Complex evolutionary systems in behavioural finance

Heterogeneous agent models

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Dynamics in Games and Economics

Recapitulation of first lecture

- Hog-cycle model

$$d(p_t) = s(p_t^e)$$

- Economic decisions are determined by expectations
- Expectations feed back into the dynamics of the system
- Key problem: how are expectations formed?
  - Rational expectations?
  - Learning?
  - If not rational, what else — wilderness of bounded rationality

Cobweb experiment I

Hommes, Sonnemans, Tuinstra & van der Velden (2007)

Six sellers $i = 1, \cdots, 6$ making price predictions

Reward for correct predictions

$$\Pi_{i,t} = \max\left\{1300 - 260(p_t - p^e_{i,t})^2, 0\right\}$$

Individual supply

$$s(p^e_{i,t}) = 1 + \tanh \lambda(p^e_{i,t} - 6)$$

Linear demand (demand shocks $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$)

$$d(p_t) = a - bp_t + \varepsilon_t$$

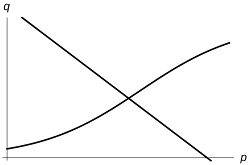
Equilibrium

$$d(p_t) = \sum_{i=1}^{6} s(p^e_{i,t})$$

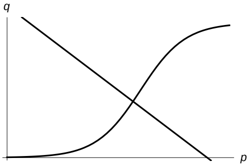
Cobweb experiment II

Treatments

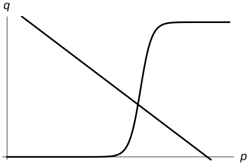
	λ	$s'(\bar{p})/d'(\bar{p})$
Stable	0.22	-0.87
Unstable	0.50	-1.96
Strongly unstable	2.00	-7.75



$\lambda = 0.22$



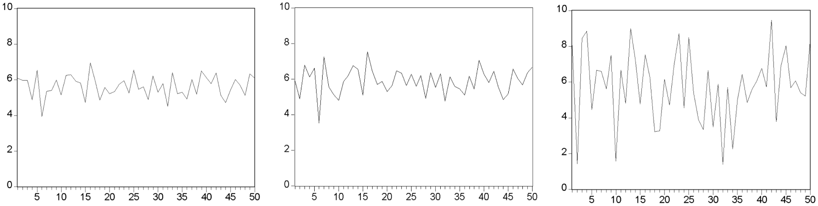
$\lambda = 0.50$



$\lambda = 2.00$

Cobweb experiment III

Experimental results (one group out of six)



stable: $\lambda = 0.22$ unstable: $\lambda = 0.50$ strongly unstable: $\lambda = 2.00$

Both in the unstable and in the strongly unstable treatment, the volatility is significantly higher than expected under full rationality

Evolutionary competition of beliefs

Brock & Hommes (1997, 1998)

Central idea: let different expectation rules compete

- Successful expectation rules get more followers
- If agents try hard, small differences in success rates matter
- Take information costs into account

Asset pricing model

Wealth dynamics of an investor

$$W_{t+1} = R(W_t - p_t z_{ht}) + (p_{t+1} + y_{t+1}) z_{ht}$$

Variables

- W_t: wealth at time t
- $R = 1 + r$: gross rate of risk-free return
- p_t: price of risky asset at time t
- z_t: number of shares
- y_{t+1}: dividends at time $t + 1$

Assets II

Utility of investor (of type h)

$$\mathscr{U}_{ht} = \mathbf{E}_{ht} W_{t+1} - \frac{a}{2} \mathbf{Var}_{ht} W_{t+1}$$

where

- $\mathbf{E}_{ht} X_{t+1} = \mathbf{E}_h(X_{t+1}|\mathscr{F}_t)$ expected value of X_{t+1} of a type-h investor, based on information $\mathscr{F}_t$ up to and including time t
- $\mathbf{Var}_{ht} X_{t+1} = \mathbf{Var}_h(X_{t+1}|\mathscr{F}_t)$ variance of X_{t+1} that is expected by a type-h investor

Assumptions:

$$\mathbf{E}_{ht} p_{t+1} = p^e_{ht+1}, \quad \mathbf{E}_{ht} y_{t+1} = \bar{y},$$

$$\mathbf{Var}_{ht} p_{t+1} = 0, \quad \mathbf{Var}_{ht} y_{t+1} = \sigma^2$$

Assets III

Utility can be written as

$$\mathscr{U}_{ht} = RW_t + (p^e_{ht+1} + \bar{y} - p_t)z_{ht} - \frac{a\sigma^2}{2} z^2_{ht}$$

Maximising U_{ht} with respect to z_{ht} yields

$$z_{ht} = \frac{p^e_{ht+1} + \bar{y} - Rp_t}{a\sigma^2}$$

The function $z_{ht} = z_{ht}(p_t)$

- is the demand function of a type-*h* investor;
- depends on expectation of *future* price;
- has the same sign as the expected excess return

Heterogeneous expectations I

H competing boundedly rational expectation rules

A fraction n_{ht} of the sellers believes p^e_{ht} to be correct

Aggregate demand $d(p_t)$ at time t

$$d(p_t) = \sum_{h=1}^{H} n_{ht} z_{ht}(p_t) = \sum_{h=1}^{H} n_{ht} \frac{p^e_{ht+1} + \bar{y} - Rp_t}{a\sigma^2}$$

Fixed aggregate supply $s(p_t) = z^s$

Market equilibrium

$$Rp_t = \bar{y} + \sum_{h=1}^{H} n_{ht} p^e_{ht+1} - a\sigma^2 z^s$$

Fundamental price

What if all investors are fully rational?

Market equilibrium

$$Rp_t = \bar{y} + p_{t+1} - a\sigma^2 z^s$$

Only one bounded solution

$$\bar{p} = \frac{\bar{y} - a\sigma^2 z^s}{R-1} = \frac{\bar{y} - a\sigma^2 z^s}{r}$$

Note: $\bar{p}$ is the *net present value* of the dividend stream $\{y_t\}$ defining the risky asset

Risk premium $=$ expected return $-$ risk free rate

$$RP = \frac{\bar{y}}{\bar{p}} - r = \frac{a\sigma^2 z^s}{\bar{y} - a\sigma^2 z^s} r$$

Henceforth: $\boxed{z^s = 0}$

Heterogeneous expectations II

Price deviations

$$x_t = p_t - \bar{p}$$

Market equilibrium in price deviations

$$Rx_t = \sum_{h=1}^{H} n_{ht} x^e_{ht+1}$$

Expectations of the form

$$x^e_{ht+1} = f_{ht} = f_h(x_{t-1}, \cdots, x_{t-L})$$

To close the model, the evolution of the fractions n_{ht} has to be specified

Heterogeneous expectations III

Agents are assumed to be pragmatic about beliefs, picking those that perform best according to some *fitness criterion* U_{ht-1}, e.g.

- last realised profit
- risk-adjusted profit
- (average) prediction error

Last realised profit

$$\begin{aligned} U_{ht-1} &= (p_{t-1} + y_{t-1} - Rp_{t-2})z_{ht-2} - C_h \\ &= (x_{t-1} - Rx_{t-2} + \delta_{t-1})\frac{f_{ht-2} - Rx_{t-2}}{a\sigma^2} - C_h \end{aligned}$$

where

- C_h: information cost for obtaining predictor h
- $\delta_t = y_t - \bar{y}$

Heterogeneous expectations III

Individual agents obtain a noisy signal $\tilde{U}_{ht-1}$ about the fitnesses U_{ht-1} with

$$\tilde{U}_{ht-1} = U_{ht-1} + \varepsilon_{iht-1};$$

the ε_{iht-1} are iid and $\mathbf{E}\varepsilon_{iht-1} = 0$

Extreme cases

- If $\mathbf{Var}\varepsilon_{iht-1} = 0$, agents choose h such that U_{ht-1} is maximal
- If $\mathbf{Var}\varepsilon_{iht-1} = \infty$, agents choose a predictor at random

For doubly exponentially iid ε_{iht-1}, we have

$$n_{ht} = \frac{e^{\beta U_{ht-1}}}{\sum_{h=1}^{H} e^{\beta U_{ht-1}}}$$

Intensity of choice β is inversely related to $\mathbf{Var}(\varepsilon_{iht})$

Heterogeneous expecations IV

Evolution law

$$x_t = \frac{1}{R} \sum_{h=1}^{H} n_{ht} f_h(x_{t-1}, \cdots)$$

where

$$n_{ht} = \frac{e^{\beta U_{ht-1}}}{Z_{t-1}}$$

$$U_{ht} = \frac{1}{a\sigma^2}(x_{t-1} - Rx_{t-2} + \delta_{t-1})(f_{ht-2} - Rx_{t-2}) - C_h$$

Fundamentalists against trend followers I

Costly fundamentalists (type 1) vs cheap trend followers (type 2)

$$f_{1t} = 0 \qquad C_1 = 1$$
$$f_{2t} = g x_{t-1} \qquad C_2 = 0$$

Evolution (with small supply shocks ε_t)

$$x_t = \frac{g}{R} n_{2t} x_{t-1} + \varepsilon_t$$

with ($a\sigma^2 = 1$)

$$U_{ht-1} = (x_{t-1} - R x_{t-2})(f_{ht-2} - R x_{t-2}) - C_h$$

and

$$n_{2t} = \frac{e^{\beta U_{2t-1}}}{e^{\beta U_{1t-1}} + e^{\beta U_{2t-1}}}$$

Fundamentalists against trend followers II

Only fitness differences are relevant

$$n_{2t} = \frac{e^{\beta U_{2t-1}}}{e^{\beta U_{1t-1}} + e^{\beta U_{2t-1}}} = \frac{1}{e^{\beta(U_{1t-1} - U_{2t-1})} + 1}$$

Compute

$$\beta U_{1t-1} - \beta U_{2t-1} = -\beta g x_{t-3}(x_{t-1} - R x_{t-2}) - \beta$$

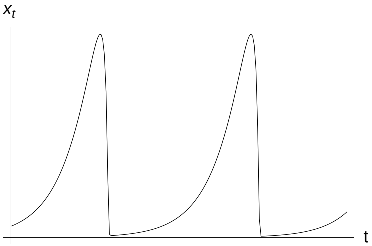
Evolution law

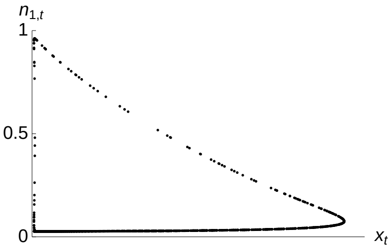
$$x_t = \frac{g}{R} \frac{x_{t-1}}{e^{-\beta g x_{t-3}(x_{t-1} - R x_{t-2}) - \beta} + 1} + \varepsilon_t$$

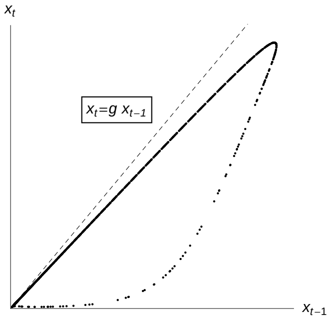
Note: symmetry $x_t \mapsto -x_t$

Fundamentalists against trend followers III

Var$\varepsilon_t = 0$: deterministic skeleton ($\beta = 3.6$, $g = 1.2$, $R = 1.1$)

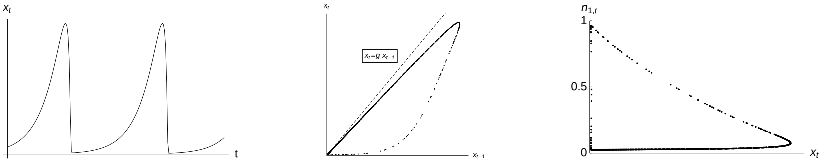




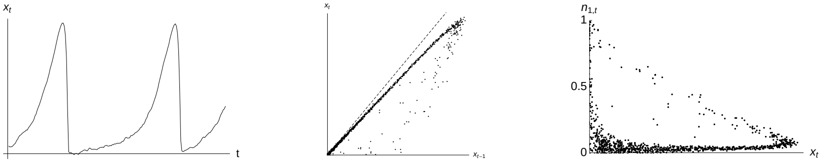


Fundamentalists against trend followers IV

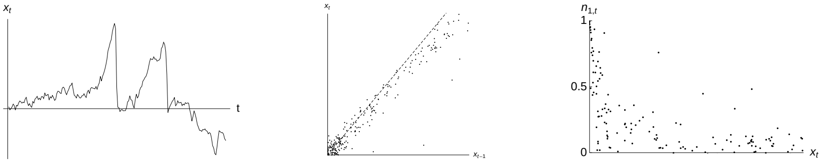
Var$\varepsilon_t = 0$: deterministic skeleton ($\beta = 3.6$, $g = 1.2$, $R = 1.1$)



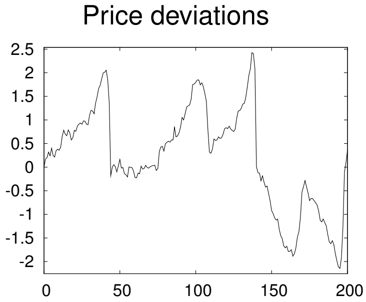
Var$\varepsilon_t = (0.01)^2$

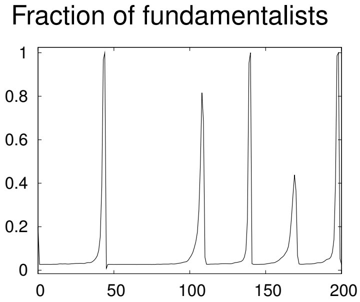


Var$\varepsilon_t = (0.1)^2$



Fundamentalists against trend followers IV





- Investors switch between expectation rules
- Long periods of building up overconfidence
- Sharp crashes

Expectations feedback system

Perceived dynamics (several types)

$$x^e_{ht+1} = f_h(x_{t-1}, \cdots)$$

Actual dynamics

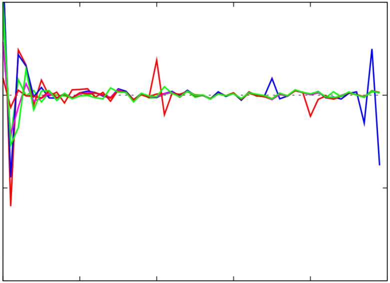
$$x_t = \sum_{h=1}^{H} n_{ht} x^e_{ht+1}$$

- Hog-cycle
  - negative feedback
  - tends to stabilise
- Stock market
  - positive feedback
  - tends to destabilise

Experiment: comparing negative and positive feedback

Negative feedback

$$p_t = \frac{20}{21}(123 - \langle p_t^e \rangle) + \varepsilon_t$$

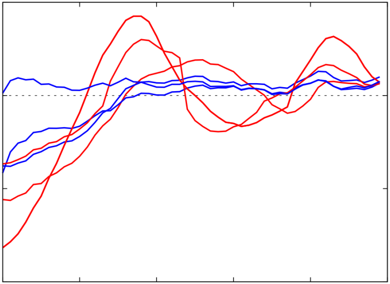


where $\langle p_t^e \rangle = (1/6)\sum_{i=1}^{6} p_{it}^e$

Group averages of six groups

Positive feedback

$$p_t = \frac{20}{21}(3 + \langle p_t^e \rangle) + \varepsilon_t$$



Summary

- Goods market (Cobweb)
  - expectations about current prices
  - negative feedback
  - tends to stabilisation
- Stock market
  - expectations about future prices
  - positive feedback
  - long persistent deviations from fundamental
- Rational beliefs cannot drive out boundedly rational beliefs
- Heterogeneous agent model with two types can describe price crashes