MA424 Example Sheet 4

4 November 2015

1. For the doubling map $f(x) = 2x \bmod 1$:

   (a) What is the orbit of $\frac{1}{3}$?

   (b) What is the orbit of $\frac{1}{4}$?

   (c) What is the orbit of $\frac{1}{12}$?

2. Let $f_2 : \mathbb{R}/\mathbb{Z} \to \mathbb{R}/\mathbb{Z}$ be defined by $f_2(x) = 2x \bmod 1$. Show that a point has a finite orbit for f_2 if and only if it is rational. When is a point periodic?

3. Let Σ be the shift space:

   $$\Sigma = \{(\omega_k)_{k=0}^{\infty} : \omega_k \in \{0,1\}\}.$$

   Let d be the distance given by

   $$d(\omega, \omega') = 2^{-\min\{k:\omega_k \neq \omega'_k\}}$$

   if $\omega \neq \omega'$ and $d(\omega, \omega) = 0$.

   (a) Show that (Σ, d) is a complete metric space.

   (b) Show that it is compact.

4. Let $q_c(x) = x^2 + c$ and $f_\lambda(x) = \lambda x(1-x)$. Show that there is a function of the form $h(x) = \alpha x + \beta$ which topologically conjugates q_c and f_μ. Find an expression for μ, α, β in terms of c.