MA3D9 Example Sheet 3

Without otherwise mentioned, all curves are smooth and regular.

1. Complete all the exercises mentioned in the class and in the lecture notes.

2. Verify the set $\{(x, y, z) \in \mathbb{R}^3; x^2 + y^2 - z^2 = 0\}$ is not a regular surface.

3. Show the ellipsoid
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$
is a regular surface.

4. Compute the first fundamental forms of the following surfaces
   a. $\sigma(u, v) = (au \cos v, bu \sin v, u^2)$; elliptic paraboloid.
   b. $\sigma(u, v) = (au \cosh v, bu \sinh v, u^2)$; hyperbolic parablooid.

5. Calculate the first fundamental form, second fundamental form and area of the torus:
$$\sigma(u, v) = ((a + b \cos u) \cos v, (a + b \cos u) \sin v, b \sin u); 0 < b < a, 0 \leq u, v \leq 2\pi$$

6. Calculate the first fundamental form, second fundamental form and area of explicitly given surface $z = f(x, y)$.

7. Let $\phi : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $\phi(x, y) = (u(x, y), v(x, y))$, where u and v and differentiable functions satisfy the Cauchy-Riemann equations
$$u_x = v_y, u_y = -v_x.$$
Show that ϕ is a conformal map from $\mathbb{R}^2 - Q$ into $\mathbb{R}^2$, where $Q = \{(x, y) \in \mathbb{R}^2 | u_x^2 + u_y^2 = 0\}$.