Course Outline

MA475 Riemann Surfaces

1. Riemann Surfaces and Atlases

   (a) Holomorphic functions and the Cauchy-Riemann equations

   (b) Holomorphic and smooth atlases in arbitrary dimensions

   (c) Example: The sphere in $\mathbb{R}^3$ and $\mathbb{CP}^1$

   (d) Holomorphic functions on surfaces

   (e) The tangent bundle and the chain rule for surfaces

   (f) Quotient constructions, free and non-free actions

   (g) Plane curve examples in degree 2

   (h) Cut and paste construction of the octagon

2. Algebraic Curves (Plane curves)

   (a) Real and complex plane algebraic curves non-singular examples

   (b) Construction of $\mathbb{RP}^2$ and $\mathbb{CP}^2$, homogeneous polynomials and projective curves

   (c) Atlases for non-singular algebraic curves and the holomorphic implicit function theorem

3. Holomorphic Maps Between Riemann Surfaces

   (a) Meromorphic functions on $\mathbb{CP}^1$

   (b) Automorphism groups of the plane, sphere and disk

   (c) The holomorphic inverse function theorem

   (d) Local form of holomorphic maps

   (e) Open mapping theorem, non-constant holomorphic functions

   (f) Proper maps, local homeomorphisms, covering maps and branched covers

   (g) The degree formula for proper maps

   (h) The Riemann-Hurwitz Theorem

4. The Weierstrass $\mathcal{P}$-function and Elliptic Curves
   (a) Meromorphic functions on tori of the form $\mathbb{C}/\Lambda$
   (b) Using meromorphic functions on tori to parametrise cubic curves
   (c) Elliptic integrals and holomorphic 1-forms
   (d) Determining a lattice from a cubic curve

5. References and Sources

   (a) "Riemann Surfaces" by Simon Donaldson (Parts I and II)
   (b) "Complex Algebraic Curves" by Frances Kirwan (Chapters 1-6) This book is available electronically through the Warwick Library.
   (c) "Complex Analysis" by Lars Ahlfors (Chapter 7)
   (d) Curt McMullen online notes for Math 213a.
   (e) Curt McMullen online notes for Math 213b. (Pages 1-22) He discusses the regular octagon surface on page 8 and the irregular cover example on page 14.
   (f) "A primer on Riemann Surfaces" by Alan Beardon. Chapter 9 proves the Uniformization Theorem.
   (g) "A Scrapbook of Complex Curve Theory" by Herb Clemens
   (h) "Part IID Riemann Surfaces" by Alexei Kovalev (Cambridge) online.