A hyperbolic pertubation of the Navier-Stokes equations

(joint work with M. Paicu)

Y. Brenier, R. Natalini and M. Puel have considered a singular perturbation of the Euler equations in $\mathbf{R}^2$. After an approriate scaling, they have obtained the following hyperbolic version of the Navier-Stokes equations, which is similar to the hyperbolic version of the heat equation introduced by Cattaneo ([1]),

$$\varepsilon u^\varepsilon_{tt} + u^\varepsilon_t - \Delta u^\varepsilon + P(u^\varepsilon \nabla u^\varepsilon) \,=\, Pf \;, \quad (u^\varepsilon(.,0), u^\varepsilon_t(.,0)) \,=\, (u_0(.), u_1(.)) \;, \tag{1}$$

where P is the classical Leray projector and ε is a small, positive number. Under adequate hypotheses on the forcing term f, we prove global existence and uniqueness of a mild solution $(u^\varepsilon, u^\varepsilon_t) \in C^0([0,+\infty), H^1(\mathbf{R}^2) \times L^2(\mathbf{R}^2))$ of (1), for large initial data (u_0, u_1) in $H^1(\mathbf{R}^2) \times L^2(\mathbf{R}^2)$, provided that $\varepsilon > 0$ is small enough. We thus improve the global existence results of [2]. We also show that $(u^\varepsilon, u^\varepsilon_t)$ converges to (v, v_t) on finite intervals of time $[t_0, t_1]$, $0 < t_0 \leq t \leq t_1 < +\infty$, when ε goes to 0, where v is the solution of the corresponding Navier-Stokes equations

$$v_t - \Delta v + P(v \nabla v) \,=\, Pf \;, \quad v(.,0) \,=\, u_0 \;. \tag{2}$$

We also consider Equation (1) in the three-dimensional case. Here we expect global existence results for small data. Under appropriate assumptions on the forcing term, we prove global existence and uniqueness of a mild solution $(u^\varepsilon, u^\varepsilon_t) \in C^0([0,+\infty), H^{1+\delta}(\mathbf{R}^3) \times H^\delta(\mathbf{R}^3))$ of (1), for initial data (u_0, u_1) in $H^{1+\delta}(\mathbf{R}^3) \times H^\delta(\mathbf{R}^3)$ (where $\delta > 0$ is a small positive number), provided that $\varepsilon > 0$ is small enough and that u_0 and f satisfy a smallness condition.

References

[1] C. Cattaneo, *Sulla conduzione del calore*, Atti Sem. Mat. Fis. Univ. Modena, vol. 3, 1949, pages 83–101.

[2] Y. Brenier, R. Natalini et M. Puel, *On a relaxation approximation of the incompressible Navier-Stokes equations*, Proc. Amer. Math. Soc., vol. 132, 2004, pages 1021–1028.