Complex evolutionary systems in behavioural finance

Application: financial instruments and market stability

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Overview

Brock, Hommes & Wagener (2009)

Extension of the Brock & Hommes (1998) heterogeneous agents asset pricing model by adding hedging instruments (Arrow securities)

Main Question
Is it true that giving traders more trading options will improve the market outcome?

BH asset pricing model with Arrow securities

Stochastics

In the next period ("tomorrow") one of S possible states of the world occurs with probability α_s

Investment possibilities

- Bonds, totally elastically supplied, price 1, return R
- Risky asset, total supply ζ^0, price p_t^0, return $p_{t+1}^0 + y_{t+1}^s$
- n Arrow securities, total supply 0, price p_t^j, $j = 1, .., n$, return

$$\delta_j^s = \begin{cases} 1 & \text{if} \quad s = j, \\ 0 & \text{otherwise} \end{cases}$$

Profits

Trader of "type h" expects price p^0_{ht+1} of the risky asset; excess return of portfolio $(z^0_t, z^1_t, \cdots, z^n_t)$ is

$$\pi^s_{ht+1} = (p^0_{ht+1} + y^s_{t+1} - Rp^0_t)z^0_t + \sum_j (\delta^s_j - Rp^j_t)z^j_t$$

Demands and prices written as vectors

$$\mathbf{z}_t = (z^0_t, \tilde{\mathbf{z}}_t) = (z^0_t, z^1_t, \cdots, z^n_t)$$
$$\mathbf{p}_t = (p^0_t, \tilde{\mathbf{p}}_t) = (p^0_t, p^1_t, \cdots, p^n_t)$$

Expected excess returns in state s

$$\mathbf{B}^s_{ht} = \begin{pmatrix} p^0_{ht+1} + y^s_{t+1} - Rp^0_t \\ \delta^s - R\tilde{\mathbf{p}}_t \end{pmatrix}, \qquad \mathbf{B}_{ht} = \sum_s \mathbf{B}^s_{ht}\alpha_s$$

Excess return of portfolio $\mathbf{z}_t$ expected by trader type h

$$\pi_{ht+1} = \langle \mathbf{B}_{ht}, \mathbf{z}_t \rangle$$

Demands

Traders maximise expected risk-corrected profits

$$\mathscr{U}_{ht}(\mathbf{z}) = \mathbf{E}_{ht}\, \pi_{ht+1} - \frac{a}{2}\, \mathbf{Var}\, \pi_{ht+1} = \langle \mathbf{B}_{ht}, \mathbf{z} \rangle - \frac{1}{2} \langle \mathbf{z}, V_n \mathbf{z} \rangle,$$

where

- a: coefficient of risk aversion
- V_n: (dividend) risk structure

$$V_n = a\, \mathbf{Cov}(y^s_{t+1}, \boldsymbol{\delta}^s) = a\, \mathbf{Cov}(y^s_{t+1}, \delta^s_1, \cdots, \delta^s_n)$$

Key assumption: risk structure is common knowledge

Demands

$$\mathbf{z}_{ht} = V_n^{-1} \mathbf{B}_{ht}$$

Benchmark: homogeneous market

Single rational trader

- Outside supply of risky assets: ζ^0
- No outside supply of Arrow securities
- Total outside supply of assets $\boldsymbol{\zeta} = (\zeta^0, 0, \cdots, 0)$

Market equilibrium at steady state prices p_*

$$\zeta = V_n^{-1}\mathbf{B}$$

implies

$$\mathbf{B} = \begin{pmatrix} (1-R)p_*^0 + \mathbf{E}y_{t+1}^s \\ \alpha - R\,\tilde{\mathbf{p}}_* \end{pmatrix} = V_n\boldsymbol{\zeta},$$

determines homogeneous benchmark prices $\mathbf{p}_* = (p_*^0, \mathbf{p}_*)$

Heterogeneous agents market equilibrium

- n_{ht}: market fraction of traders of type h
- $\mathbf{x}_t = (x_t^0, \tilde{\mathbf{x}}_t) = \mathbf{p}_t - \mathbf{p}_*$: deviation from benchmark price
- Expectations of type h on the price deviation of the risky asset

$$x_{ht+1}^0 = p_{ht+1}^0 - p_*^0 = f_{ht}(x_{t-1}, \cdots)$$

Demands in deviation from the homogeneous demands ζ

$$\mathbf{z}_{ht}(\mathbf{x}_t) = \zeta + V_n^{-1} \begin{pmatrix} f_{ht} - Rx_t^0 \\ - R\,\tilde{\mathbf{x}}_t \end{pmatrix}$$

Market clearing $\boxed{\zeta = \sum_h n_{ht}\mathbf{z}_{ht}(\mathbf{x}_t)}$ implies

$$x_t^0 = \frac{1}{R}\sum n_{ht} f_{ht}, \quad \tilde{\mathbf{x}}_t = 0.$$

Reinforcement learning

Agents flock to strategies having good fitness scores

- U_{ht-1} is the fitness of type h after trading round $t-1$
- The intensity of choice β is inversely related to noise when observing U_{ht-1}
- Discrete choice model

$$n_{ht} = \sum \frac{e^{\beta U_{ht-1}}}{Z_t}, \qquad \sum_h n_{ht} = 1$$

We take as fitness measure the average realised risk-adjusted profits

$$\begin{aligned} U_{ht+1} &= \langle \mathbf{B}_t, z_{ht} \rangle - \frac{1}{2} \langle \mathbf{z}_{ht}, V_n \mathbf{z}_{ht} \rangle \\ &= -\frac{1}{2} \langle \mathbf{e}_0, V_n^{-1} \mathbf{e}_0 \rangle (x_{t+1}^0 - f_{ht})^2 + C_t \end{aligned}$$

Risk measure

σ_n^2: measure of risk in presence of n Arrow securities

$$\frac{1}{\sigma_n^2} = a\langle \mathbf{e}_0, V_n^{-1}\mathbf{e}_0\rangle$$

Theorem: $\sigma_0^2 > \sigma_1^2 > \cdots > \sigma_{S-2}^2 > \sigma_{S-1}^2 = 0$

$$\mathbf{z}_{ht} = \begin{pmatrix} \zeta^0 \\ 0 \end{pmatrix} + (f_{ht} - Rx_t^0)\begin{pmatrix} \frac{1}{\sigma_n^2} \\ \tilde{w}_n \end{pmatrix}$$

Less (percieved) risk means more demand $\Rightarrow$ leveraging

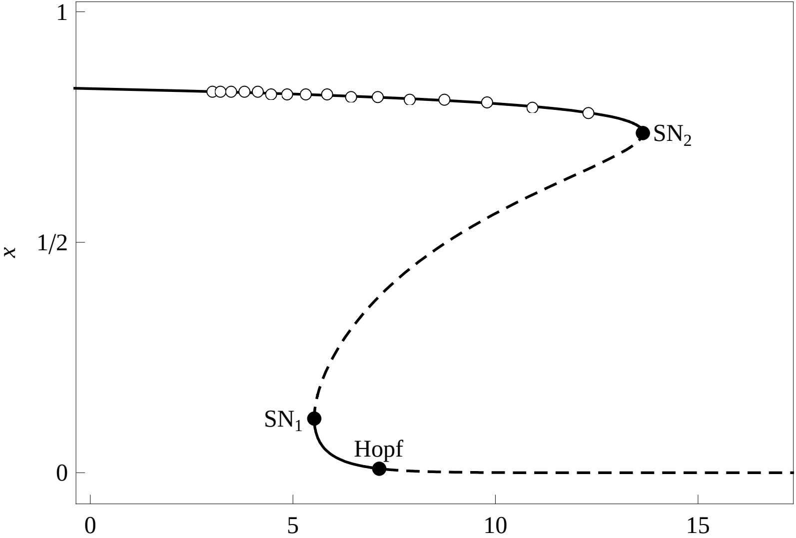
Price dynamics

$$Rx_t^0 = \frac{\sum e^{-\boxed{\frac{\beta}{\sigma_n^2}}(x_{t-1}^0 - f_{ht-2})^2} f_{ht}}{\sum e^{-\boxed{\frac{\beta}{\sigma_n^2}}(x_{t-1}^0 - f_{ht-2})^2}}$$

Example

Two-type model: near-fundamentalists versus trend chasers

$$f_{1t} = 1, \quad f_{2t} = x_{t-1} + g(x_{t-1} - x_{t-2})$$



Welfare

Welfare: population average of realised utility

$$W_t = \sum n_{ht-1} U_{ht}$$

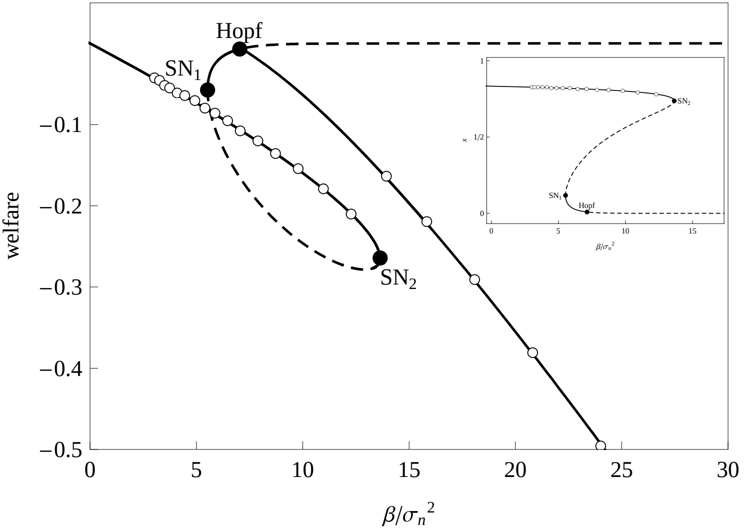
With prediction errors $\varepsilon_{ht} = x_t^0 - f_{ht-1}$ of type h, welfare can be written as

$$W_t = \underbrace{\frac{1}{2} a^2 \sigma_0^2 (\zeta^0)^2}_{\text{risk premium}} + \underbrace{(x_t^0 - R x_{t-1}^0) a \zeta^0}_{\text{irrationality bias}}$$

$$- \underbrace{n_{0,t-1} C}_{\text{perfect foresight costs}} - \underbrace{\frac{1}{2a} \frac{\mathbf{Var}\, \varepsilon_{ht}}{\sigma_n^2}}_{\text{variance of prediction errors}}$$

Example II

Average welfare $\bar{W} = T^{-1} \sum W_t$



Theorem: $\lim_{\sigma_n \to 0} \bar{W} = -\infty$

Adding a perfect foresight type

- Demand of perfect foresight type has to clear the market at the previously predicted price
- First round prediction should be such that local rational bubbles are avoided
- Perfect foresight induces an additional cost C

Market clearing equation:

$$Rx_t = n_{0t}x_{t+1} + \sum_h n_{ht}f_h(x_{t-1}, \cdots)$$

Fitness (no prediction error)

$$U_{0t} = -C$$

Typically $C \gg 1$

Singular perturbation theory

Introduce $\varepsilon = \mathrm{e}^{-\beta C}$, rewrite dynamics

$$\varepsilon x_{t+1} = R x_t \sum_h \mathrm{e}^{-\beta U_{ht}} - \sum_h \mathrm{e}^{-\beta U_{ht}} f_h(x_{t-1}, \cdots, x_{t-L})$$

Here $0 < \varepsilon \ll 1$: singular perturbation (increases order)

Reduction to centre-stable manifold at stability loss

- rules out rational bubbles
- “correct” limit behaviour as $C \to \infty$

Effects of perfect foresight traders

If all boundedly rational types in the market are biased in the fundamental equilibrium, then, if Arrow securities are added to the market

- all boundedly rational types are driven out
- the price is driven to the fundamental
- welfare stabilises at forecasting costs

If some types are nonbiased in the fundamental equilibrium (trend chasers, naive expectations), then

- nonbiased types are not driven out
- prices may remain volatile
- welfare may still decrease towards minus infinity

Summary

In markets with heterogeneous, adaptively learning traders, adding hedging instruments

- decreases risk
- amplifies effect of forecast errors
- destabilises rather than stabilises the market
- increases volatility
- decreases welfare

Adding perfect foresight traders can counteract the last three conclusions in some cases

But:

No, adding trading options is not automatically a good idea