MA3H2 Markov Processes and Percolation theory Example Sheet 2

2010, term 2
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Students should hand in solutions by 1pm Monday of week 5 to the maths pigeonloft.

Exercise 5: Let $S_n = (X_n, Y_n)_{n\in\mathbb{N}_0}$ be a simple symmetric random walk in $\mathbb{Z}^2$, starting from $(0,0)$, and set $T = \inf\{n \geq 0 \colon \max\{|X_n|, |Y_n|\} = 2\}$. Determine the quantities $\mathbb{E}(T)$ and $\mathbb{P}(X_T = 2 \text{ and } Y_T = 0)$.

Exercise 6: Markov chains

(a) Show that any sequence of independent random variables taking values in the countable set I is a Markov chain. Under what condition is this chain homogeneous (in time)?

(b) A die is rolled repeatedly. Which of the following are Markov chains? For those that are, supply the transition matrix.

  (i) The number N_n of sixes in n rolls.

  (ii) At time r, the time C_r since the most recent six.

Exercise 7: Exponential distribution

(a) Let X and Y be independent random variables having the exponential distribution with parameters λ and μ respectively. Find the probability density function of the sum $X + Y$.

(b) Let X have an exponential distribution. Show that $\mathbb{P}(X > s + x | X > s) = \mathbb{P}(X > x)$, for $x, s \geq 0$.

Exercise 8: Poisson process

(a) *Superposition.* Flies and wasps land on your dinner plate in the manner of independent Poisson processes with respective intensities λ and μ. Show that the arrivals of flying objects form a Poisson process with intensity $\lambda + \mu$.

(b) *Thinning.* Insects land in the soup in the manner of a Poisson process with intensity λ, and each insect is green with probability $p \in [0,1]$, independently of the colours of all other insects. Show that the arrivals of green insects form a Poisson process with intensity λp.