

Joint CDT Colloquium 2019



Interplay of Probability, Statistics and Analysis University of Warwick, 1st - 5th April 2019



CCA • MASDOC • MIGSAA • OxpDE • SAMBa

Timetable

| TIME | Monday | Tuesday | Wednesday | Thursday | Friday |
|----------|--------------------------------------|-----------------------------------|-------------------------------------|-------------------------------------|--------------------------------|
| 09:30 AM | | Mini course (Vassili Kolokoltsov) | Mini course (Vassili Kolokoltsov) | Mini course (Marie-Therese Wolfram) | Matteo Giordano |
| 10:00 AM | | | | | Giorgos Vasdekis |
| 10:30 AM | | David Woodford | Syafiq Johar | Greg Maierhofer | Jaro Sant |
| 11:00 AM | | Coffee break | | | |
| 11:30 AM | | Daniel Heydecker | Angeliki Manegaki | Philip Herbert | Paolo Grazieschi |
| 12:00 PM | | Ellie Archer | Luke Hatcher | Mildred Aduamoah | Plenary talk (Wilfrid Kendall) |
| 12:30 PM | | Lunch | | | |
| 01:00 PM | | | | | |
| 01:30 PM | | | | | |
| 02:00 PM | | Mini course (Vassili Kolokoltsov) | Mini course (Marie-Therese Wolfram) | Mini course (Marie-Therese Wolfram) | |
| 02:30 PM | | | | | |
| 03:00 PM | Registration/coffee | Dominic Brockington | Craig Robertson | Jack Thomas | |
| 03:30 PM | Plenary talk (Jane Hutton) | Coffee break | | | |
| 04:00 PM | | Peter Taylor | Nikos Athanasiou | Simon Becker | |
| 04:30 PM | Poster session with wine and nibbles | Trish Gunaratnam | Christian Scharrer | Hamed Masaood | |
| 05:00 PM | | | | | |
| 06:00 PM | | | Conference dinner | | |

All talks will be in MS.04 of the Zeeman Building. Registration, coffee, the poster session and the conference dinner will all be in the maths common room, on the first floor. This will be signposted from the main entrance on the Monday afternoon.

We will provide lunch on Tuesday in the maths common room. For other days, there are a range of food outlets by the SU building on central campus.

MS.05 and B1.01 will also be available for private study throughout the week.

Abstracts

Plenary Talks

Professor Jane Hutton (University of Warwick): Mathematical models meet (missing) patient data

As mathematicians, we enjoy the power of equations to express patterns and condense information. Statisticians consider how to connect equations with the world, collecting data and deciding on what is a good fit. Differential equations provide a plausible model for recovery from sprained ankle, but what data can we get from people? Stochastic process models for the effects of drugs

on epileptic seizures make sense, but can people record when seizures occur? I will discuss some successes and failures in fitting.

I have enjoyed working in medical statistics for 35 years. Information is not available on what I did about missing data in my early analysis of a study of artificial lung surfactant for premature babies. I progressed through regarding missing data as a separate category to considering sensitivity analyses for missing data in meta-analysis and survival models. I am now interested in using chain event graphs to inform choices of assumptions in models for non-response.

Professor Wilfrid Kendall (University of Warwick): Poisson processes and Spatial Transportation Networks: network geodesics and Rayleigh random flights

Scale-invariant random spatial networks (SIRSN) are remarkable random structures which provide patterns of random routes that are scale-invariant, thus modelling apparent scale-invariance in on-line maps. My talk will review the rather non-trivial theory establishing the existence of the Poisson line SIRSN and the known properties of network geodesics. In order to develop a good intuition about the behaviour of these geodesics, attention has turned to random scattering processes on the Poisson line SIRSN. This in turn leads to an axiomatization of abstract scattering processes (Markov chains which algebraically look like scattering processes), perhaps of wider interest in reliability theory. Ergodic theory (in particular a continuum version of the famous range theorem of Kesten, Spitzer and Whitman) can then be applied to produce insight into the behaviour of a randomly broken geodesic on the SIRSN.

Mini-courses

Professor Vassili Kolokoltsov (University of Warwick): Probabilistic methods for solving fractional PDEs and generalized fractional PDEs

We shall discuss recent achievements in the probabilistic interpretation and extensions of the fractional PDEs of Caputo and Riemann-Liouville type and on the path integral representations for their solutions arising from this point of view. The main advantage of these path integral representations is their universality allowing to cover a variety of different problems in a concise unified way, and the possibility to yield solutions in a compact form that is explicitly stable with respect to the initial data and key parameters and is directly amenable to numeric schemes (Monte-Carlo simulation). The starting point for this development is the observation that from the point of view of stochastic analysis the Caputo and Riemann-Liouville derivatives of order $\alpha \in (0, 2)$ can be viewed as (regularized) generators of stable Levy motions interrupted on crossing a boundary. This interpretation naturally suggests fully mixed, two-sided or even multidimensional generalizations of these derivatives, as well as a probabilistic approach to the analysis of the related equations. As application of these ideas we shall discuss wide classes of generalized fractional equations giving probabilistic interpretations of their solutions in terms of the Dynkin type martingales and/or chronological operator-valued extensions of the Feynman-Kac formulas.

Dr. Marie-Therese Wolfram (University of Warwick): On Boltzmann-type equations in socio-economic applications

Kinetic equations have become an indispensable tool to provide a quantitative description of diverse phenomena, such as semi-conductors, plasma physics, price and opinion formation, and more recently animal herding and pedestrian dynamics. In this talk we discuss how kinetic or so-called Boltzmann type models, can be used to describe the dynamics of many interacting particle systems

using laws of statistical mechanics in pedestrian dynamics and opinion formation. Starting with the microscopic interaction laws, we derive the corresponding Boltzmann-type equations and discuss the behavior of solutions in different limits. Numerical simulations complete the picture of the rich dynamics and confirm the emergence of complex pattern, such as lane formation in bidirectional flows.

Student talks

Mildred Aduamoah (University of Edinburgh): A new PDE-constrained Optimization Framework for Multiscale Particle Dynamics

A vast number of important applications in mathematics and engineering are governed by mathematical optimization problems. One crucial class of these problems that have gained significant recent attention are PDE-constrained optimization problems, which can be used to model problems from fluid flow, industrial processes, and biological mechanisms. However, to date, relatively little has been done by way of devising a systematic approach to tackle multiscale PDE-constrained optimization problems arising from particle dynamics. In this talk, we will introduce particle dynamics optimization models, discuss the real life applications they arise from, and explain the new numerical methods that we have developed to solve them.

Ellie Archer (University of Warwick): Brownian motion on looptree-like structures

Stable looptrees are a class of random objects that can informally be thought of as the dual graphs of stable trees. In this talk, we start by introducing discrete looptrees via pictures and then explain how one can construct Brownian motion on their scaling limits. In the α -stable case (for $\alpha \in (1, 2)$), we also give some properties of this Brownian motion, and compare and contrast to similar results for Brownian motion on stable trees. Finally, we discuss applications to random walks on related random map models, such as outerplanar maps and critical percolation clusters.

Nikos Athanasiou (University of Oxford): Formation of singularities for the relativistic Euler equations

An archetypal phenomenon in the study of hyperbolic systems of conservation laws is the development of singularities (in particular shocks) in finite time, no matter how smooth or small the initial data are. A series of works by Lax, John et al confirmed that for some important systems, when the initial data is a smooth small perturbation of a constant state, singularity formation in finite time is equivalent to the existence of compression in the initial data. Our talk will address the question of whether this dichotomy persists for large data problems, at least for the system of the Relativistic Euler equations in (1+1) dimensions. We shall also give some interesting studies in (3+1) dimensions. This is joint work with Dr. Shengguo Zhu.

Simon Becker (University of Cambridge): Discrete random Schrödinger operators

I will start by discussing the rich mathematical properties of discrete Schrödinger operators in homogeneous magnetic fields. After reviewing some results on the spectral theory of quasi-periodic operators and semiclassical analysis, I will explain how these techniques are used in the study of such quantum graphs to prove the existence of Cantor spectra and Landau levels. I will then discuss how randomness can be included in the model and how it affects the spectral properties. This will include a discussion of localization, delocalization, and the quantum hall effect.

References: [1] *S. Becker, R. Han, S. Jitomirskaya, Cantor spectrum in graphene, preprint, 2018.*
 [2] *S. Becker, M. Zworski, Cantor spectrum in graphene, preprint, 2018.*

Dominic Brockington (University of Warwick): Sticky Brownian motions and Stochastic flows

We define a consistent family of sticky Brownian motions, and the related stochastic flow of kernels. Finally we describe the Bethe ansatz and show it solves the backwards equations for this system.

Matteo Giordano (University of Cambridge): Bernstein-von Mises Theorems and Uncertainty Quantification for Linear Inverse Problems

The talk will present asymptotic results for the statistical inverse problem of approximating an unknown function f from a linear measurement corrupted by additive Gaussian white noise. We employ a nonparametric Bayesian approach with standard Gaussian priors, for which the posterior-based reconstruction of f corresponds to a Tikhonov regulariser with a Cameron-Martin space norm penalty. We prove a semiparametric Bernstein-von Mises theorem for a large collection of linear functionals of f , implying that semi-parametric posterior estimation and uncertainty quantification are valid and optimal from a frequentist point of view.

For the problem of recovering the source function in elliptic partial differential equations, we also obtain a nonparametric version of the theorem that entails the convergence of the posterior distribution to a fixed infinite-dimensional Gaussian probability measure with minimal covariance in suitable function spaces. As a consequence, we show that the distribution of the Tikhonov regulariser is asymptotically normal and attains the information lower bound, and that credible sets centred at the regulariser have correct frequentist coverage and optimal diameter.

Paolo Grazieschi (University of Bath): Convergence of the three-dimensional Ising-Kac model to ϕ_3^4

We will introduce the Ising model and then change the way particles behave by introducing a long range interaction. The resulting system, called Ising-Kac model, is studied in its scaling properties and ultimately the goal is to prove that a suitably rescaled version of it converges to the ϕ_3^4 equation. We will also discuss about the regularity difficulties that this equation poses and how to define a concept of solution.

Trishen Gunaratnam (University of Bath): Very Low Temperature Φ^4

We describe some recent results regarding the phase transition and its effect on the dynamics for Φ^4 models in 3D. Joint work with Ajay Chandra and Hendrik Weber.

Luke Hatcher (University of Warwick): Sharp and diffuse interface problems relating to Canham-Helfrich equilibria

Small micro-domains known as lipid rafts have been posited to form on biological membranes and are thought to play a role in many cellular processes, such as signal transduction, membrane trafficking and protein sorting. The small scales on which lipid rafts are believed to form (10-200nm) has meant their existence remains controversial. However, some support for the raft hypothesis can be found in the experimental observation of such micro-domains in artificial membranes known as giant unilamellar vesicles. It is therefore interesting to study the processes by which these rafts

could be produced. In this talk we investigate whether the geometry of the membrane could be a mechanism which drives the formation of these lipid rafts. We will derive a model for these two-phase membranes by considering the Canham-Helfrich energy with phase-dependent spontaneous curvature, and by making the simplifying assumption that the membrane remains approximately spherical in shape. We will consider the diffuse interface problem and motivate this by a formal asymptotic analysis. Finally we will discuss some numerical experiments obtained by considering a corresponding gradient flow.

Philip Herbert (University of Warwick): Biological membranes with deformations induced by point particles

We motivate a fourth order surface PDE which arises in the study of near spherical biological membranes, where we consider the geometry is made non-trivial by point particle inclusions. One may consider two models for study, one where the constraints are strictly obeyed, and another where the constraint is penalised (so called hard constraints and soft constraints respectively). We discuss the well posedness of these problems along with the well posedness of an appropriate splitting schemes in order to develop numerical experiments. This is joint work with Charlie Elliott.

Daniel Heydecker (University of Cambridge): From Newton to Boltzmann: Propagation of Chaos for the homogenous Boltzmann Equation

Imagine a container of warm gas as a collection of many (very many: typically $O(10^{23})$) small billiard balls bouncing off each other. What can we say about the dynamics of the system? In principle, we could determine everything that goes on from Newton's laws of motion; in practice, this is not only intractable, and instead, a popular idea is to consider a probabilistic alternative, which leads to Boltzmann's equation. The main technical aspect in deriving Boltzmann's equation is to justify the assumption of *molecular chaos*, which quantifies the non-independence of particles. I will present my work leading to quantitative chaoticity estimates, and some of the counterintuitive results which occur in connection with this problem.

Syafiq Johar (University of Oxford): Ricci Flow in Milnor Frames

We are going to discuss the Type I singularity on 4-dimensional manifolds foliated by homogeneous S^3 evolving under the Ricci flow. We review the study on rotationally symmetric manifolds done by Angenent, Isenberg, Knopf and Sesum. In their study, a global frame for the tangent bundle, called the Milnor frame, was used to set up the problem. This frame arises from the symmetries of the manifold, derived from the Lie group structure of S^3 . Numerical simulations of the Ricci flow on these manifolds are done, providing some insight on their behaviour. Some analytic results will be proven for the manifolds S^1S^3 and S^4 using maximum principles from parabolic PDE theory and some sufficiency conditions for a neckpinch singularity will be provided. Finally, we will pose a problem from general relativity with similar symmetries but endowed on a manifold with different topology, called the Taub metrics.

Georg Maierhofer (University of Cambridge): Wave scattering and highly oscillatory integrals

The efficient modelling of wave propagation plays an important role in modern science and technology. In many applications, such as prediction and control of noise from turbomachinery, the fast solution of wave scattering problems is of crucial importance. Motivated by recently developed hybrid-numerical asymptotic methods we will consider a class of quadrature techniques that

are especially powerful for the numerical evaluation of highly oscillatory integrals. For decades, these integrals were regarded as notoriously difficult to compute for large frequencies, but intensive research over the past fifteen years has helped develop methods that overcome this challenge. In this talk we will consider so-called Filon methods, and present new error estimates that provide a deeper understanding of the interplay between frequency and number of quadrature points. We will also introduce a new Filon-based method for the singular case, which provides the first step towards further enhancing the asymptotic performance of algorithms for wave scattering problems. Throughout we will assess the practical performance of these algorithms based on a number of numerical examples.

Hamed Masood (University of Cambridge): Scattering Theory for the Teukolsky Equation and Linearized Gravitational Perturbations in a Schwarzschild Background

In scattering theory perturbations to an isolated physical system are evolved over a much larger scale compared to the scale of their interaction with the system, such that data is set up away from the effects of the system where the equations simplify and could allow for a study of the system in terms of asymptotic information. The Teukolsky equations are second order linear wave equations that arise in the study of linear perturbations to solutions of the Einstein equations and govern certain components of the perturbed curvature. We develop a physical-space scattering theory for these equations on a Schwarzschild black hole background identifying the function spaces for the asymptotic past and future data parametrizing the solutions relevant to the study of linear perturbations, and construct a scattering matrix defining a Hilbert space-isomorphism between past and future radiation fields. Finally we discuss scattering for the remaining curvature components covering the full system of linear perturbations to the Schwarzschild black hole.

Angeliki Menegaki (University of Cambridge): Hypocoercivity through calculus for a model describing heat diffusion

Equilibrium states are already well understood in statistical mechanics but what is more challenging and there is still a lot of work to be done, are the out of equilibrium systems. A long-standing issue in the study of these systems is the heat conductivity and the validity of Fourier's law. In this talk we will discuss about the problems arising in deriving microscopically Fourier's law and we will present a model created for this purpose, i.e. to describe properly heat diffusion. It consists of a 1-dimensional chain of oscillators, where one starts with a classical Hamiltonian description of the particles (oscillators) of the chain and then perturbs the system by putting two heat baths at both ends of the chain at different temperatures.

We will talk about the particular case of a perturbation of the harmonic chain (the chain is harmonic when all the potentials are quadratic). Then we will show how it is possible to prove exponential convergence to the non-equilibrium steady state (NESS) in Wasserstein-2 distance and in Entropy as well. The method we follow is to combine a generalised version of the well-known theory of Γ calculus thanks to Bakry-Emery and some known properties of the harmonic chain. This new method has the advantage to give quantitative results, and so, if time permits we will talk about how one can get estimates of the rates in terms of the number of the particles.

Craig Robertson (University of Oxford): Finite-time degeneration for Teichmüller harmonic map flow

Teichmüller harmonic map flow is a geometric flow designed to evolve combinations of maps and metrics on a surface into minimal surfaces in a Riemannian manifold. I will introduce the flow and

describe known existence results, and discuss recent joint work with M. Rupflin that demonstrates how singularities can develop in the metric component in finite time.

Jaro Sant (University of Warwick): Bayesian Estimation for Diffusions in Genetics

A number of discrete time, finite population size models in genetics describing the dynamics of allele frequencies are known to converge (subject to suitable scaling) to a diffusion process limit termed the Wright-Fisher diffusion. This diffusion lives on the unit interval and encompasses within it important information regarding genetic evolution. In particular, phenomena such as selection and mutation are reflected in the drift coefficient of the process, whereas the diffusion coefficient is an a-priori known function of the process. Thus in most cases the aim of conducting inference on such models is to estimate some feature present in the drift, as this can help shed light onto how certain factors influence genetic variation. In Watterson (1979), the author constructed an MLE for the selection parameter in the diffusive limit and proceeded to analyse its asymptotic properties. The aim of this talk is to consider a Bayesian estimator in the parametric setup for the selection coefficient when both selection and mutation are acting on the population being observed. Through the use of two general results from Ibragimov & Hasminskii (1981), it is shown that this estimator has several desirable properties, namely uniform consistency over compacts, uniform asymptotic normality and uniform convergence of moments on compacts. No prior knowledge of biology or genetics is required.

Christian Scharrer (University of Warwick): Existence and regularity of surfaces minimising the Canham-Helfrich energy

Most cell membranes of living organisms are made of lipid bilayer, which is a thin polar membrane made of two opposite oriented layers of lipid molecules. In the early 70s, Canham and Helfrich introduced their energy. It consist of the integrated squared mean curvature, called Willmore energy, the first moment of the mean curvature, the integrated Gauss curvature, the area, and the volume. In contrast to the Willmore energy, the Canham-Helfrich energy changes under conformal transformation which is the reason why existence of minimisers was only known in the axisymmetric case. We prove existence and regularity of minimisers in the class of weak branched conformal immersions using techniques introduced by Mondino and Riviere.

Peter Taylor (University of Cambridge): Local Limit Theorems for the Random Conductance Model and Connections to Stochastic Interface Models

The Random Conductance Model is a random walk on the Euclidean lattice which moves according to random weights on the edges of the lattice. Recent results on the local limit theorem, which describes how the heat kernel of this process scales to that of Brownian motion, will be discussed. We extend existing quenched results to a class of general speed measures for the walk, where the random weights can be unbounded but satisfy some integrability conditions. Annealed local limit theorems, for both static and time-dynamic environments, will also be presented and applied to prove a scaling limit for the covariance of random heights in a class of Ginzburg-Landau grad ϕ models. Joint work with Sebastian Andres.

Jack Thomas (University of Warwick): Zero Temperature Limit of the Tight Binding Model for Point Defects

The tight binding model is a minimalist electronic structure model for predicting properties of molecules and materials. We review (Chen, Lu, Örtner, 2016) to introduce a limiting model as

domain size grows to infinity and then extend these results to the more common zero temperature case. In particular, we formulate zero temperature models and prove these are consistent with taking zero temperature and thermodynamic limits in the finite temperature models. As an application of the aforementioned results, we obtain qualitatively sharp estimates on the interaction range for the zero temperature electronic structure model.

Joint work with Christoph Örtner.

Giorgos Vasdekis (University of Warwick): On the Exponential Ergodicity of the ZigZag Process

Piecewise Deterministic Markov Processes have recently drawn the attention of the Markov Chain Monte Carlo community. The first reason for this is that, in general, one can simulate exactly the entire path of such a process. The second is that these processes are non-reversible, which sometimes leads to faster mixing (Diaconis et al 2000). Most of the Piecewise Deterministic Algorithms constructed nowadays are based on two processes: The ZigZag and the Bouncing Particle Sampler Processes. The ZigZag Process moves linearly in the space \mathbb{R}^d in specific directions for a random period of time, changing direction one coordinate at a time. An important question related to these samplers is the existence of a Central Limit Theorem which is closely connected to the property of Exponential Ergodicity. For both of the processes this property has been verified in (Deligiannidis et al 2017) and (Bierkens et al 2018) when the target distribution has light tails. In this talk we will prove that the ZigZag Sampler is not Exponentially Ergodic in a heavy tails scenario and will briefly explain how one could try to correct this by allowing the ZigZag process to take more velocities than ± 1 .

David Woodford (University of Warwick): Exponential Functionals of Markov Additive Processes

There are many scenarios where the evolution of a stochastic process depends on some underlying state which changes randomly. We will discuss a particular class of examples, Markov additive processes, where the underlying state is given by the value of a continuous time discrete state space Markov chain and the stochastic process of interest is a Levy process. This is an example that naturally arises in queueing, finance and theoretical probability.

Posters

Luke Hatcher (University of Warwick), *Phase field models for small deformations of biomembranes arising as Helfrich energy equilibria*

Philip Herbert (University of Warwick), *Biological membranes with deformations induced by point particles*

Syafiq Johar (University of Oxford), *Stability of Taub Metrics under the Ricci Flow*

Giorgos Vasdekis (University of Warwick), *A Generalised ZigZag Process*

David Woodford (University of Warwick), *Exponential Functionals of Markov Additive Processes*

Participants

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Campus map

