Complex evolutionary systems in behavioural finance I

Expectations feedback systems

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Dynamics in Games and Economics

The Marshallian cross

Demand $d(p)$ decreases
Supply $s(p)$ increases
At equilibrium $p = \bar{p}$ and

$$\bar{q} = d(\bar{p}) = s(\bar{p})$$

Walrasian auctioneer: “As if” dynamics

$$\dot{p} = d(p) - s(p)$$

Only when $\dot{p} = 0$ the good is traded

Surplus I

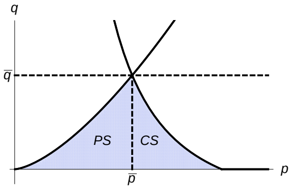
Consumer's surplus

$$CS = \int_0^{\bar{q}} \left(d^{-1}(q) - \bar{p}\right) \mathrm{d}q = \int_{\bar{p}}^{p_0} d(p)\mathrm{d}p$$

Producer's surplus

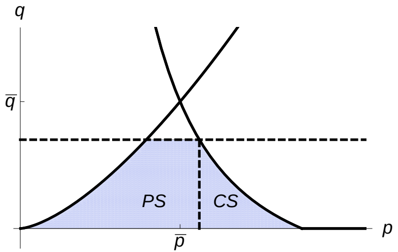
$$PS = \int_0^{\bar{q}} \left(\bar{p} - s^{-1}(q)\right) \mathrm{d}q = \int_0^{\bar{p}} s(p)\mathrm{d}p$$

Equilibrium $\bar{p}$ maximises *total surplus (= welfare)*

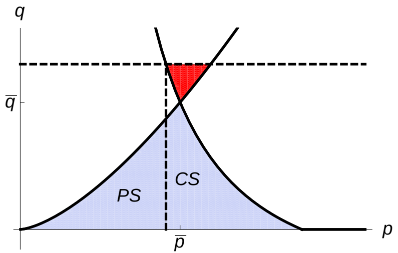


Surplus II

Surplus is lower if $p > \bar{p}$



or if $p < \bar{p}$



The hog-cycle

Production usually costs time

Production decisions have to taken in time period $t-1$, based on the *expected price* p_t^e, instead of the yet unknown market price p_t

Supply in period t is independent of p_t

Expected price is based on known data

$$p_t^e = f(p_{t-1}, p_{t-2}, \cdots)$$

Market equilibrium

$$d(p_t) = s(p_t^e)$$

Evolution

$$p_t = d^{-1}\left(s(p_t^e)\right) = F(p_t^e)$$

Expectations feedback system

The system

$$\begin{aligned} p_t^e &= f(p_{t-1}, p_{t-2}, \cdots) && \text{perceived dynamics} \\ p_t &= F(p_t^e) && \text{actual dynamics} \end{aligned}$$

forms an *expectations feedback system*

Very common situation in economics

Central question: how are expectations formed?

Naive expectations: the cobweb

A very simple expectation rule is "nothing changes"

$$p_t^e = p_{t-1}$$

The expectations feedback system then reads as

$$p_t = F(p_{t-1}) = d^{-1}\left(s\left(p_{t-1}\right)\right)$$

Since

$$F'(p) = \frac{s'(p)}{d'(F(p))}$$

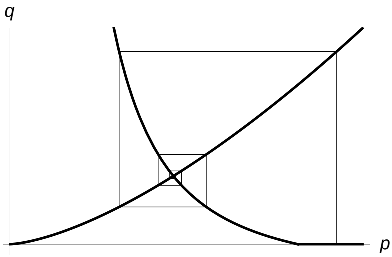
there is at most one fixed point $p = \bar{p}$, the *fundamental price*

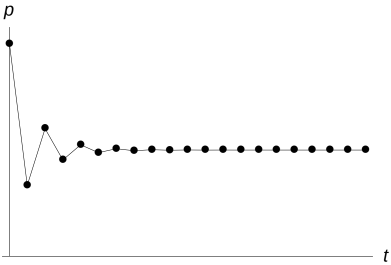
This point is asymptotically stable if $|F'(\bar{p})| < 1$, that is, if

$$|s'(\bar{p})| < |d'(\bar{p})|$$

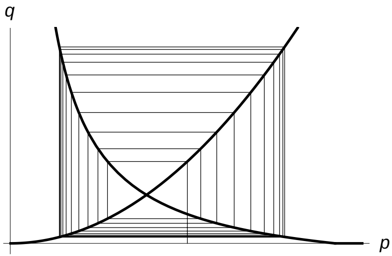
Dynamics under naive expectations

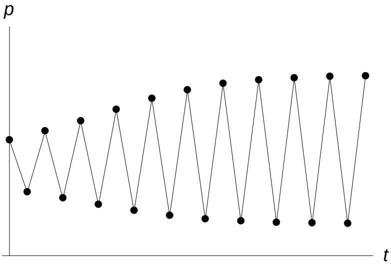
Stable cobweb dynamics





Unstable cobweb dynamics





Linear expectation rules I

A more general family of expectation rules is

$$p_t^e = \alpha + \beta(p_{t-1} - \alpha)$$

Example: subfamily that forecasts correctly if $p = \bar{p}$:

$$p_t^e = \bar{p} + \beta(p_{t-1} - \bar{p})$$

Note: $\beta = 1$ is the naive case

Expectations feedback system

$$p_t = F\left(\bar{p} + \beta(p_{t-1} - \bar{p})\right)$$

Linear expectation rules II

Introduce *price deviations* from $\bar{p}$

$$x_t = p_t - \bar{p}$$

Expectation rule

$$x_t^e = \beta x_{t-1}$$

Expectations feedback system

$$\begin{aligned} x_t &= F(\bar{p} + \beta x_{t-1}) - \bar{p} \\ &= \tilde{F}(\beta x_{t-1}) \end{aligned}$$

with $\tilde{F}(x) = F(\bar{p} + x) - \bar{p}$ and

$$\tilde{F}(0) = 0$$

Linear expectation rules III: example

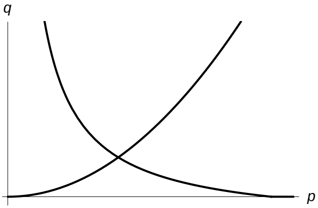
Price evolution is determined by the parametrised family of dynamical systems

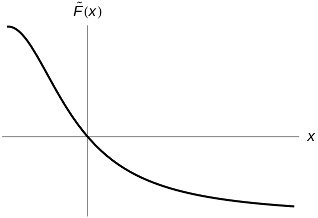
$$x_t = \tilde{F}(\beta x_{t-1})$$

depending on the parameter β

Example:

$$s(p) = 3p^2,$$
$$d(p) = \max\left\{\frac{0.3}{p} - 0.325,\ 0\right\}$$

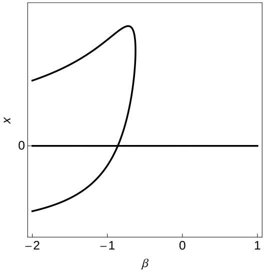




Linear expectation rules IV: stability analysis

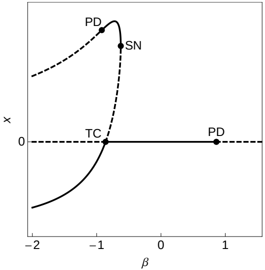
Fixed points are solutions of

$$x = \tilde{F}(\beta x)$$



If $\quad |\beta \tilde{F}'(\beta x)| < 1 \quad (> 1)$

the fixed point is asymptotically stable (unstable)



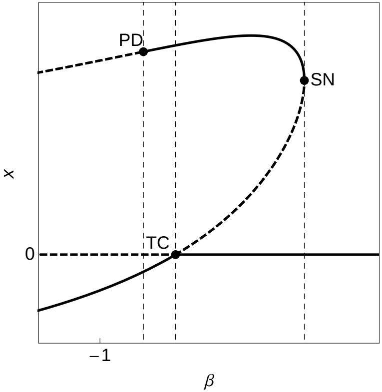
If $\quad |\beta F'(\beta x)| = 1$

usually a *bifurcation* occurs

Linear expectation rules V: bifurcations

Types of one-dimensional bifurcations

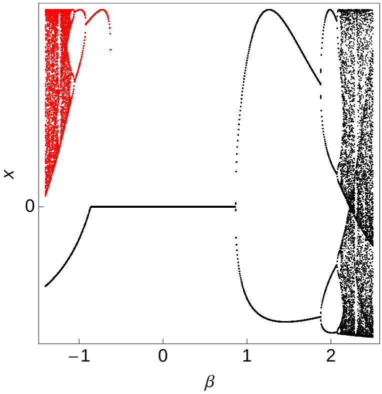
- Saddle-node (SN): two fixed point are created, one stable, one unstable
- Period-doubling (PD): a fixed point changes its stability, period-2 cycle is created
- Transcritical (TC): two fixed points exchange stability



Linear expectation rules VI: dynamics

Types of dynamics

- $\beta > \beta_{\text{PD}}$: Period-doubling cascade, resulting in chaotic dynamics
- $\beta_{\text{SN}} < \beta < \beta_{\text{PD}}$: Globally attracting fundamental fixed point
- $\beta_{\text{TC}} < \beta < \beta_{\text{SN}}$: Fundamental coexisting with non-fundamental fixed point
- $\beta_{\text{PD}} < \beta < \beta_{\text{TC}}$: Two non-fundamental fixed points coexisting
- $\beta < \beta_{\text{PD}}$: Non-fundamental fixed point coexisting with period-doubling cascade and chaotic dynamics



Rational expectations I

Most fixed expectation rules are criticised

- Systematic forecasting errors
- Welfare losses ("Market knows best")

Radical solution: *rational expectations*

$$p_t^e = p_t$$

The expectations feedback system for rational expectations reads as

$$p_t = F(p_t)$$

which has as its unique solution

$$p_t = \bar{p}$$

Rational expectations II

Rationale (not a proof!)

- By *learning* the system, agents minimise their forecasting errors down to 0
- Other beliefs should do worse and should be outcompeted

For

- Maximises welfare
- No forecasting errors

Against

- Is the rationale correct?
- Adequate description of reality?

Learning I

We have now two expectation formation rules

- Linear
  - $x_t^e = \beta x_{t-1}$
  - Easy to implement
  - Too many parameters
- Rational
  - $x_t^e = x_t$
  - Good properties
  - Hard to implement

One way to reconcile them is to assume that agents learn β using econometric techniques (Evans & Honkapohja, 2001)

Learning II

Given the time series $x_0, x_1, \cdots, x_t$, sellers estimate the model

$$x_t = \beta x_{t-1}$$

using least squares, by minimising

$$\beta \mapsto \sum_{s=1}^{t} (x_s - \beta x_{s-1})^2$$

The minimiser is

$$\hat{\beta}_t = \frac{a_t}{b_t} = \frac{\frac{1}{t}\sum_{s=1}^{t} x_{s-1} x_s}{\frac{1}{t}\sum_{s=1}^{t} x_{s-1}^2}$$

Learning III

At each time step, sellers use the best available model:

$$x_t = \tilde{F}\left(\frac{a_{t-1}}{b_{t-1}} x_{t-1}\right)$$

The quantities a_t and b_t can be computed recursively

$$\begin{aligned} a_t &= \frac{1}{t}\sum_{s=1}^{t} x_{s-1}x_s \\ &= \frac{t-1}{t}\frac{1}{t-1}\sum_{s=1}^{t-1} x_{s-1}x_s + \frac{1}{t}x_{t-1}x_t \\ &= \left(1-\frac{1}{t}\right)a_{t-1} + \frac{1}{t}x_{t-1}\tilde{F}\left(\frac{a_{t-1}}{b_{t-1}}x_{t-1}\right) \end{aligned}$$

Likewise

$$b_t = \left(1-\frac{1}{t}\right)b_{t-1} + \frac{1}{t}x_{t-1}^2$$

Learning IV

Learning system

$$x_t = \tilde{F}\left(\frac{a_{t-1}}{b_{t-1}} x_{t-1}\right)$$
$$a_t = \left(1 - \frac{1}{t}\right) a_{t-1} + \frac{1}{t} x_{t-1} \tilde{F}\left(\frac{a_{t-1}}{b_{t-1}} x_{t-1}\right)$$
$$b_t = \left(1 - \frac{1}{t}\right) b_{t-1} + \frac{1}{t} x_{t-1}^2$$

Again an expectational feedback system:

- Perceived dynamics

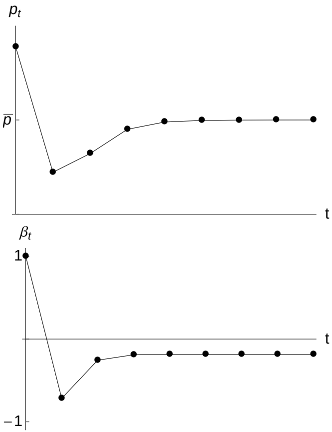
$$x_t^e = \beta_t x_{t-1}$$

- Actual dynamics

$$x_t = \tilde{F}(x_t^e)$$

Learning V

Results



It seems that learning solves the expectation problem

However

- Assumes homogeneity of agents
- Can converge to non-rational equilibria
- Can fail to converge
- What about stock markets?

Summary

- Economic decisions are determined by expectations
- Expectations feed back into the dynamics of the system
- Key problem: how are expectations formed?