

Introduction to Functional Analysis – Errata

With thanks to Robin Chapman, Juan David González Cobas, Wojciech Ożański, Richard Rivero, and Shamindra Shrotriya.

p24 The first line of the proof of Lemma 2.17 should read ‘for some $\varepsilon > 0$ ’ (rather than ‘for every $\varepsilon > 0$ ’).

p31 Exercise 2.12: S must also be bounded above (otherwise the supremum need not exist).

p29 The displayed equation in the proof of Proposition 2.29 is incomplete: it should read

$$\sup\{y : y \in f(K)\} \in f(K)$$

p37 In the statement of Lemma 3.4, it should be $N : X \rightarrow [0, \infty)$. Before the final displayed equation in the proof, property (ii) from the statement of the lemma is used ($N(\lambda x) = |\lambda|N(x)$), not property (ii) from Definition 3.1.

p42 The left-hand side of the final displayed equations in the proof of Example 3.13 should be

$$\|\lambda f + (1 - \lambda)g\|_{L^p}^p$$

p75 The definitions of g and p in the proof of Corollary 6.3 should be $g(x) := f(a + x(b - a))$ and $p(x) := q((x - a)/(b - a))$.

p85 The space $\text{Lip}_L(X; \mathbb{K})$ and the similar space on p86 are not compact as defined. We need to include a boundedness condition (as required by Corollary 6.13): so, for example, the space

$$\{f \in C(X; \mathbb{K}) : \|f\|_\infty \leq B, |f(x) - f(y)| \leq L|x - y| \text{ for all } x, y \in X\}$$

is compact.

p94ff When defining L^p spaces as completions, a little more care is needed than here. There are two issues: when Ω is unbounded, and when the boundary of Ω has non-zero measure. In the first case (which includes \mathbb{R}^n , of course) there may be functions in $C(\overline{\Omega})$ that are not in $L^p(\Omega)$, so here one must take the completion of functions in $C(\overline{\Omega})$ such that $\int_\Omega |f|^p < \infty$. In the second case one has to take the completion of continuous functions with compact support in Ω .

p149 The proof of surjectivity of $T + S$ uses the Contraction Mapping Theorem, so the result guarantees that there is a *unique* x such that $(T + S)x = y$,

so $T + S$ is not only surjective, it is also injective. Given a different ordering of the material in the book, one could now appeal to the Inverse Mapping Theorem (Theorem 23.2) to deduce that $(T + S)^{-1}$ is bounded.

p158 Exercise 12.3 (iii) - should read $B(x, x) \geq b\|x\|^2$ for some $b > 0$.

p158 Exercise 12.4 (iii) - should read $f \in H^*$

p170 Third paragraph of proof of Theorem 14.9 is unnecessary: it is an immediate consequence of Proposition 11.18 that $\beta_i I - T$ is invertible for each i , and hence $\beta_i \notin \sigma(T)$.

p178 The spectrum of a bounded operator on a Banach space is also non-empty; but this requires the theory of operator-valued complex functions (see Theorem X in Kreyszig, 1978, for example).

p179 Exercise 15.7: summation in the definition of $S\mathbf{x}$ should be over j

p184 After the first sentence of the proof of Theorem 16.6, define λ_1 : “set $\lambda_1 = \pm\|T\|$, so that $Tw_1 = \lambda_1 w_1$ ”.

p192 Lemma 17.3. For general functions u_1 and u_2 , having zero Wronskian $W_1(u_1, u_2)$ does not imply that u_1 and u_2 are linearly dependent: for example, $u_1(x) = x|x|$ and $u_2(x) = x^2$ have $u_1'(x) = 2|x|$ and $u_2'(x) = 2x$, so $W(u_1, u_2)(x) = 0$ but these functions are linearly independent.

However, if $L[u_1] = L[u_2] = 0$ then zero Wronskian *does* imply linear dependence. To see this, fix any $x_0 \in [a, b]$; then, as written in the current proof, there must exist $\alpha, \beta \in \mathbb{R}$ such that $\alpha u_1(x_0) + \beta u_2(x_0) = 0$ and $\alpha u_1'(x_0) + \beta u_2'(x_0) = 0$. The issue – not addressed in the proof as printed – is that a priori α and β could depend on x . However, if we consider the function $\phi(x) := \alpha u_1(x) + \beta u_2(x)$ then

$$L[\phi] = 0 \quad \text{with} \quad \phi(x_0) = \phi'(x_0) = 0.$$

Uniqueness of solutions for this initial-value problem now shows that $\phi(x) \equiv 0$ for all $x \in [a, b]$.

p193 Theorem 17.5: u_1, u_2 here should be in $C^2([a, b])$, not in \mathcal{D} . [If they were in \mathcal{D} then Lemma 17.1 would imply that they were both zero.]

p194 The functions u_1 and u_2 constructed in this way satisfy

$$L[u_1] = 0, \quad u_1(a) = 0, \quad u_1'(a) = 1 \quad \text{and} \quad L[u_2] = 0, \quad u_2(b) = 0, \quad u_2'(b) = 1.$$

They are linearly independent, since otherwise $u_1(a) = u_1(b) = 0$, which

coupled with the fact that $L[u_1] = 0$ would imply (by Lemma 17.1) that $u_1 = 0$ (and $u_2 = 0$ too).

p196 That $\mathcal{T}g = u$ follows from the uniqueness of \mathcal{D} -valued solutions of $L[u] = g$: if $L[u] = L[v] = g$ then $L[u - v] = 0$, so clearly

$$(L[u - v], u - v) = 0$$

and Lemma 17.1 implies that $u = v$.

p196 The final sentence of the penultimate paragraph should say “in particular, zero is not an eigenvalue of \mathcal{T} ”.

p198 The first case should be when $\lambda < 0$, in which case $\sqrt{\lambda}$ should be replaced by $\sqrt{|\lambda|}$. The final case is then $\lambda > 0$.

p234 line 5 of proof of Lemma 21.6, should read $|\phi(x_n)| \geq n$ since X could be a complex space.

p260 in the displayed equation following (24.1), the denominator should be $\|x_{n_j} - z_{n_j}\|$ (z_{n_j} not y_{n_j}).

p262 Proof of Proposition 24.6: if we set $E_0 = \{0\}$ then it is possible to take $n > m \geq 1$ in the penultimate line.

p294 Theorem 27.12: in the second paragraph of the proof, the separability of Y^* is required in order to invoke Theorem 27.11, which is applied with $X = Y^*$ and $X^* = (Y^*)^* = Y^{**}$.

p297 Exercise 27.10: X here is once again (as in Exercise 27.9) a real Banach space.

p301 “An element $b \in P$ is an *upper bound*...” [‘is’ not ‘in’]

p366 solution of Exercise 12.3, the coefficient of $B(v, v)$ in first equation after (S.20) should be $\frac{t^2}{2}$.

p369 line -4: ℓ^∞ should be ℓ^2 (twice)

p382 solution of Exercise 20.9: in first line ϕ should read ϕ_n