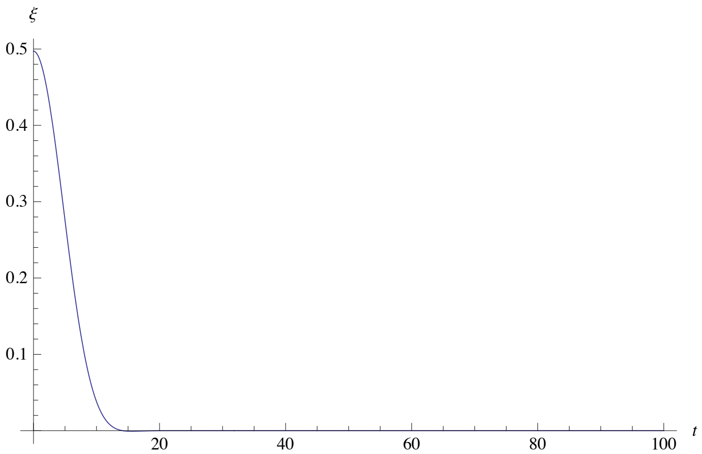
```
(* Full Screen: Command-Option-F *)
(* Evaluate Cells: Shift-Enter *)

(* TOWARDS A SPECTRAL PROOF OF RIEMANN'S HYPOTHESIS *)

(* Zeta[s] = Sum[n^{-s},{n,1,Infinity}] | Re[s]>1 *)
(* Gamma[s] = Integrate[x^{s-1}Exp[-x],{x,0,Infinity}] | Re[s]>0 *)
```

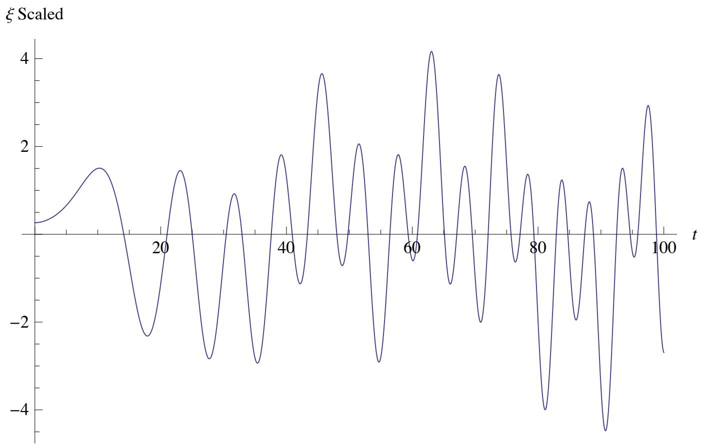
```
Riemannxi[t_] := -(t^2 + 1 / 4) Gamma[I t / 2 + 1 / 4] Zeta[I t + 1 / 2] / (2 Pi^(I t / 2 + 1 / 4))
TraditionalForm[Riemannxi[t]]
Plot[Re[Riemannxi[t]], {t, 0, 100},
 AxesOrigin → {0, 0}, PlotRange → All, AxesLabel → {t, ξ}]
```

$$\frac{1}{2}\pi^{-\frac{1}{4}-\frac{i t}{2}}\left(-t^2-\frac{1}{4}\right)\zeta\left(i t+\frac{1}{2}\right)\Gamma\left(\frac{i t}{2}+\frac{1}{4}\right)$$

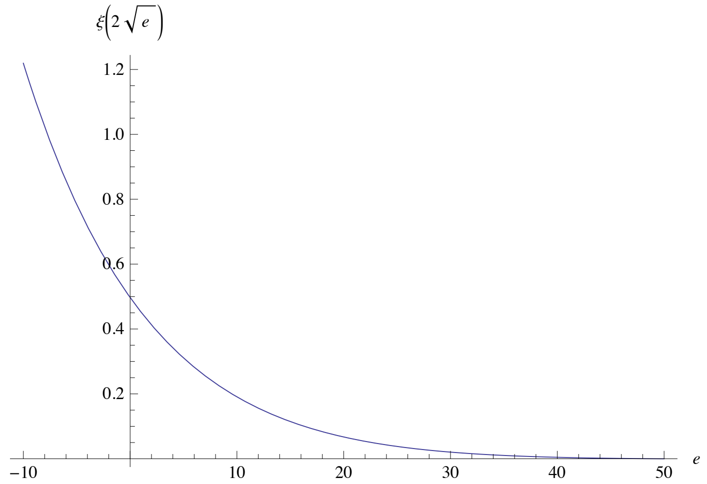



```
ScalingR[t_] := 2^{5 / 4} Cosh[Pi t / 4] / (Pi^{1 / 4} (t^2 + 4)^{7 / 8})
TraditionalForm[ScalingR[t]]
Plot[ScalingR[t] Re[Riemannxi[t]],
 {t, 0, 100}, AxesLabel → {t, Scaled ξ}, PlotRange → All]
```

$$\left\{\frac{2\sqrt[4]{\frac{2}{\pi}}\cosh\left(\frac{\pi t}{4}\right)}{(t^2+4)^{7/8}}\right\}$$



```
Plot[Re[Riemannxi[2 Sqrt[e]]], {e, -10, 50}, AxesLabel → {e, ξ[2 Sqrt[e]]}]
```



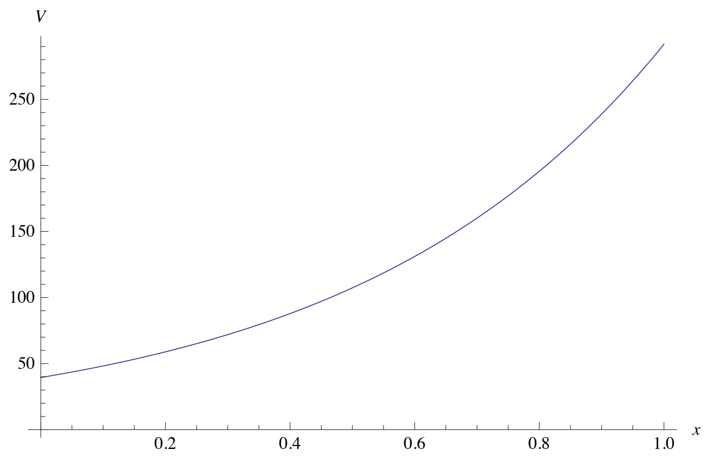

```
Density[e_] := Log[Sqrt[e] / Pi] / Sqrt[e]
TraditionalForm[Density[e]]
```

$$\frac{\log\left(\frac{\sqrt{e}}{\pi}\right)}{\sqrt{e}}$$

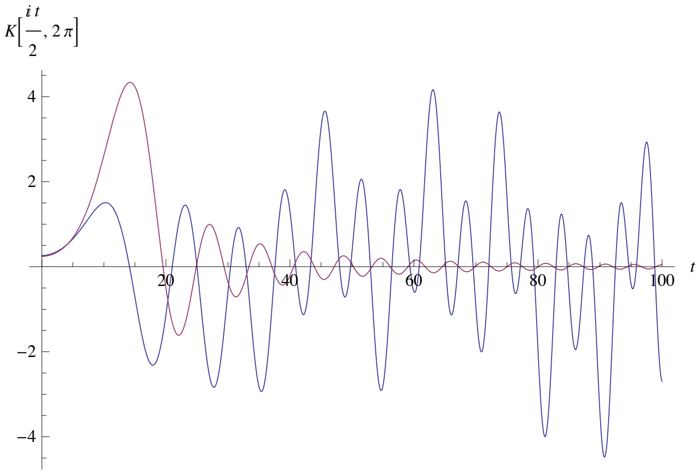
```
heldint =
  HoldForm[Integrate[Density[e] / Pi / Sqrt[v - e], {e, 0, v}, Assumptions → {v > 0}]];
int = ReleaseHold[heldint];
TraditionalForm[Row[{heldint, " = ", int}]]
```

$$\text{Integrate}\left[\frac{\text{Density}(e)}{\pi\sqrt{v-e}}, \{e, 0, v\}, \text{Assumptions} \to \{v > 0\}\right] = \frac{1}{2}\log\left(\frac{v}{4\pi^2}\right)$$

```
Plot[4 Pi^2 Exp[2 x], {x, 0, 1}, AxesLabel -> {x, V}, AxesOrigin → {0, 0}]
```



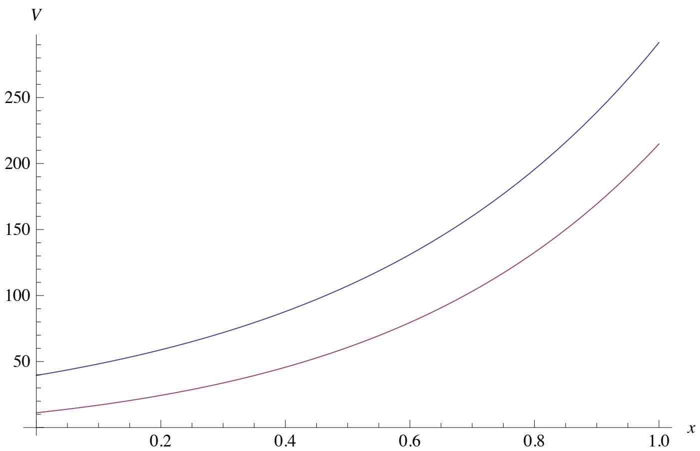
```
Plot[{ScalingR[t] Re[Riemannxi[t]], ScalingR[t] 500 BesselK[I t / 2, 2 Pi]},
 {t, 0, 100}, PlotRange → All, AxesLabel → {t, K[I t / 2, 2 Pi]}]
```



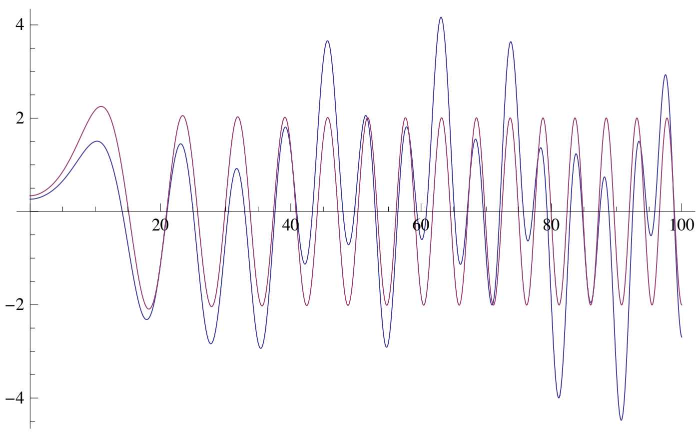

```
TraditionalForm[
 Row[{"Riemannxi[-I tau]  ~  ", (tau / (2 Pi E)) ^ {tau / 2} tau ^ {7 / 4} (Pi / 2) ^ {1 / 4}}]]
```

$$\text{Riemannxi[−I tau]} \sim \left\{2^{-\frac{\text{tau}}{2}-\frac{1}{4}}\, e^{-\text{tau}/2}\, \pi^{\frac{1}{4}-\frac{\text{tau}}{2}}\, \text{tau}^{\frac{\text{tau}}{2}+\frac{7}{4}}\right\}$$

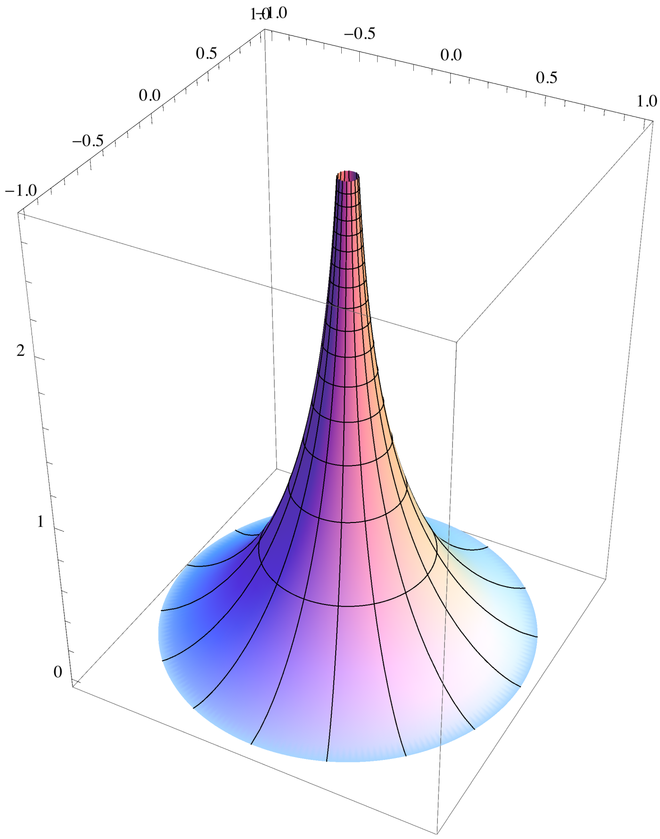
```
Plot[{4 Pi^2 Exp[2 x], 4 Pi^2 Exp[2 x] - 9 Pi Exp[x]},
 {x, 0, 1}, AxesOrigin → {0, 0}, AxesLabel → {x, V}]
```




```
Plot[{ScalingR[t] Re[Riemannxi[t]],
  ScalingR[t] 2 Pi^{-1 / 4} WhittakerW[9 / 4, I t / 2, 4 Pi]}, {t, 0, 100}]
```



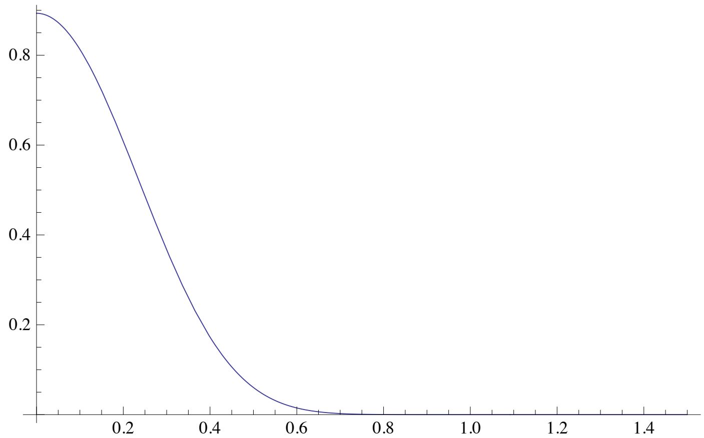
```
ParametricPlot3D[{Sin[2 Pi x] / y, Cos[2 Pi x] / y, Log[y + Sqrt[y^2 - 1]] - Sqrt[1 - y^{-2}]},
 {y, 1, 20}, {x, 0, 1}, PlotRange → All, PlotPoints → 100]
```



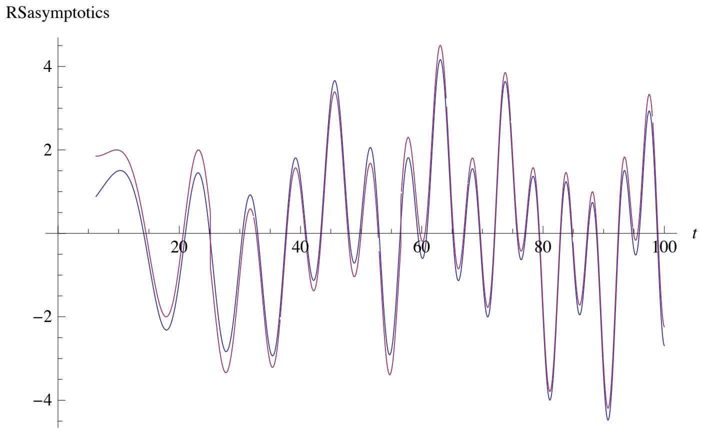

```
(* Magnetic Laplacian Δ_B = -y^2 (d_x^2 + d_y^2) + 2iBy d_x + B^2 *)
(* ϕ(y)Exp[-2πimx] eigenfunction for eigenvalue (t^2+1)/4 + B^2 *)
(* iff (-u^2 d_u^2 + (u^2-1-t^2)/4 \pm Bu)ϕ = 0 with u = 4π|m|y *)
(* Solution decaying as y ->Infinity is WhittakerW[\pm B,it/2,4π|m|y] *)
(* So general solution evaluated at x=0,y=1, is:
 Sum[a[m]WhittakerW[9/4,it/2,4πm]+a[-m]WhittakerW[-9/4,it/2,4πm],{m,1,Infinity}] *)
(* How to choose the coefficients a? *)

PolyaPhi[v_, NN_] :=
 Sum[(4 Pi^2 n^4 Exp[9 v / 2] - 6 Pi n^2 Exp[5 v / 2]) Exp[-Pi n^2 Exp[2 v]],
  {n, 1, NN}] (* even! *)
TraditionalForm[PolyaPhi[v, Infinity]]
Plot[PolyaPhi[v, 8], {v, 0, 1.5}]
```

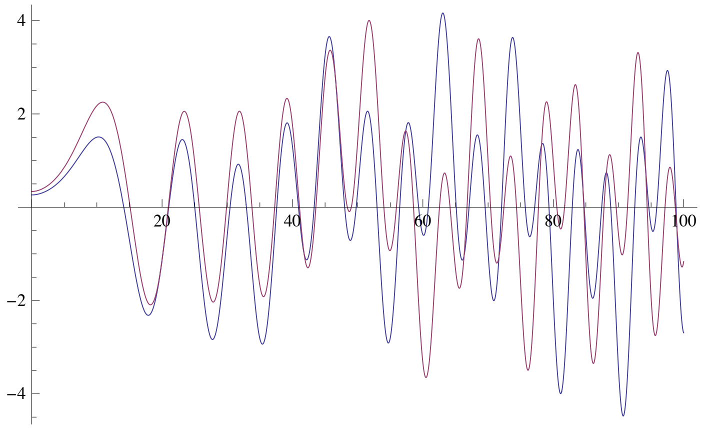
$$\sum_{n=1}^{\infty} e^{\pi n^2 (-e^{2v})} \left(4 \pi^2 n^4 e^{9v/2} - 6 \pi n^2 e^{5v/2}\right)$$



```
RSasymptotics[t_] :=
 -2 Sum[n^{-1/2} Cos[Arg[Gamma[1/4 + I t/2]] - t Log[Pi]/2 - t Log[n]],
   {n, 1, Sqrt[t/(2 Pi)]}]
Plot[{ScalingR[t] Re[Riemannxi[t]], RSasymptotics[t]}, {t, 2 Pi, 100},
 AxesOrigin → {0, 0}, AxesLabel → {t, RSasymptotics}, PlotPoints → 300]
```




```
Plot[{ScalingR[t] Re[Riemannxi[t]], ScalingR[t] 2 Pi^{-1/4}
   Sum[n^{-3/2} Re[WhittakerW[9/4, I t/2, 4 Pi n^2]], {n, 3}]}, {t, 0, 100}]
```



```
p = ParametricPlot3D[{Sin[2 Pi x] / y, Cos[2 Pi x] / y,
     Log[y + Sqrt[y^2 - 1]] - Sqrt[1 - y^{-2}]}, {y, Pi Sqrt[2.5], 20},
    {x, 0, 1}, PlotRange → All, PlotPoints → 100, BoxRatios → {1, 1, 1}];
c1 = ParametricPlot3D[{0, 1 / y, Log[y + Sqrt[y^2 - 1]] - Sqrt[1 - y^{-2}]},
    {y, 2 Pi, 20}, PlotStyle → Thick];
c2 = ParametricPlot3D[{0, -1 / y, Log[y + Sqrt[y^2 - 1]] - Sqrt[1 - y^{-2}]},
    {y, Pi Sqrt[3], 20}, PlotStyle → Thick];
c3 = ParametricPlot3D[{Sin[2 Pi Cos[t]] / (2 Pi Sin[t]), Cos[2 Pi Cos[t]] / (2 Pi Sin[t]),
     Log[2 Pi Sin[t] + Sqrt[(2 Pi Sin[t])^2 - 1]] - Sqrt[1 - (2 Pi Sin[t])^{-2}]},
    {t, Pi / 3, 2 Pi / 3}, PlotStyle → Thick];
Show[
 {p,
  c1,
  c2,
  c3}]
```