

MA424 Example Sheet 6

2 December 2015

1. Let $f_r(x) = rx(1-x)$ with $r \geq 4$. Show that f_r^n has 2^n fixed points for every $n \geq 1$.
2. Let G and G' be the directed graphs associated to the matrices

$$A_G = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad A_{G'} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

Show that the associated shift maps on G_∞ and G'_∞ are topologically conjugate.

3. Let $f : X \rightarrow X$ be a continuous map from a compact metric space X to itself. Define $d_n(x, y) = \max_{0 \leq \ell \leq n-1} d(f^\ell(x), f^\ell(y))$. Show that d_n satisfies the axioms of a metric. Show that d_n defines the same topology on X as d .
4. Show that a compact metric space possesses a finite (n, ϵ) -spanning set. Show that a compact metric space possesses a finite (n, ϵ) -separated set.
5. Compute the entropy of the Smale horseshoe.
6. Compute the entropy of a C^2 -diffeomorphism of the circle with an irrational rotation number.
7. Compute the entropy of the map $f : [0, 1] \rightarrow [0, 1]$ defined by $f(x) = x(1-x)$.
8. Give an example of a continuous map with $h(f) > 0$ and no periodic points.