MA3H2 Markov Processes and Percolation theory
Example Sheet 6 - Solutions

2010, term 2
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Please find below the revised solution for question 3 (b) (i).

Question 3: (b) Consider the Q-matrix on $I = \{1, 2, 3, 4\}$ given by

$$Q = \begin{pmatrix} -1 & 1/2 & 1/2 & 0 \\ 1/4 & -1/2 & 0 & 1/4 \\ 1/6 & 0 & -1/3 & 1/6 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

(i) Calculate the probability of hitting 3 when starting from 1: We write $h_i := h_i^{\{3\}} = \mathbb{P}_i(D^{\{3\}} < \infty)$. Clearly, $h_3 = 1$ and $h_4 = 0$. Moreover, we get

$$\begin{aligned} -h_1 + \frac{1}{2}h_2 + \frac{1}{2} &= 0 \\ \frac{1}{4}h_1 - \frac{1}{2}h_2 &= 0. \end{aligned}$$

Henceforth we get $h_1 = \frac{2}{3}$.