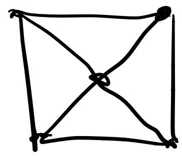
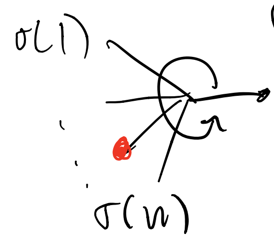
From lecture 1

Ex:  $=0$ for odd or even orientation.

Ex: There are only finitely many connected graphs with $\chi = 1-r$ with all vertices at least trivalent.

Ex: Let P be the operad with:

$P(n) =$ linear orders on $\{1, \dots, n\}$ with one distinguished slot

Picture: 

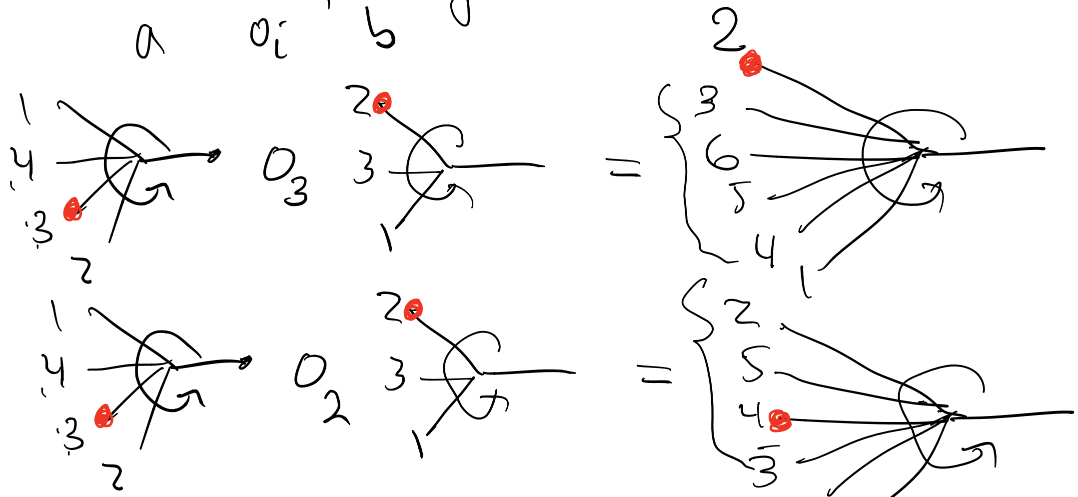
$\in P(n)$

Composition $P(k) \circ_i P(\ell) \to P(k+\ell-1)$

inserts a into b at position i

If position i is the marked slot, use the marked slot in a

Otherwise, forget the slot in a



Show P is not cyclic.

Lecture 2:

The bracket on $\mathcal{O}$-spiders was given by
$$[S,T] = \sum_{\substack{\lambda \in S \\ \mu \in T}} (ST)_{\lambda\mu}$$

Exercise Show this is Anti-symmetric, and satisfies the Jacobi identity

Exercise: For $\mathcal{O} = \mathrm{Comm}$,

show $[S,T] = \{S,T\}$
(Poisson bracket)

we showed 0-spiders give derivations

Exercise In general,

Derivations form a Lie algebra:

$$[D_1, D_2] = D_1 \circ D_2 - D_2 \circ D_1$$

Identify this bracket w/ the above bracket.

Lecture 3

$A \in \mathfrak{sp}_K$ acts on $\mathfrak{h}_K$.

Exercise: Show

$$A \cdot [S_1, S_2] = [A \cdot S_1, S_2] + [S_1, A \cdot S_2]$$

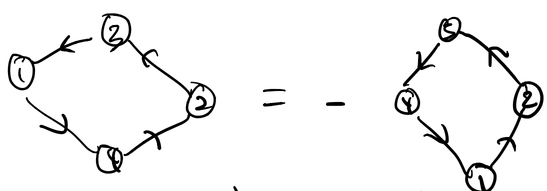
Exercise: Check $A \cdot d^{CE}(x) = d^{CE}(A \cdot x)$

Lecture 4

exercise: Compute $PH_*(\Lambda^* \mathfrak{h}^{(2)})$

Hint: Show A generator of $cC_* \mathcal{G}^{(2)}$ with k vertices is zero unless $k \equiv 3 \pmod 4$

eg



$\Rightarrow (G, or) = 0$

$$(G, or) = -(G, or)$$

exercise: The action of $Out(F_n)$ on CV_n is well-defined

Exercise: Find a graph with no odd symmetries, (ie odd edge-permutations) describe $\sigma(G, \theta)/_{Isom(G)}$

Lecture 5

Exercise Compute $H_*(\mathcal{M}\mathcal{G}_n^*, \partial \mathcal{M}\mathcal{G}_n^*)$

for $n = 2, 3, 4$

Hint: Use $\mathcal{C}\mathcal{G}_*^{(n)}$

Exercise Show $\partial^2 = 0$ in the forested graph complex

$$\partial(G, \Phi) = \sum_{\Phi \cup e \text{ a forest}} (G, \Phi \cup e)$$

Exercise $0 \to U \to V \to W \to Z \to 0$

a short exact seq of finite-dim'l v. spaces

$\Rightarrow \exists$ canonical isomorphism

$$\det U \otimes \det W \cong \det V \otimes \det Z$$

Hint: split the sequence into two short exact sequences)

Lecture 6

Exercise: Compute the quotient $K_3/\mathrm{Out}(F_3)$ (using the cube complex structure)

Lecture 7

Exercise Let $B \subset X$ be a full subcomplex of a flag complex.
If $v \in X \setminus B$ is a vertex with $\mathrm{lk}\, v \cap B \neq \emptyset$, and $J \subset B$ is the subcomplex spanned by $\mathrm{lk}\, v \cap B$ then the subcomplex $\langle B, v \rangle$ spanned by B and v is equal to $B \cup_J c(J)$.

Exercise Let K^S be the cube complex associated to a surface $S = S_{g,s}$ and δ_S the "vertical" coboundary operator that splits vertices.

Identify the vertical (co)chain complex as a direct sum of cochain complexes of a sphere:
(look at the trees you can get by IH splitting, starting at a single vertex v of valence $|v|$.)

Lecture 8

Exercise Adapt the proof that $C_*(B_n)$ is acyclic to prove that $C_* \Delta_n^{\infty}$ is acyclic.

Exercise If $\deg(G) = v(G) - rk(G) - 1$, show

$$\deg(G_1 \cdot G_2) = \deg G_1 + \deg G_2$$

Exercise $[\cdot, \cdot]\big|_{C_0}$ is anti-symmetric and satisfies the Jacobi identity

Exercise: Show $\delta G = [\bullet\!\!-\!\!\bullet, G]$

Conclude $[\cdot,\cdot]$ induces a Lie algebra structure on $H^0(C\mathfrak{g}_*) = \ker \delta_0$