineering and Physical Sciences
Research Council

THE UNIVERSITY OF
WARWICK

$25^{th} - 27^{th}$ March 2013
University of Warwick

CCA-MASDOC Conference Timetable

All talks are in MS.04. Lunch and dinner are both served in the maths common room.

	25th March	26th March	27th March
9.00 – 9.30		CCA Talk 4	MASDOC Talk 9
9.30 – 10.00		MASDOC Talk 4	CCA Talk 10
10.00 – 10.30		Tea/Coffee Break	Tea/Coffee Break
10.30 – 11.00		CCA Talk 5	MASDOC Talk 10
11.00 – 11.30		MASDOC Talk 5	CCA Talk 11
11.30 – 12.00		CCA Talk 6	End of Conference
12.00 – 12.30	Arrival in Warwick	Lunch	Lunch
12.30 – 13.00	Lunch	Lunch	Lunch
13.00 – 13.30	Lunch	Lunch	Lunch
13.30 – 14.00	CCA Talk 1	MASDOC Talk 6	Lunch
14.00 – 14.30	MASDOC Talk 1	CCA Talk 7	Departure from Warwick
14.30 – 15.00	CCA Talk 2	MASDOC Talk 7	
15.00 – 15.30	Tea/Coffee Break	Tea/Coffee Break	
15.30 – 16.00	MASDOC Talk 2	CCA Talk 8	
16.00 – 16.30	CCA Talk 3	MASDOC Talk 8	
16.30 – 17.00	MASDOC Talk 3	CCA Talk 9	
17.00 – 18.00			
18.00 – 20.00	Dinner	Dinner	

MASDOC Talk 1: Maha Al Hajri (Mathematical Biology)
MASDOC Talk 2: Amal Alphonse (PDEs)
MASDOC Talk 3: Abhishek Shukla (Data Assimilation)
MASDOC Talk 4: Simon Bignold (Density Functional Theory)
MASDOC Talk 5: Matthew Thorpe (Multiscale Methods)
MASDOC Talk 6: Barnaby Garrod (Statistical Mechanics)
MASDOC Talk 7: Alex Kister (Statistical Mechanics)
MASDOC Talk 8: Chin Lun (Stochastic Integrable Systems)
MASDOC Talk 9: Yuchen Pei (Stochastic Integrable Systems)
MASDOC Talk 10:Owen Daniel (Statistical Mechanics)

CCA Talk 1: Alberto Gil Ramos (Spectral Theory)
CCA Talk 2: Clarice Poon (Generalized sampling)
CCA Talk 3: Eoin Devane (Optimization and control)
CCA Talk 4: Kornel Maczynski (Variational Problems)
CCA Talk 5: Martin Taylor (General Relativity)
CCA Talk 6: Luca Calatroni (Image Processing)
CCA Talk 7: Alberto Cabrero (Statistical Inference)
CCA Talk 8: Alex Jones (Sensing and sampling)
CCA Talk 9: Kevin Crooks (PDEs)
CCA Talk 10: Lukas Vermach (Stability Theory)
CCA Talk 11: Guolong Li (Stochastic Analysis)

-13:30-13:55, Alberto Ramos, CCA, (Spectral Theory).

-Spectral analysis of the Fox–Li operator-

In this talk I present some recent work with Prof. Arieh Iserles on the asymptotic behaviour of a generic eigenvalue and eigenfunction pair of the Fox–Li operator in the asymptotic regime of large oscillations. This Fox–Li eigenvalue and eigenfunction problem is particularly relevant to the science of laser beams and resonators, as well as a very appealing theme for research given that it is (mildly) non-normal and falls outside of standard spectral theory. The talk will concern two of our different approaches and conclude with current thoughts on the problem.

-14:00-14:25, Maha Al Hajri, MASDOC, (Mathematical Biology).

-On the analysis of a reaction diffusion systems defined on a partitioned domain with nonlinear conditions at the interface-

This is work in progress to address a global existence and uniqueness of a strong solution for semi-linear reaction diffusion systems. The system is defined on two adjoining portions of a domain $\Omega_1 \cup \Omega_2 = \Omega \subset \mathbb{R}^N$ with nonlinear conditions on the interface $\Gamma = \partial\Omega_1 \cup \partial\Omega_2$. We will use the strong maximum principle, Hope boundary Lemma, the Schauder fixed point theorem and the fundamental solutions of linear parabolic partial differential equations.

-14:30-14:55, Clarice Poon , CCA, (Generalized sampling).

-A stable and consistent approach to generalized sampling-

We consider a stable and consistent reconstruction scheme for generalized sampling and its application to wavelet reconstructions from Fourier samples. The implications of this algorithm for variable density sampling schemes in compressed sensing will also be presented.

-15:30-15:55, Amal Alphonse, MASDOC, (PDEs).

-Existence and Uniqueness of solutions to certain parabolic PDEs on evolving domains-

I will talk about some of the issues involved in the existence and uniqueness of solutions to certain parabolic partial differential equations on evolving domains. This will involve setting up relevant function spaces and the application of some results from functional analysis.

-16:00-16:25, Eoin Devane, CCA, (Optimization and Control).

-Distributed solution of the optimal power flow problem-

Power flow studies investigate the flow of electrical power through networks with topologies that represent real-world power grids, incorporating known power loads and losses together with unknown amounts of power generation. The classical Optimal Power Flow (OPF) problem involves minimising the total cost of power generation over all of the network buses capable of generating power, subject to satisfying all of the given loads and a collection of inequality constraints corresponding to physical limits on the system's operation. Power networks today are generally partitioned in structure, with different operators running disjoint components of the overall network. Given this structure, it is desirable to investigate approaches to solving the OPF problem that are distributed in nature, whereby each operator may solve its own regional optimization problem locally. In particular, each operator would prefer to be able not to disclose the exact operational details of its own subsystem to the other operators, who may be its rivals and competitors. Consequently, we will consider an algorithmic approach, through the method of dual decomposition, to obtaining a solution of the OPF problem while requiring only minimal information sharing between the separate regions of the network.

-16:30-16:55, Abhishek Shukla, MASDOC, (Data Assimilation).

-Analysis of the 3DVAR Filter for the Partially Observed Lorenz '63 Model-

The problem of effectively combining data with a mathematical model constitutes a major challenge in applied mathematics. It is particularly challenging for high-dimensional dynamical systems where data is received sequentially in time and the objective is to estimate the system state in an on-line fashion; this situation arises, for example, in weather forecasting. The sequential particle filter is then impractical and ad hoc filters, which employ some form of Gaussian approximation, are widely used. Prototypical of these ad hoc filters is the 3DVAR method. In this talk we will show the analysis of the 3DVAR method, using the Lorenz '63 model to exemplify the key ideas. The situation where the data is partial and noisy is studied, and both discrete time and continuous time data streams are considered. The theory demonstrates how the widely used technique of variance inflation acts to stabilize the filter, and hence leads to asymptotic accuracy.

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TUESDAY 26 MARCH

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-09:00-09:25, Kornel Maczynski, CCA, (Variational Problems).

-Branched minimal surfaces with prescribed asymptotic behaviour-

Minimal surfaces are the stationary points of the area functional $\mathcal{A}(S) = \mathcal{H}^n(S)$. Classical examples include plane, catenoid and helicoid. If the ambient space is a Riemannian manifold and $n = 1$ then they are just the geodesics of the given manifold.

In the study of minimal surfaces it is common to use integral varifolds — a generalisation of manifolds. The Allard's regularity theorem states that, under some conditions, a minimal integral varifold is regular ($C^{1,\alpha}$) on a dense open set. Nonetheless, there are examples of minimal surfaces with singularities (e.g. a cone over $S^1 \times S^1$ in $\mathbb{R}^4$). Singularities are in fact a phenomenon that cannot be dismissed.

I would like to present findings (with possibly small contribution of my own) of the paper [1] by Caffarelli, Hardt and Simon where the authors present a procedure of obtaining examples of minimal surfaces with isolated singularities that asymptotically look like a given stationary cone.

-09:30-09:55, Simon Bignold, MASDOC, (Density Functional Theory).

-Density Functional Theory: Deriving Phase-Field Crystals-

This talk will review the paper [Berry et al, 2007] which uses density functional theory (DFT) to provide a justification for the simpler and more numerically tractable theory of phase field crystals (PFC). The talk begins by introducing some basic techniques required for DFT, after which we develop the ideal gas as a motivating example and a starting point for PFC. We then proceed to derive the PFC functional by using a functional Taylor expansion and a gradient expansion. We conclude by demonstrating some initial results. DFT is a topic of interest in mathematics, physics and in chemistry whereas it is hoped that PFC will be more easily treated numerically . This talk reviews the link between these two interesting areas, it should hopefully be accessible to anyone with a reasonable degree of mathematical literacy.

-10:30-10:55, Martin Taylor, CCA, (General Relativity).

-On the Collapse of Small Data Self-Gravitating Massless Collisionless Matter-

General relativity is governed by the Einstein equations - a system of nonlinear hyperbolic partial differential equations whose unknown is a Lorentzian manifold. They admit a trivial solution called Minkowski space. In their monumental 1993 work Christodoulou–Klainerman proved that Minkowski space is stable to vacuum perturbations. A similar result can be shown to hold when massless collisionless matter is present.

-11:00-11:25, Matthew Thorpe, MASDOC, (Multiscale Methods).

-Multi Target Tracking in a GSM Network-

Mobile phones communicating over the 2G Network exhibit a mixture of continuous and discrete behaviour. It is not known a-priori which detections correspond to which phones. Although we still cannot solve the data association problem we can use the characteristics of the detections when partitioning the data into mobile phone tracks. In particular the signal from the phone exhibits fast changing characteristics such as amplitude and slow changing characteristics such as location (when received across multiple sensors). This presents a multi-scale problem. In this talk I will discuss results from real world experiments and present a model that describes the data.

-11:30-11:55, Luca Calatroni, CCA, (Image Processing).

-Splitting methods and primal-dual formulations for higher order imaging models-

We present an *alternating direction implicit* method solving numerically the TV-H^{-1} flow which shows good performance when applied to various imaging problems. Some stability issues depending on the regularization of the total variation may appear, thus explicit terms need to be controlled. A primal-dual approach and its possible applications to such problems is presented as well.

-13:30-13:55, Barnaby Garrod, MASDOC, (Statistical Mechanics).

-One-dimensional interacting Brownian motions via the Brownian Web-

One of the simplest continuous time, continuous state-space interacting particle systems one can consider is independent one-dimensional Brownian motions which instantaneously annihilate or coalesce upon meeting. A natural extension is to immigrate particles as time evolves. We outline the main concepts of one approach to dealing with such models. The key construction is a system of coalescing Brownian motions, known as the Brownian Web, and its dual system. As an example, we consider the Infinitesimal Pairwise Immigration (IPI) model, which may be thought of as the continuum limit of suitably scaled domain walls for a Glauber spin chain. The construction we give is as a limit of continuous state-space processes.

-14:00-14:25, Alberto Cabrero, CCA, (Statistical Inference).

-Statistical inference on Lévy processes-

Lévy processes are a rich class of stochastic processes which form the fundamental building block for stochastic continuous-time models with jumps. They have proved to be highly useful models in many applications within mathematical finance, insurance, queueing and storage theory, biology, physics, etc. However, in these applications the data corresponds to discrete-time observations of a stochastic process. Therefore, finding consistent estimators (i.e. that converge in probability to the true value) of the underlying continuous-time process and functionals of it is of vital importance for them. It is also important to understand how fast these estimations approach the true values as the number of observations grows. These theoretical questions proved to be technically challenging which, together with the growing demand for them, explains why there were hardly any important results in the area before the last decade.

The estimators we shall present exploit the fact that it is possible to produce approximations of a Lévy process and functionals of it through approximating the characteristics that uniquely determine the process.

-14:30-14:55, Alexander Kister, MASDOC, (Statistical Mechanics).

-Large Deviations for $\Delta\phi$ interface model-

This talk gives a short introduction into interface models as they are studied in statistical mechanics and how large deviation methods are applied there to get information about macroscopic observables.

-15:30-15:55, Alex Jones , CCA, (Sensing and sampling).

-Semirandom Sampling and Incoherence-

In NMR and MRI situations, such as trying to image a Brain or determining the structure of proteins, we are often sampling Fourier Coefficients of the Object we want to acquire. Therefore the choice of coefficients to sample is of prime importance since it effects the quality of reconstruction. The talk will present current work on finding good 1D and 2D sampling schemes based on studying the incoherence of a change of two bases.

-16:00-16:25, Chin Lun , MASDOC, (Stochastic Integrable systems).

-Dyson's Brownian Motion-

We explore the two aspects of a diffusion process called Dyson's Brownian motion: as the eigenvalues of a Hermitian matrix whose entries are independent Brownian motions and as a system of Brownian motions conditioned to never collide.

-16:30-16:55, Kevin Crooks, CCA, (PDE).

-Laplace equation in a convex polygon: A numerical implementation of the weak formulation of the Fokas method-

In this talk I will describe a numerical implementation of the Fokas method to solving the Laplace equation in a convex polygon. This follows work by Fokas in [2, 3] on the method and Ashon in [4] which formulates a weak variational approach in the case of a convex polygon. This method uses strongly the analyticity of the Fourier Transform, and a powerful theorem by Paley and Wiener. The computational method uses a Galerkin approach with a natural sinc basis for the Paley–Wiener space. We discuss computational issues to this approach and possible resolutions relating to its implementation. The role of rigorous analysis cannot be underestimated in resolving these computational difficulties.

WEDNESDAY 27 MARCH

-9:00-9:25, Yuchen Pei, MASDOC, (Stochastic Integrable Systems).

-A q-weighted version of the Robinson-Schensted algorithm-

I am going to talk about a q-weighted version of Robinson-Schensted algorithm. It is based on *http://arxiv.org/abs/1212.6716* . This algorithm is closely related to q-Whittaker functions, which are Macdonald symmetric polynomials when $t = 0$. When $q \in (0, 1)$ it can be considered as a randomised version of RS correspondence. When $t \to 0$ the q-version algorithm reduces to the classical one, while when $t \to 1$ with proper rescaling it should scale to the random polymer model in O'Connell (2012). It also gives an alternative formula that is equivalent with the one in Borodin and Corwin (2013) for q-TASEP, introduced by Sasamoto and Wadati (1998).

-9:30-9:55, Lukas Vermach, CCA, (Stability Theory).

-Electrokinetic instability in the Poiseuille flow-

The talk is concerned with a flow of electrically charged liquid running through a narrow channel with charged walls. Such a problem belongs to a wider set of effects known as the electrokinetic phenomena.

We start off the discussion by analysis of the steady state solution for the pressure driven channel flow. In particular, the research is focused on the influence of the streaming potential on the underlying parabolic velocity profile. It turns out that if the electrokinetic phenomenon is strong enough, the reverse flow in the boundary region of the channel can occur.

Knowing the steady state solution, the stability of the flow is examined. It is well known that for pure pressure-driven flow with absence of electric charge the stability breaks down at certain critical Reynolds number Re_c. The modal theory predicts $\mathrm{Re}_c \approx 5772$. We present the influence of the electrokinetic phenomena on this result. Consequently, we address the question whether the presence of electric charge changes the position of the most unstable mode and we study the sensitivity to the change of the dimensionless parameters controlling the system.

-10:30-10:55, Owen Daniel, MASDOC, (Statistical Mechanics).

-Loops, Soups and Bosons-

In the 1920s, Bose and Einstein conjectured a phase transition for gasses of Bosons cooled to a very low temperature. 70 years later, scientist were able to physically show the existence of this phenomenon (known as Bose-Einstein condensation), and were duly awarded a Nobel prize. Mathematicians are still playing catch-up in this area, and except for in the simplest cases, they have been unable to prove the existence of such a phase transition. In this talk we introduce Feynman's representation of the Bose gas, and introduce its relationship with the topic of Markovian loops, soups, and fields.

-11:00-11:25, Guolong Li, CCA, (Stochastic Analysis).

-A collision-time estimate for diffusion and Ornstein-Uhlenbeck particles-

This talk will investigate the time and place distribution when two random particles collide. In the rst model, we consider two particles free motions are diusions while in the second model we think their motions as Ornstein-Uhlenbeck processes. And at the end we will see what happens as the Ornstein-Uhlenbeck processes converge to Brownian motions.

References

[1] L. Simon L. Caffarelli, R. Hardt. Minimal surfaces with isolated singularities. *Manuscripta Math*, 48.1-3:1–18, 1984.

[2] A. S. Fokas. Two-dimensional linear partial differential equations in a convex polygon. *R. Soc. Lond. Proc. Ser. A Math. Phys. Eng. Sci.*, 457(2006):371–393, 2001.

[3] A. S. Fokas and A. A. Kapaev. On a transform method for the Laplace equation in a polygon. *IMA J. Appl. Math.*, 68(4):355–408, 2003.

[4] A. Ashton. The spectral Dirichlet–Neumann map for Laplace's equation in a convex polygon. *ArXiv e-prints*, 2012.