Algebraic Multigrid

Notes by C. Wagner

Seems to be an iterative method involving approximations whose errors go away as iteration proceeds.

Not in the same spirit as I want but can be useful.

Theoretical Physics seminar 13.00 today PS 1.28 (sandwiches in PS common room at 12.40) M Bollhoefer

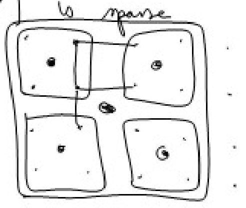
"Fast algebraic solvers for large scale linear systems & eigenvalue problems"

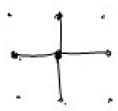
Context : linear systems $A u = f$

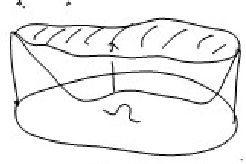
resulting from discretisation of partial differential equations e.g. $-\operatorname{div}\big(D(x)\nabla u(x)\big) = f(x)$

"Poisson equation" $u(x) = g$ on $\partial\Omega$ $\quad x \in \Omega \subset \mathbb{R}^n$

A is sparse



 $\subset \mathbb{R}^2$



I'd suggest trying an exact approach instead based on choosing new basis for vector space of values on a block $\mathbb{R}^4 = \mathbb{R} \oplus \mathbb{R}^3$

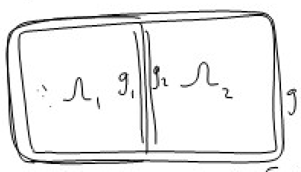
so that u_{ij} $i,j \in \{1,2\}$ $\quad = \mu + \delta_1$ (μ, δ) $\quad i,j = 1,1$

$\delta_2 \quad 1,2$

$\delta_3 \quad 2,1$

$-(\delta_1+\delta_2+\delta_3 \quad 2,2$

Derive exact eqns for $\{\underset{\sim}{\mu}, \underset{\sim}{\delta}\}$



Or top-down approach: solve Poisson on Ω subj to boundary values g

by solving Poisson $u_1[g_2]$ on Ω_1, g, g_2 generally g_2

& $u_2[g_1]$ Ω_2, g, g_1 " g_1

& then solving for "$g_1 = g_2$"

i.e. $u_1[g_2]\big|_{\substack{\text{inner}\\ \partial\Omega_1}} = g_1$ no longer a sparse problem but one dimension lower

Also DIMAP seminar last Tuesday on "Modular decomposition" speaker from OU

A hierarchical decomposition of undirected graphs

Said he couldn't do directed graphs.

Aggregation of many player games

Agents $A = \{1, 2, 3, \ldots\}$

Agent i has an action space $S_i = \{A_i, B_i, \ldots\}$

Given actions of $A \setminus i = A_{\not{i}}$

then we suppose i has a partial order on its actions e.g.

$$\begin{array}{ccc} A_i & B_i & A_i \quad B_i \\ \vee & \vee & \\ B_i & A_i & \end{array}$$

Then suppose aggregate agents 1 & 2

Derive map from $A_{\not{1}\not{2}}$ to $\mathcal{P}_{1,2}$ space of partial orders on $S_1 \times S_2$

Questions are: when does this simplify analysis?

If concentrate on maximal elements of the partial orders, when does aggregation simplify?

Compare literature on formation of coalitions, where the game involves choosing to aggregate e.g.
M Wooders & G DeMange