



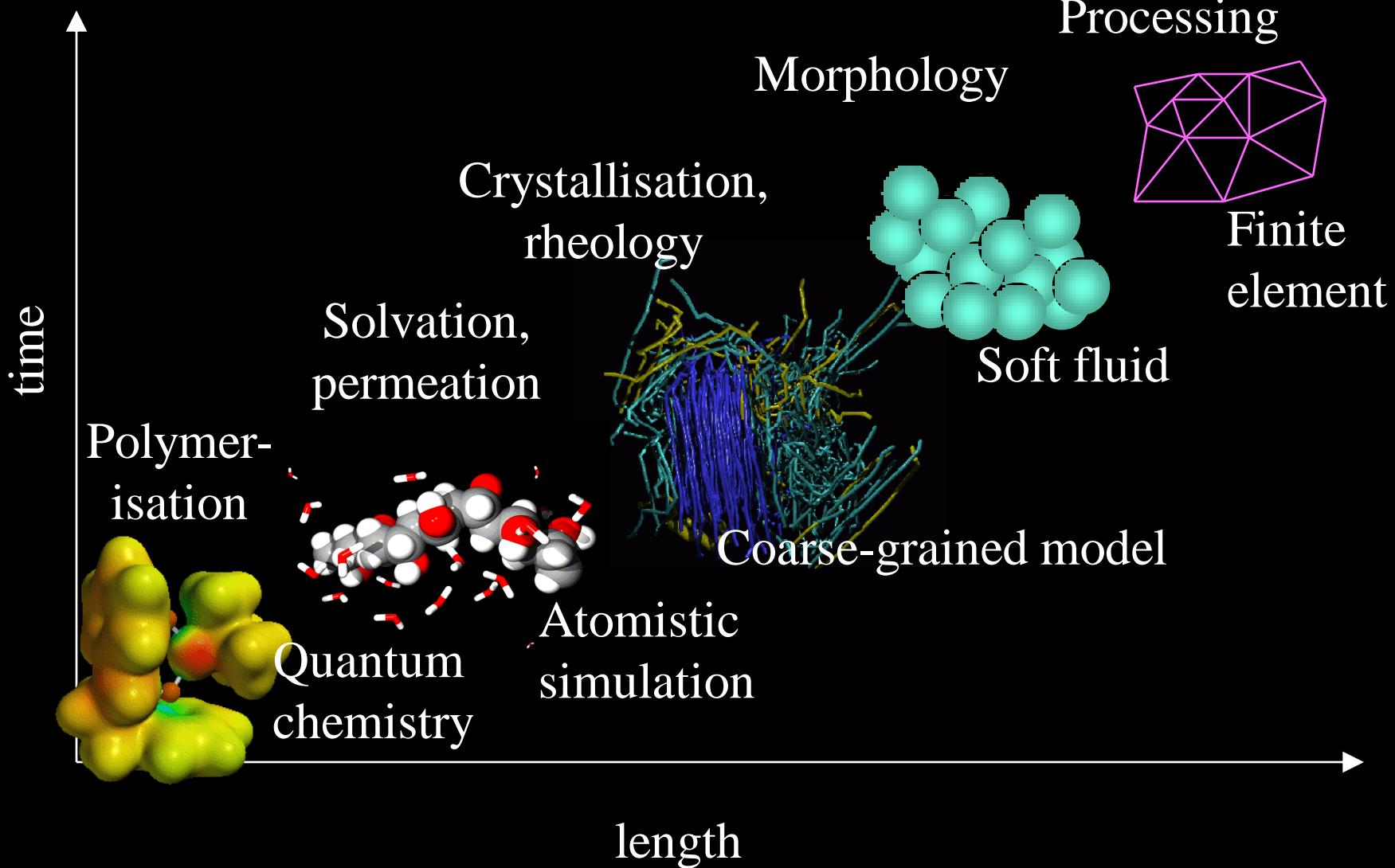
# Trends in Materials Simulation: A Molecular Dynamics Miscellany

Florian Müller-Plathe ([www.theo.chemie.tu-darmstadt.de](http://www.theo.chemie.tu-darmstadt.de))



# Polymers: Scales & Methods

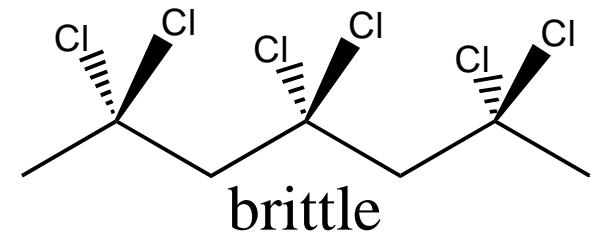
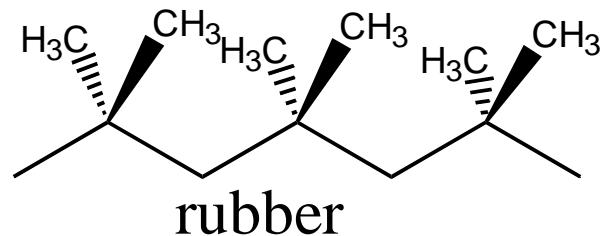
Review: FMP, Soft Materials 1, 1 (2003)



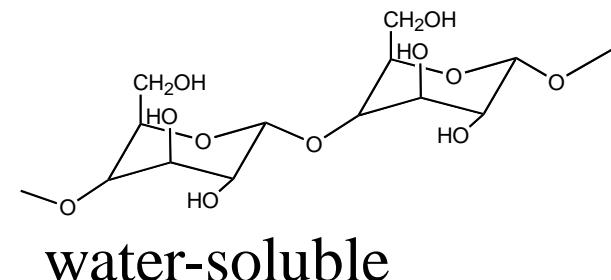
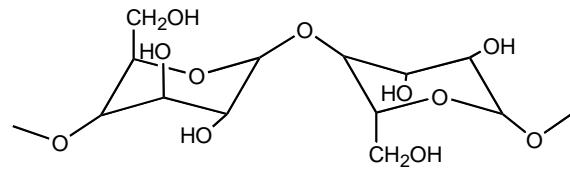


# Polymers: Structure and Properties

- Chemistry



- Tacticity

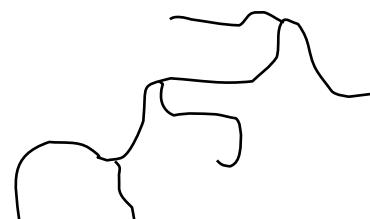


- Sequence

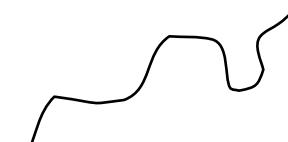
S-S-S-S-B-B-B-B  
engineering plastic

S-B-S-S-B-S-B-B  
synthetic rubber

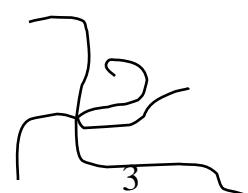
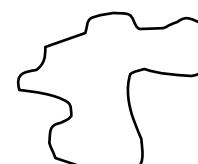
- Topology



shopping bag

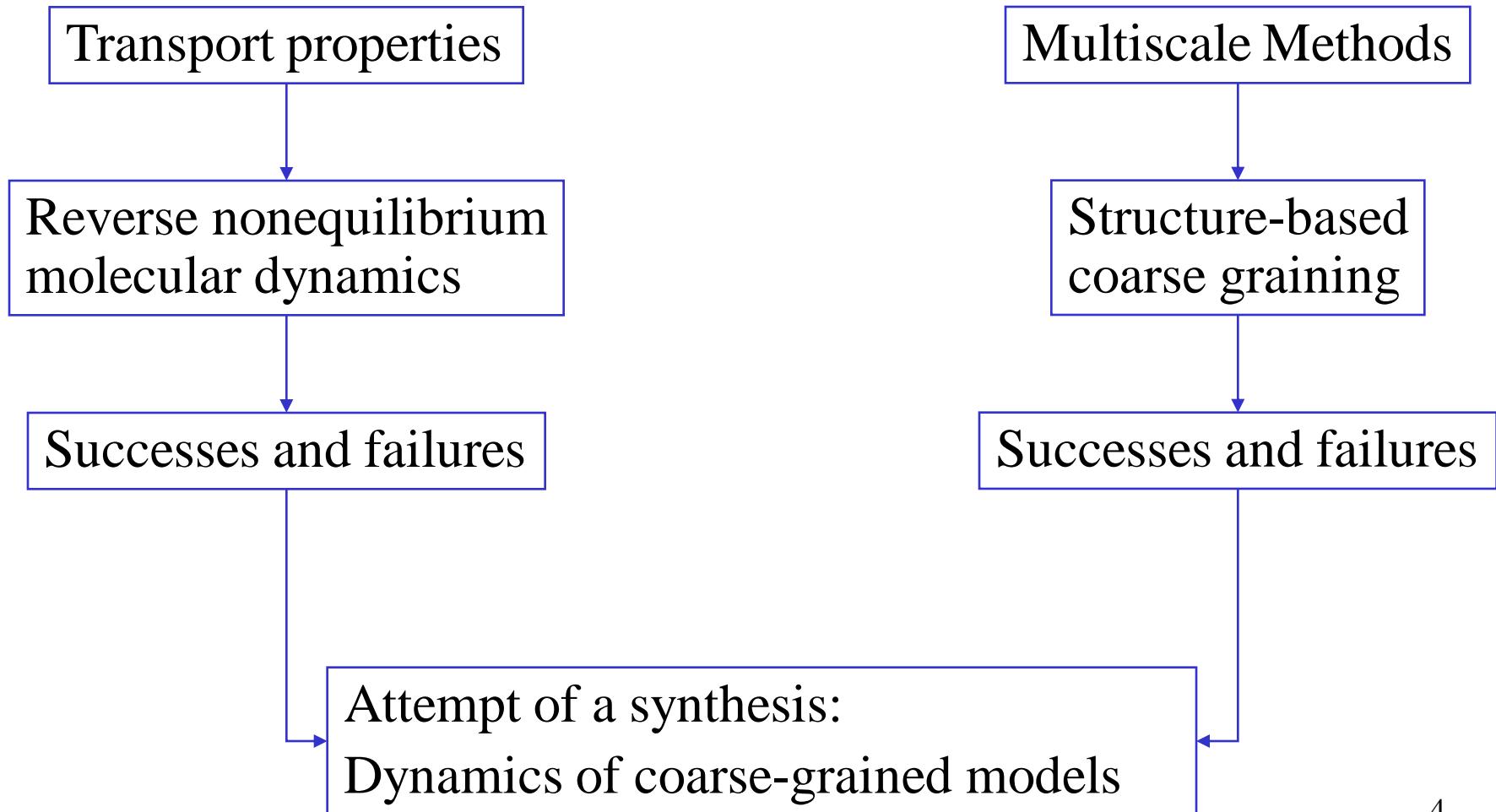


bullet-proof vest





# Multiscaling and Dynamics





# Transport Coefficients

- Transport coefficient

$$J = -\kappa E$$

flux, flow, current, ...

driving force, field,  
thermodynamic force

- Non-equilibrium situation: steady-state, periodic, ...
- Linear response: small perturbation
- Isotropic medium: liquid, melt, ...



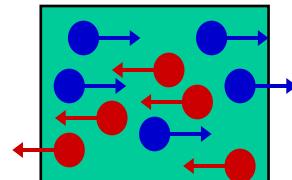
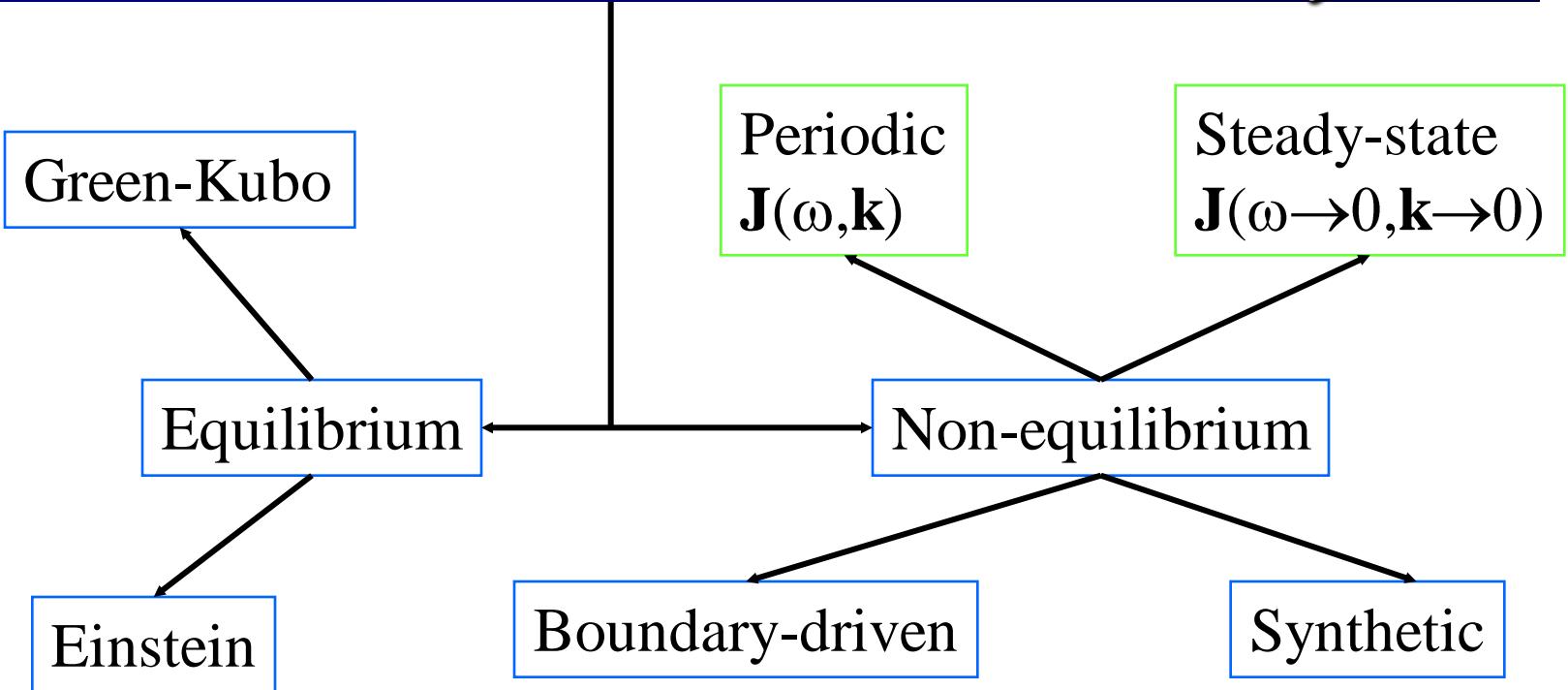
# Linear response

- **Mass**      *Berthollet 1803*  
 $J_1 = -D(\partial c / \partial z)$     *Fick 1854*
- **Charge**      *Ohm 1826*  
 $I = -(1/R)(\partial \Phi / \partial z)$
- **Energy**      *Fourier 1822*  
 $J_Q = -\lambda (\partial T / \partial z)$
- **Momentum**  
 $J(p_\perp) = -\eta (\partial v_\perp / \partial z)$   
[ $\sigma_{xy} = -\eta \dot{\gamma}$ ]

- **Coupled energy and mass**  
$$J_1 = -D_{12}\rho [ (\partial w_1 / \partial z) + S_T w_1 (1 - w_1) (\partial T / \partial z)]$$
- **$\mathbf{J} = -\mathbf{LX}$**     *Onsager 1931*  
$$J_1 = -L_{11} \frac{\nabla_T \mu}{T} - L_{1Q} \frac{\nabla T}{T^2}$$



# Calculation of Transport Coefficients by MD





# 1<sup>st</sup> Ingredient: Reverse cause and effect

## Traditional NEMD

$$\langle \mathbf{J} \rangle = -\kappa \mathbf{E}$$

↑              ↑  
**calculate**    **impose**

## Reverse NEMD

$$\mathbf{J} = -\kappa \langle \mathbf{E} \rangle$$

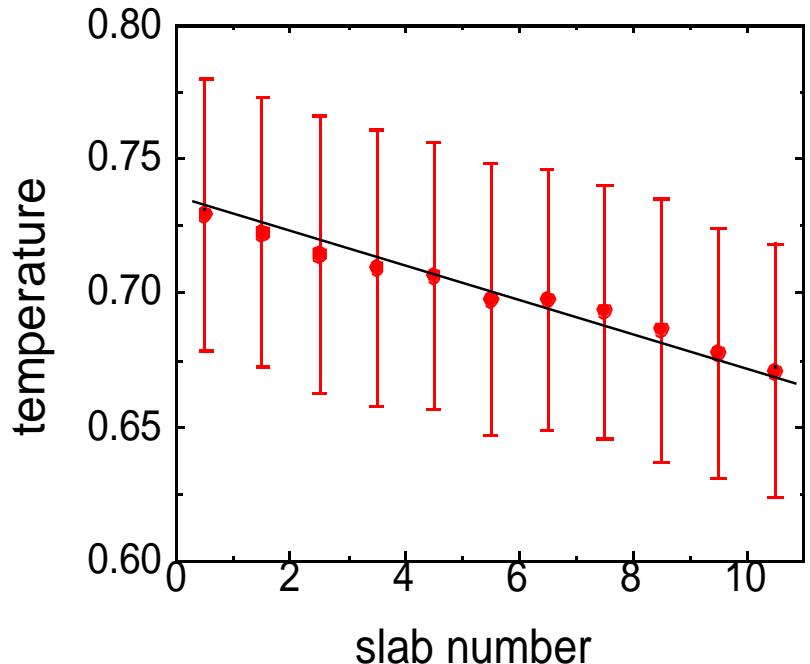
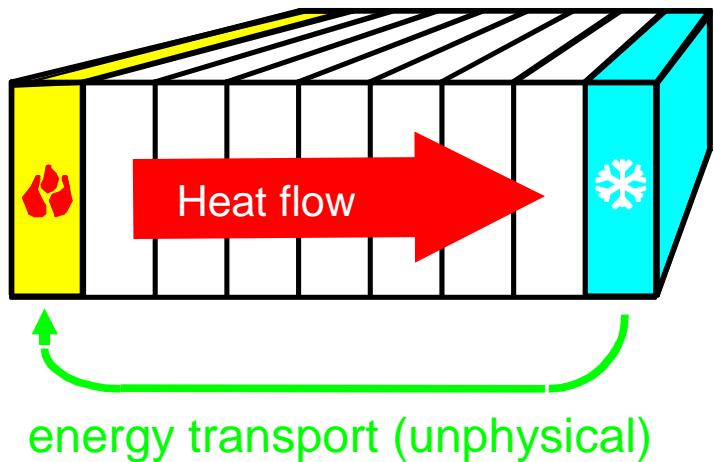
↑              ↑  
**impose**    **calculate**

Examples:

- shear viscosity  $\eta$
- thermal conductivity  $\lambda$
- Soret coefficient  $S_T$



# Thermal Conductivity



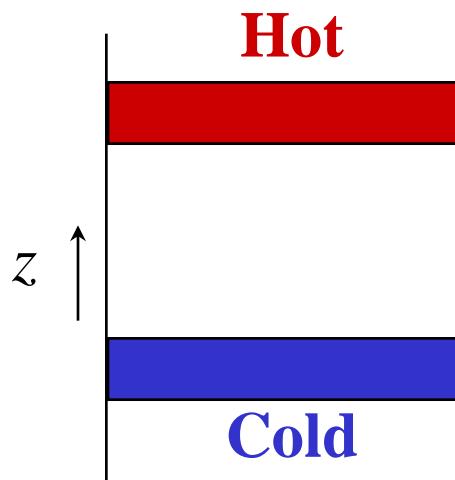
Repeat periodically, wait for steady state

Fourier's law: 
$$J_Q = -\lambda \nabla T / dz$$



# 2<sup>nd</sup> Ingredient: Unphysical velocity exchange

What we want



If  $m_1=m_2$ :

Find **hottest** particle in  
the **cold** region and  
**coldest** particle in the  
**hot** region



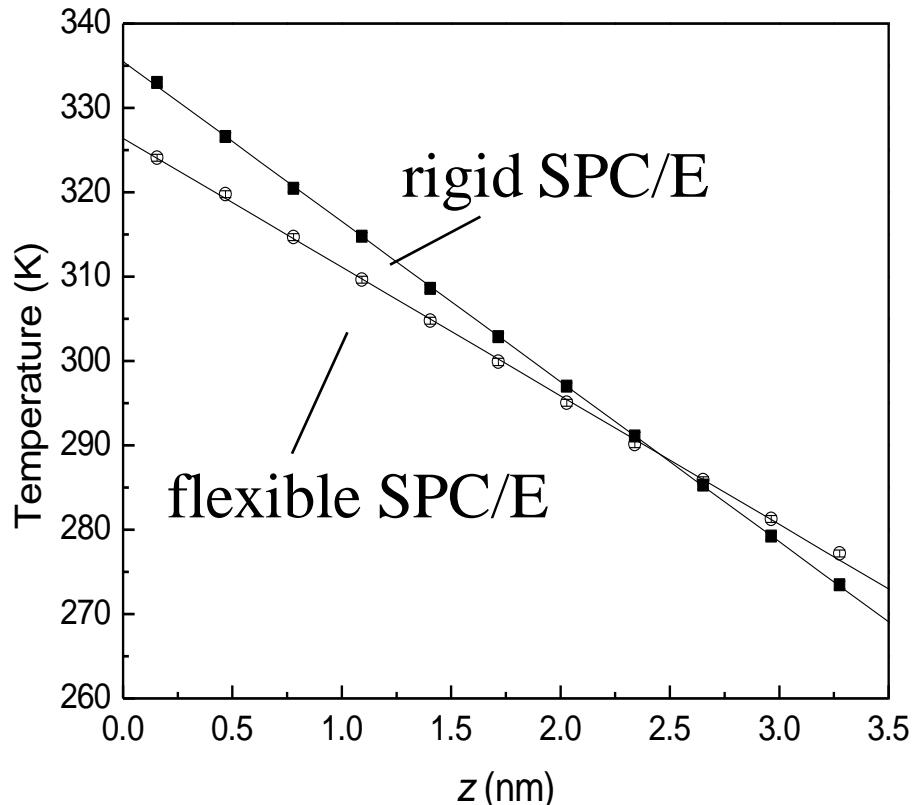
Swap their  
velocities  $\mathbf{v}$



no change in total linear momentum  
no change in total kinetic energy  
no change in total energy



# Analyse



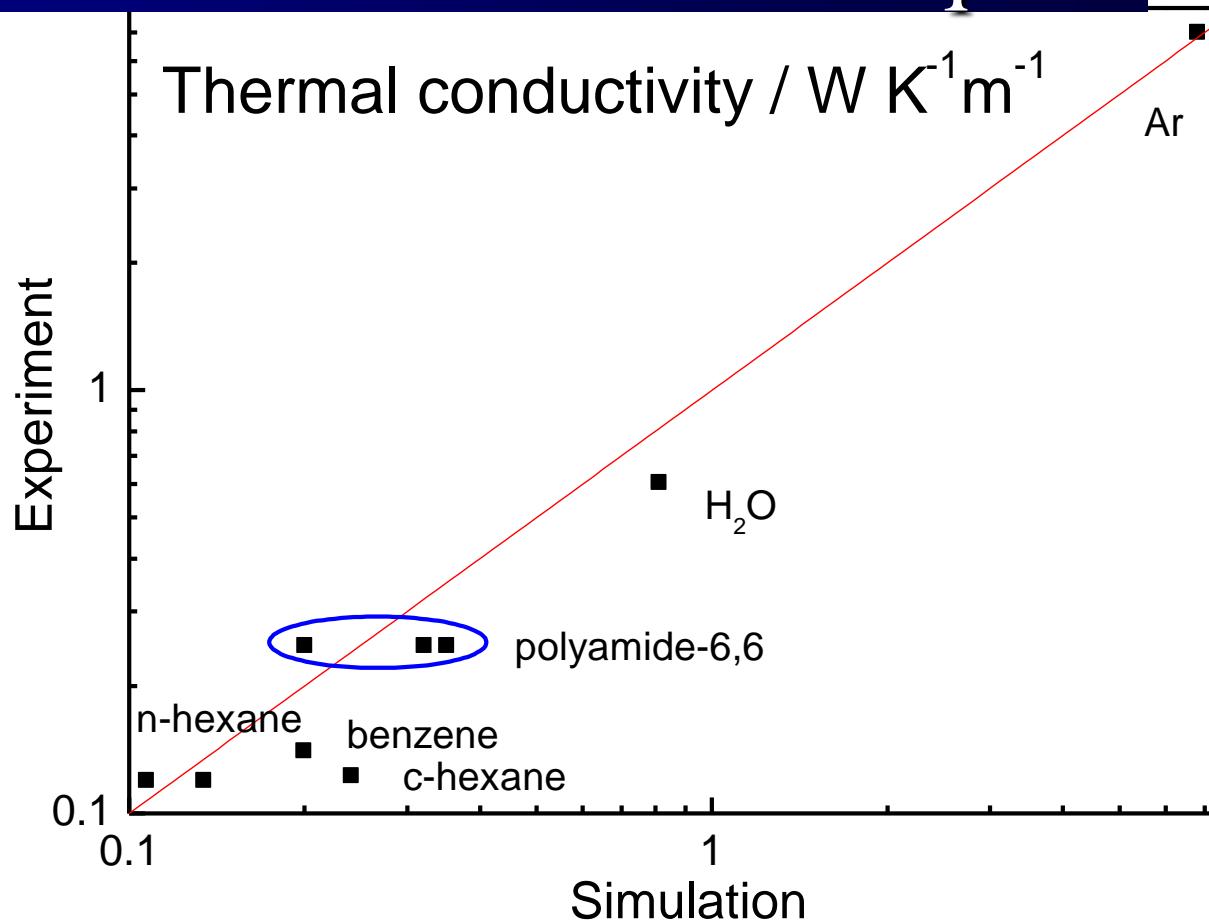
- Calculate temperature profile  $T(z)$
- Obtain gradient  $dT/dz$
- Fourier's law

$$\lambda = -\frac{J_q}{\langle dT/dz \rangle}$$

	Thermal conductivity $\lambda$ (Wm <sup>-1</sup> K <sup>-1</sup> )
rigid	$0.81 \pm 0.01$
flexible	$0.95 \pm 0.01$
experiment	0.607

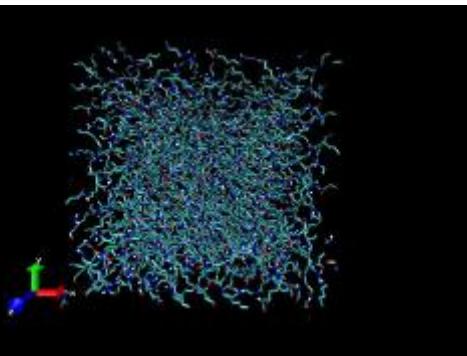


# Thermal conductivity of molecular liquids





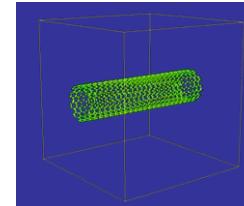
# Thermal Conductivity of Polyamide-6,6



	Simulation W/K m	Experiment W/K m
<b>amorphous (1.07 g/cm<sup>3</sup>)</b>		
(flexible model)	0.33-0.35	~0.25
(semirigid model)	0.20	
<b>stretched amorphous (0.95 g/cm<sup>3</sup>)</b>		
parallel to stretching	0.43	
perpendicular	0.20	

# Carbon nanotubes

M. Alaghemandi, E. Algaer, M. C. Böhm, FMP,  
Nanotechnology **20**, 115704 (2009).



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT

- $\lambda$  depends on tube length

- power law

$$\lambda \propto L^{0.5-0.8}$$

- Breakdown of Fourier's law

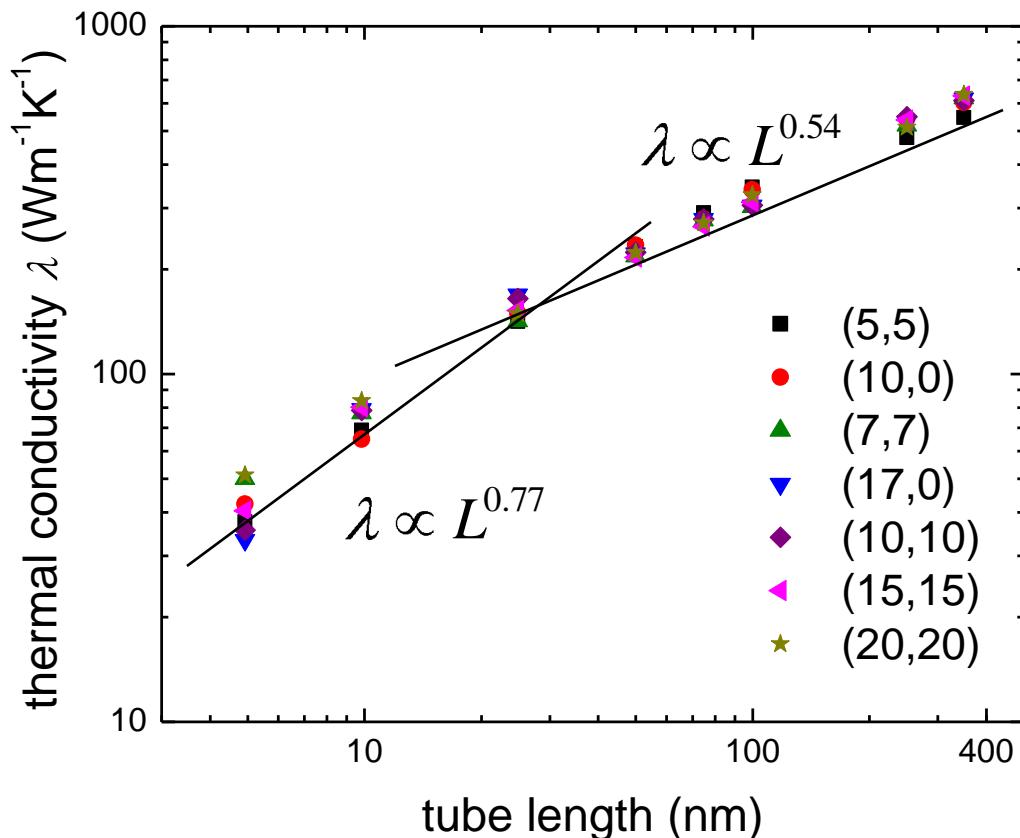
- Possibly different exponents for short and long tubes.

Theory:

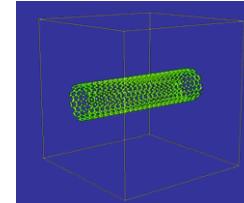
$$\alpha = 0.83 \rightarrow 0.35$$

J. Wang, J.-S. Wang, *Appl. Phys. Lett.* **88**, 111909 (2006)

- Chirality of tube unimportant

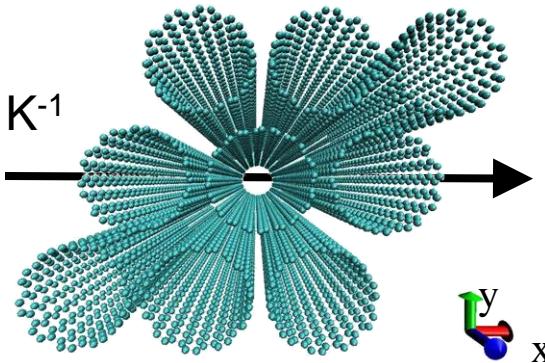


# Anisotropy of thermal conductivity



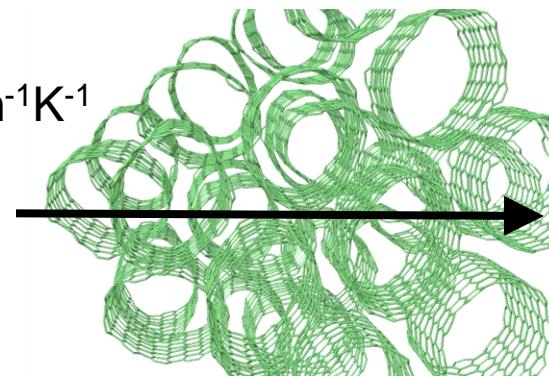
- CNT crystal

- $\lambda_{\parallel}$ :  $100 - 1000 \text{ W m}^{-1}\text{K}^{-1}$   
(better than metal)
- $\lambda_{\perp}$ :  $0.14 \text{ W m}^{-1}\text{K}^{-1}$   
(worse than polymer)



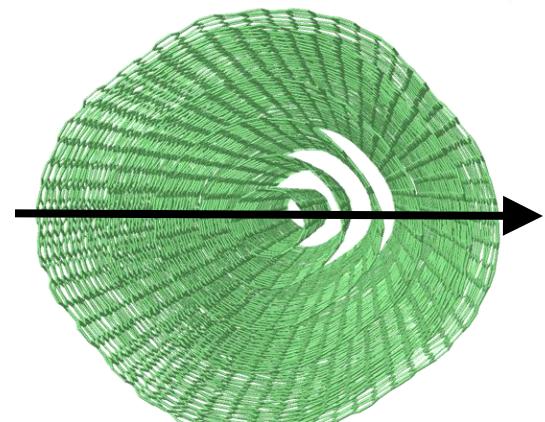
- CNT bundle

- $\lambda_{\parallel}$ :  $100 - 1000 \text{ W m}^{-1}\text{K}^{-1}$
- $\lambda_{\perp}$ :  $0.09 \text{ W m}^{-1}\text{K}^{-1}$



- Multiwall nanotube 10 nm

- $\lambda_{\parallel}$ :  $21 \text{ W m}^{-1}\text{K}^{-1}$
- $\lambda_{\perp}$ :  $0.24 \text{ W m}^{-1}\text{K}^{-1}$

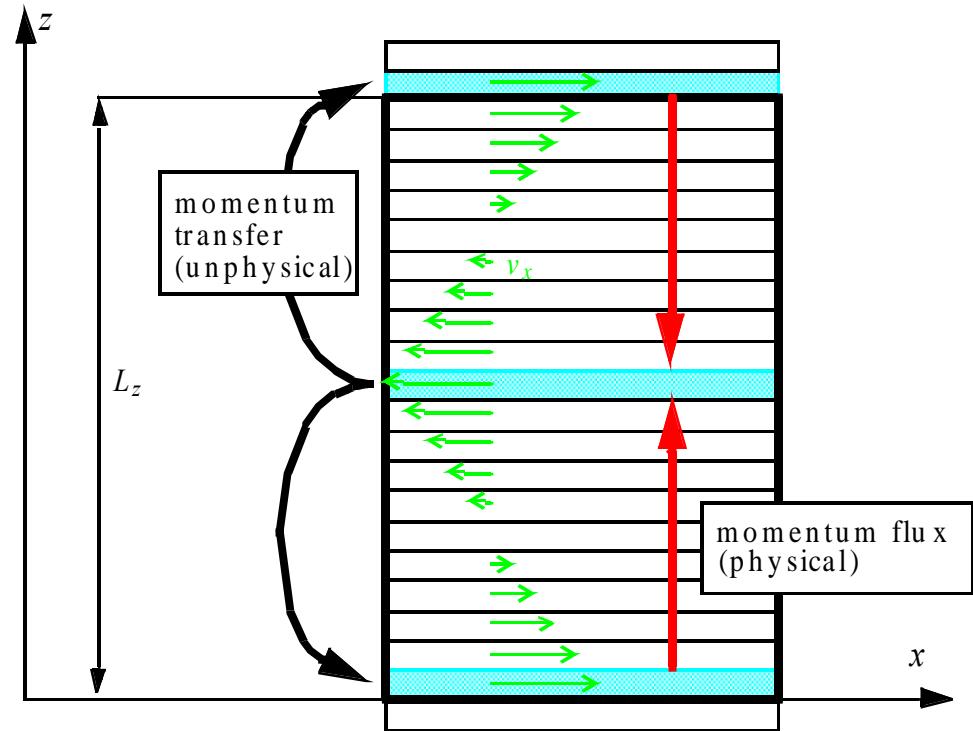
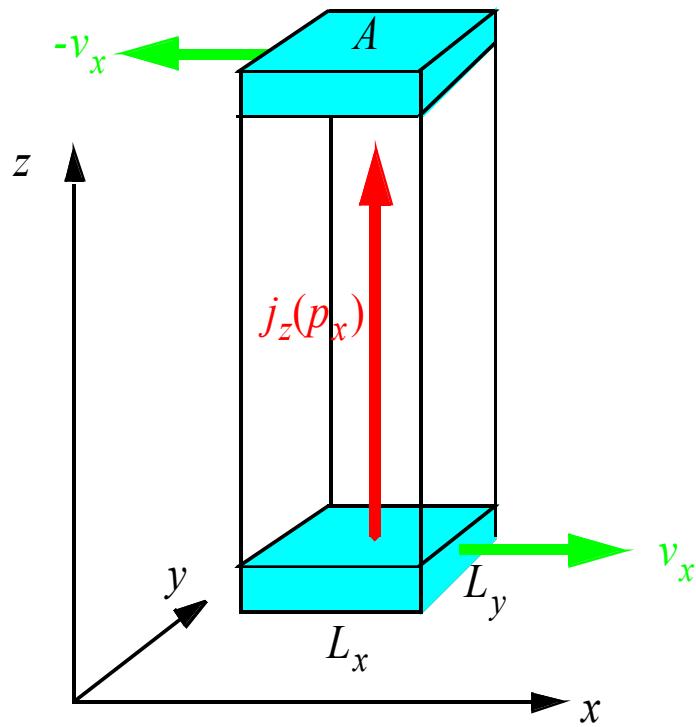


- Extreme anisotropies!

- Fast parallel mechanism (phonons)  $\leftrightarrow$  slow lateral mechanism (collisions between neighbouring tubes).



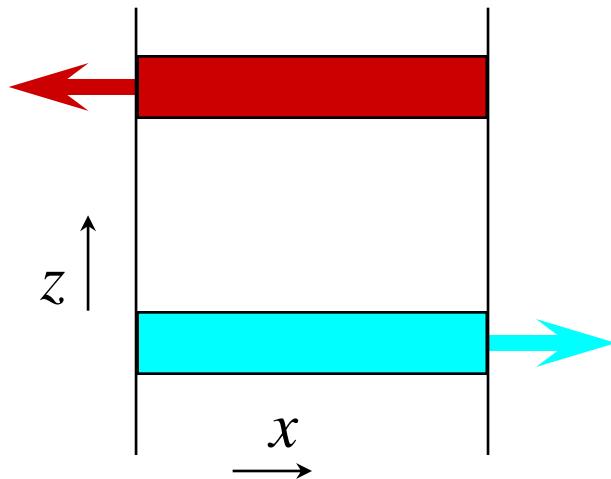
# Shear viscosity



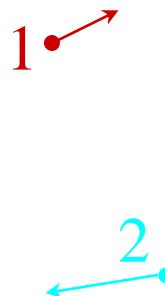


# Exchange of velocity component

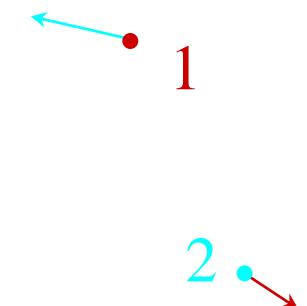
## What we want



Find 2 particles that move **against** the current



Swap their  $v_x$



If  $m_1=m_2$ :

no change in total linear momentum  
no change in total kinetic energy  
no change in total energy  
 $\rightarrow$  no thermostat!



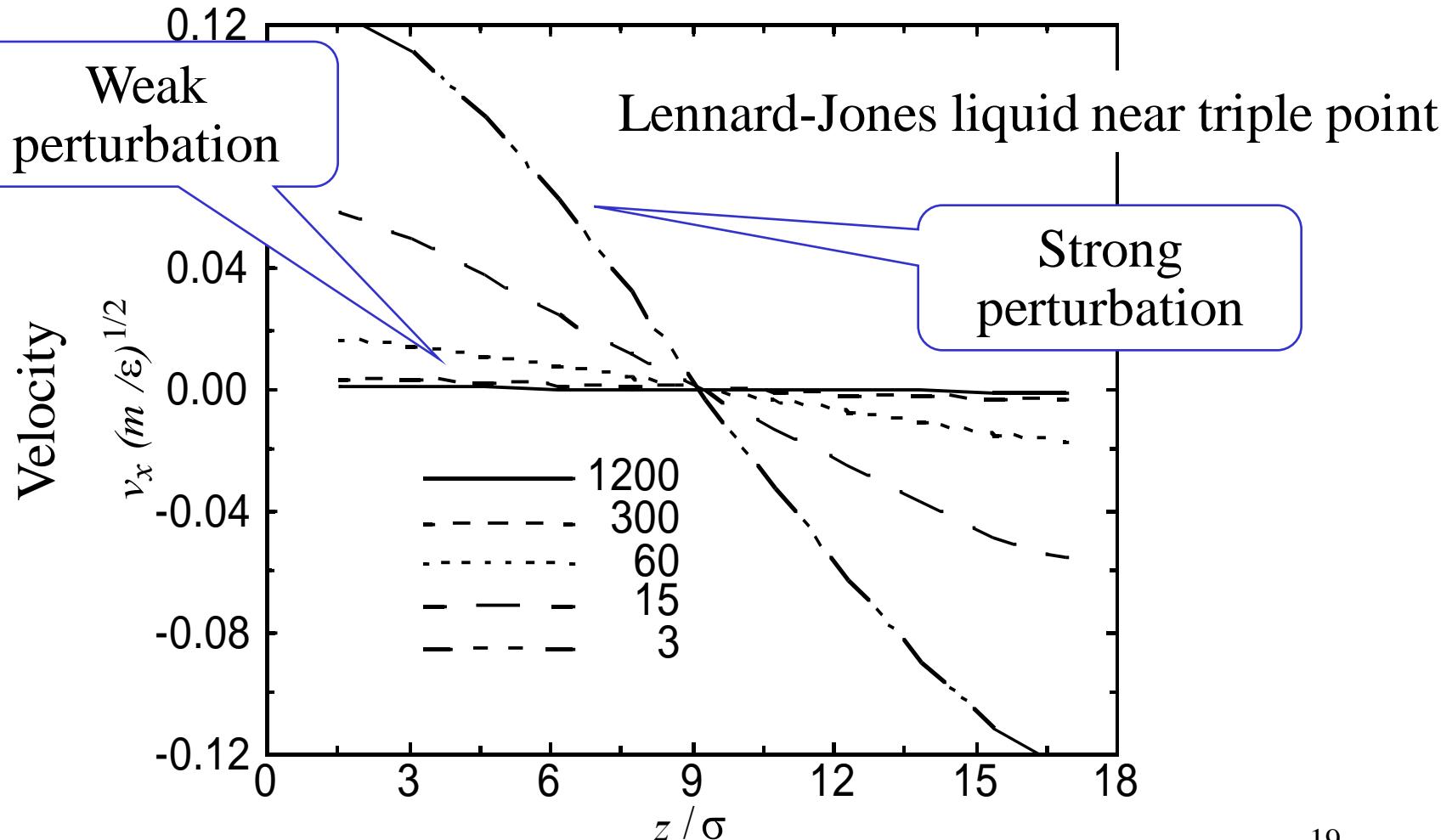
# Shear viscosity

- What have we done?
  - Impose a flux of transverse linear momentum  $J(p_\perp)$   
= apply a shear force
  - higher perturbation = swap velocities more often
- What still needs to be done?
  - Measure shear rate:  
Measure profile of flow velocity, determine  $dv_\perp/dz$
  - Viscosity  $\eta$  is proportionality constant

$$J \cancel{p}_\perp \rightleftharpoons -\eta \frac{dv_\perp}{dz}$$



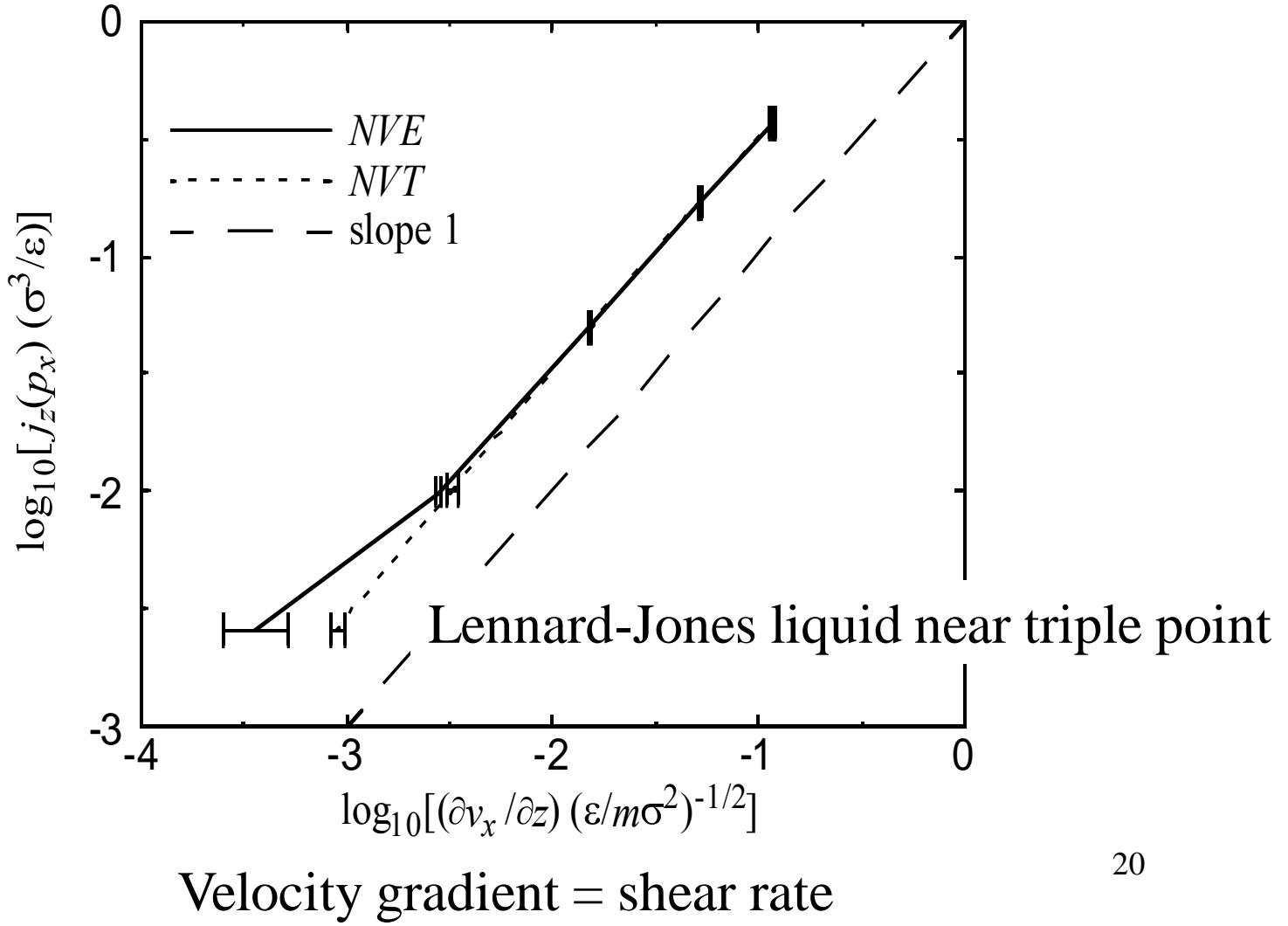
# Velocity profiles





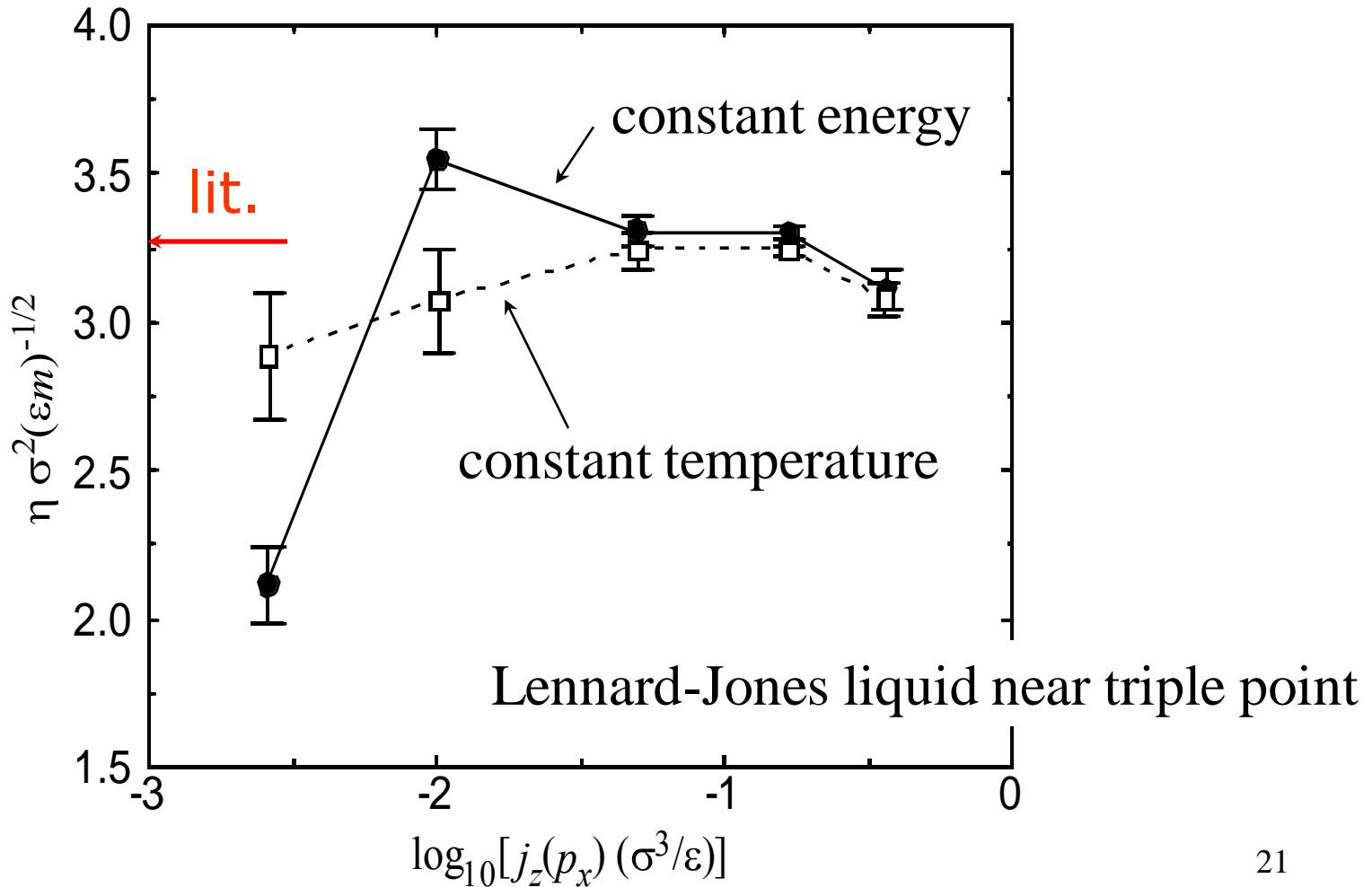
# Linear response holds

Transverse momentum flux  
= shear force





# Shear viscosity $\eta$ of the Lennard-Jones fluid





# Viscous flow without viscous heating?

**Recall:**

- Total energy is conserved: microcanonical ( $NVE$ )

**But:**

- Viscous flow  $\rightarrow$  friction  $\rightarrow$  heating !!!

**Question:**

- How is the heat removed?

**Answer:** Maxwell demon

- Undirected motion (heat)  $\rightarrow$  Directed motion (flow)
- Cooling in exchange regions

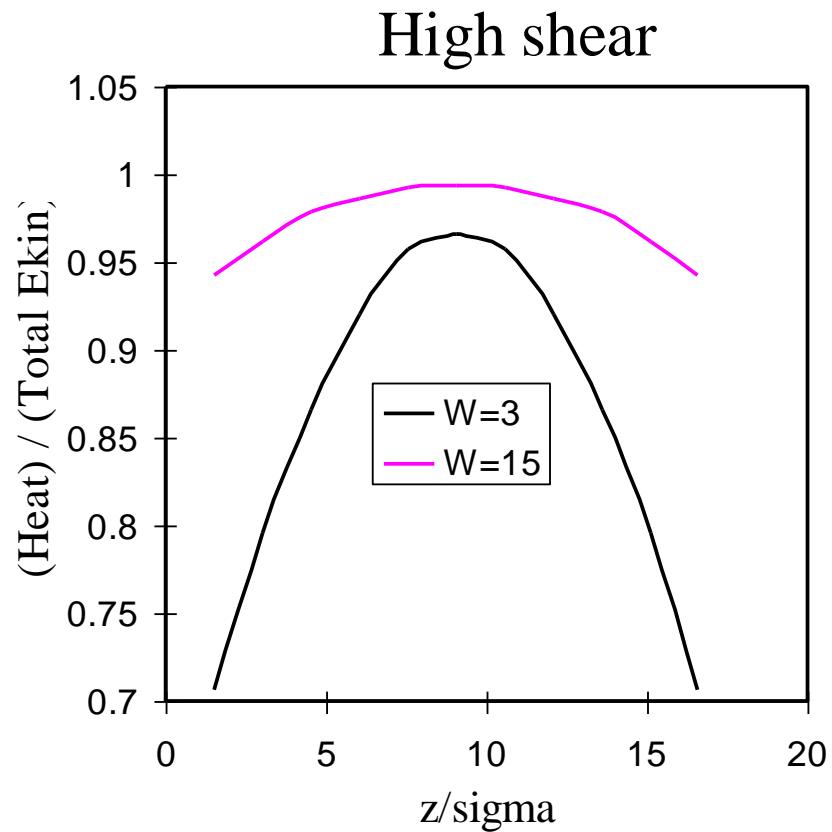
**Proof:** temperature profile

- $E_{\text{kin}}(\text{total}) = E_{\text{kin}}(\text{temperature}) + E_{\text{kin}}(\text{flow})$
- Peculiar velocity  $\mathbf{u}_i = \mathbf{v}_i - \langle \mathbf{v} \rangle$  defines temperature



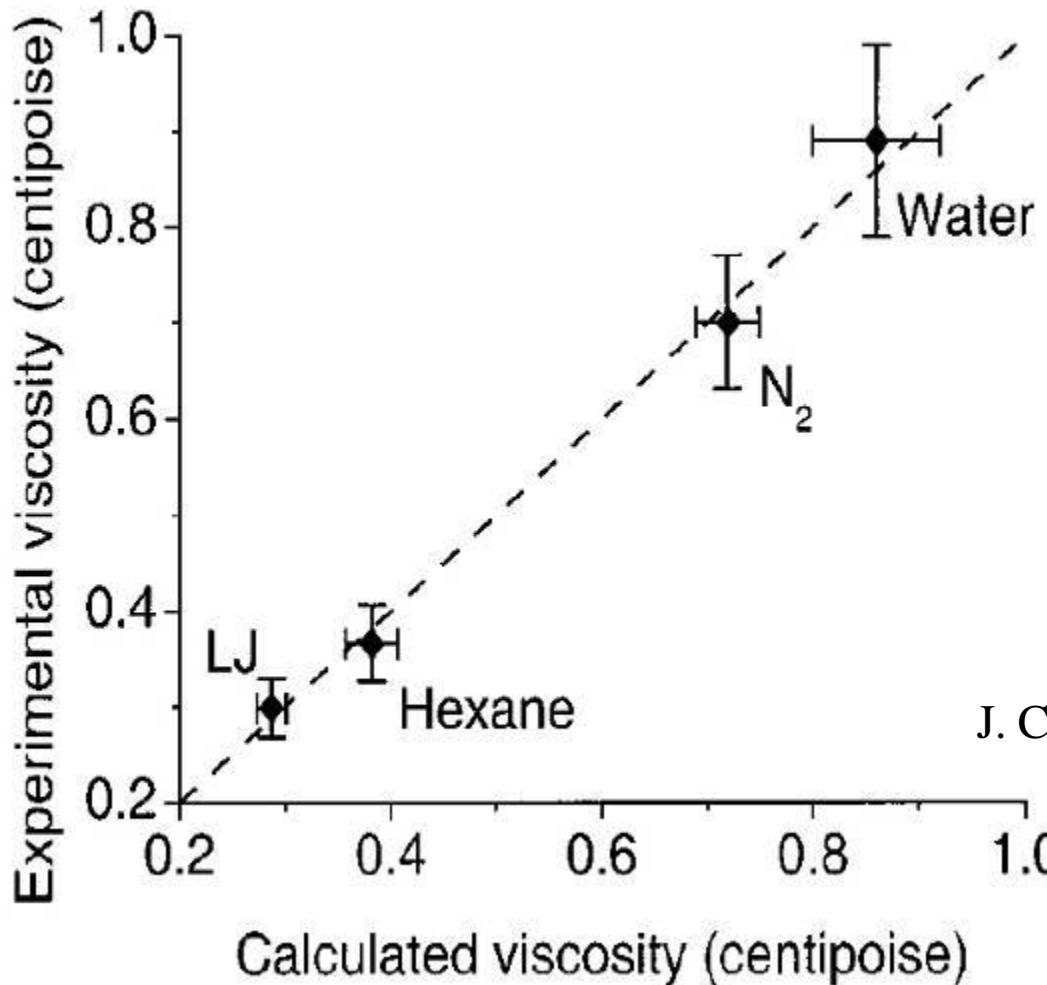


# Temperature profile from peculiar velocities





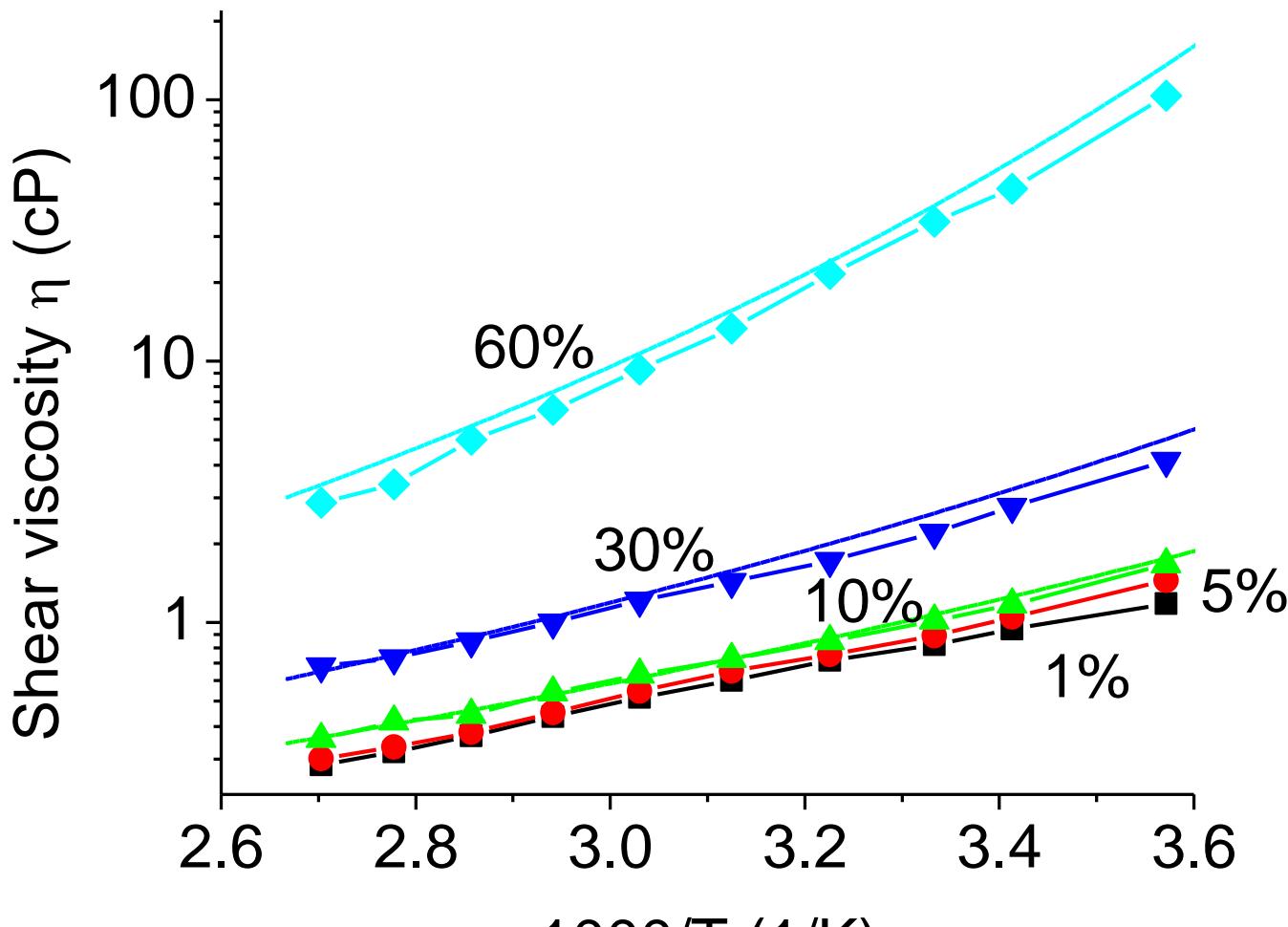
# Molecular Liquids



P. Bordat, F. Müller-Plathe,  
J. Chem. Phys. **116**, 3362 (2002)



# Aqueous solutions of saccharose



# Ionic liquids: Shear viscosity of [bmim][PF<sub>6</sub>]

Experiment:

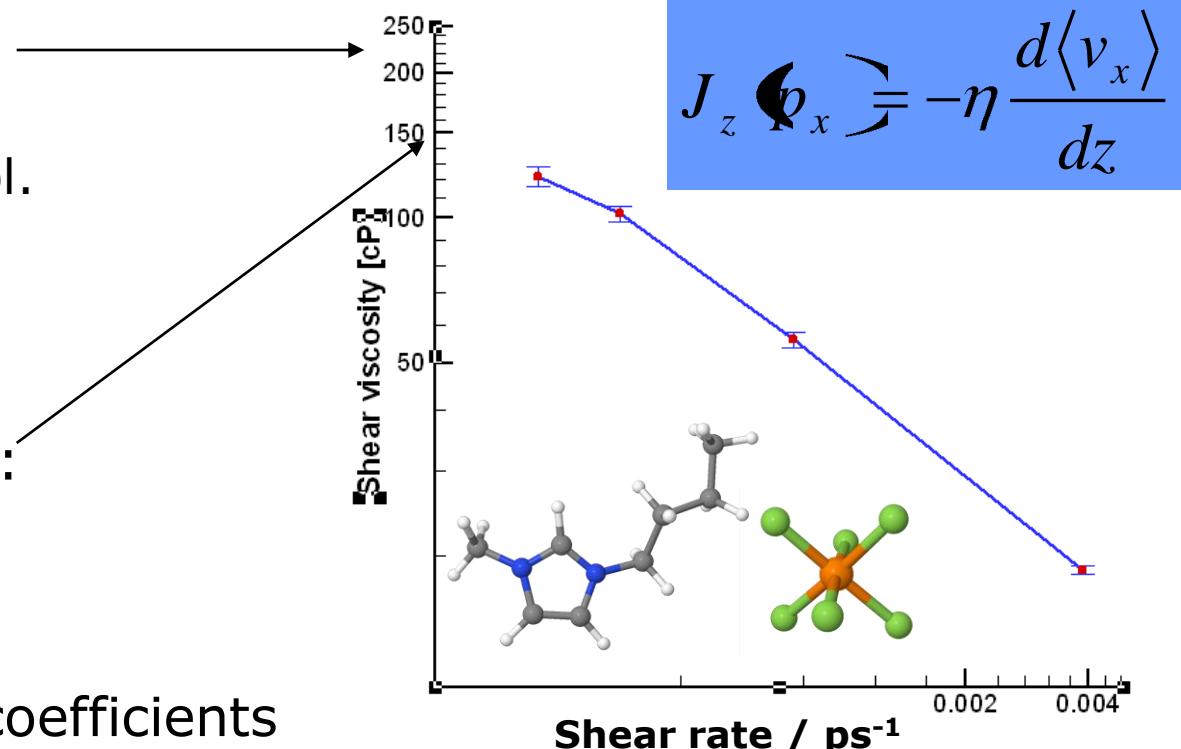
$\eta_0 = 204 \text{ cP}$  at 303K  
 [Seddon et al., Pure Appl.  
 Chem. **72**, 2275(2000)]  
 others: 173-450 cP

Simulation:  
 $\eta_0 \rightarrow 133 \text{ cP}$  at 300K

Analogous: diffusion coefficients

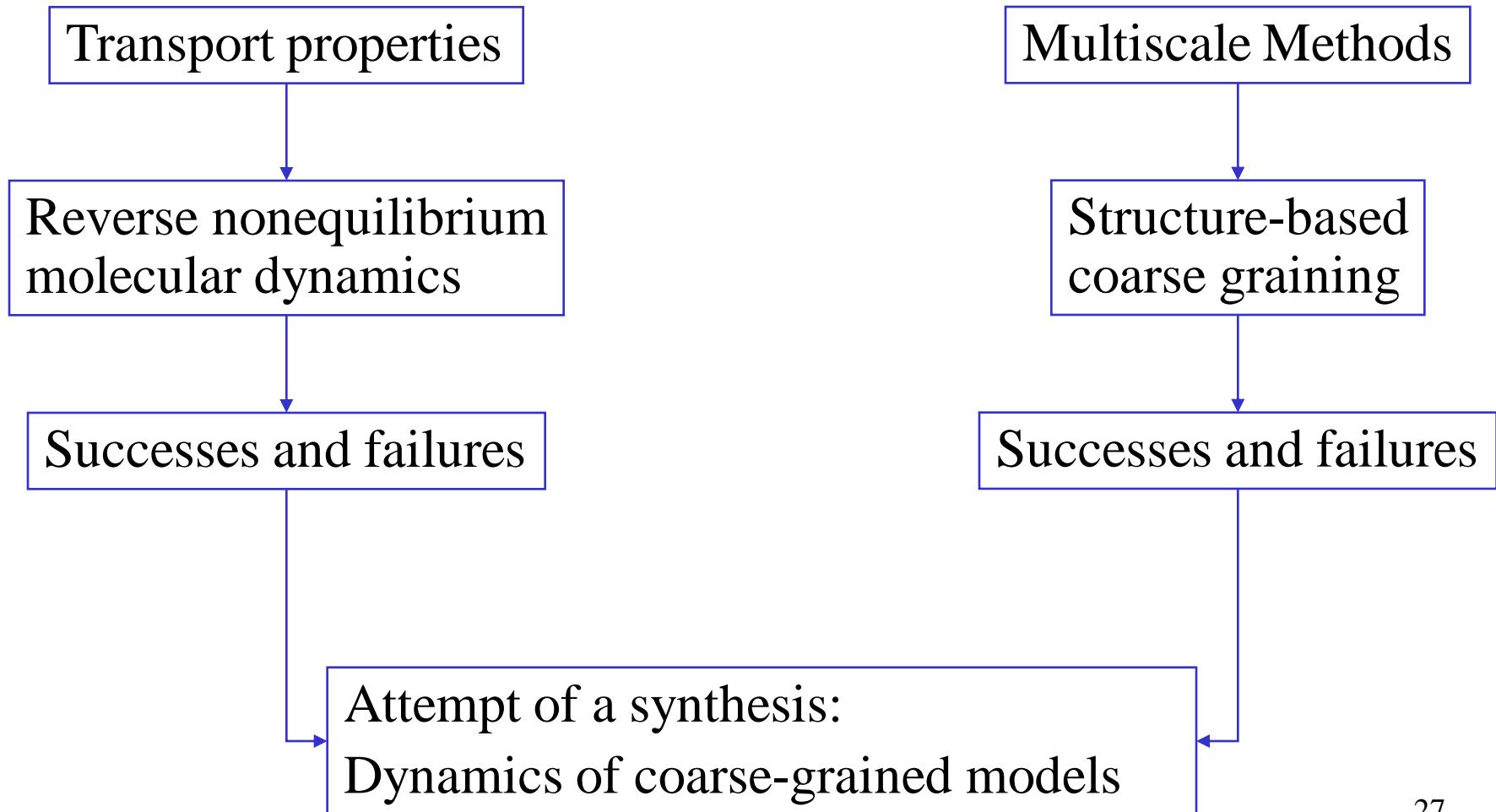
Exper.  $0.7 \times 10^{-7} \text{ cm}^2/\text{s}$

Sim.  $1.3 \times 10^{-7} \text{ cm}^2/\text{s}$



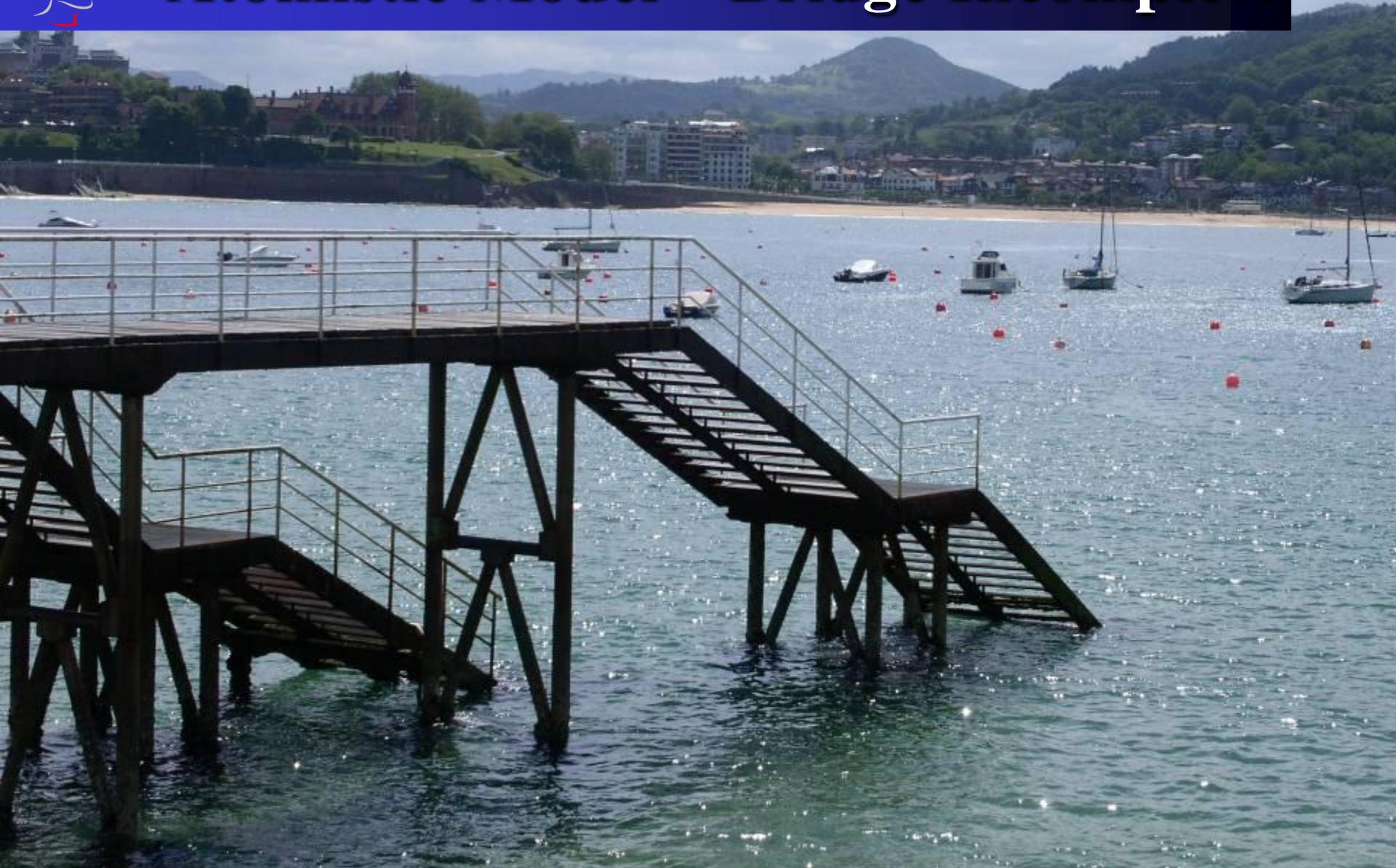


# Multiscaling and Dynamics





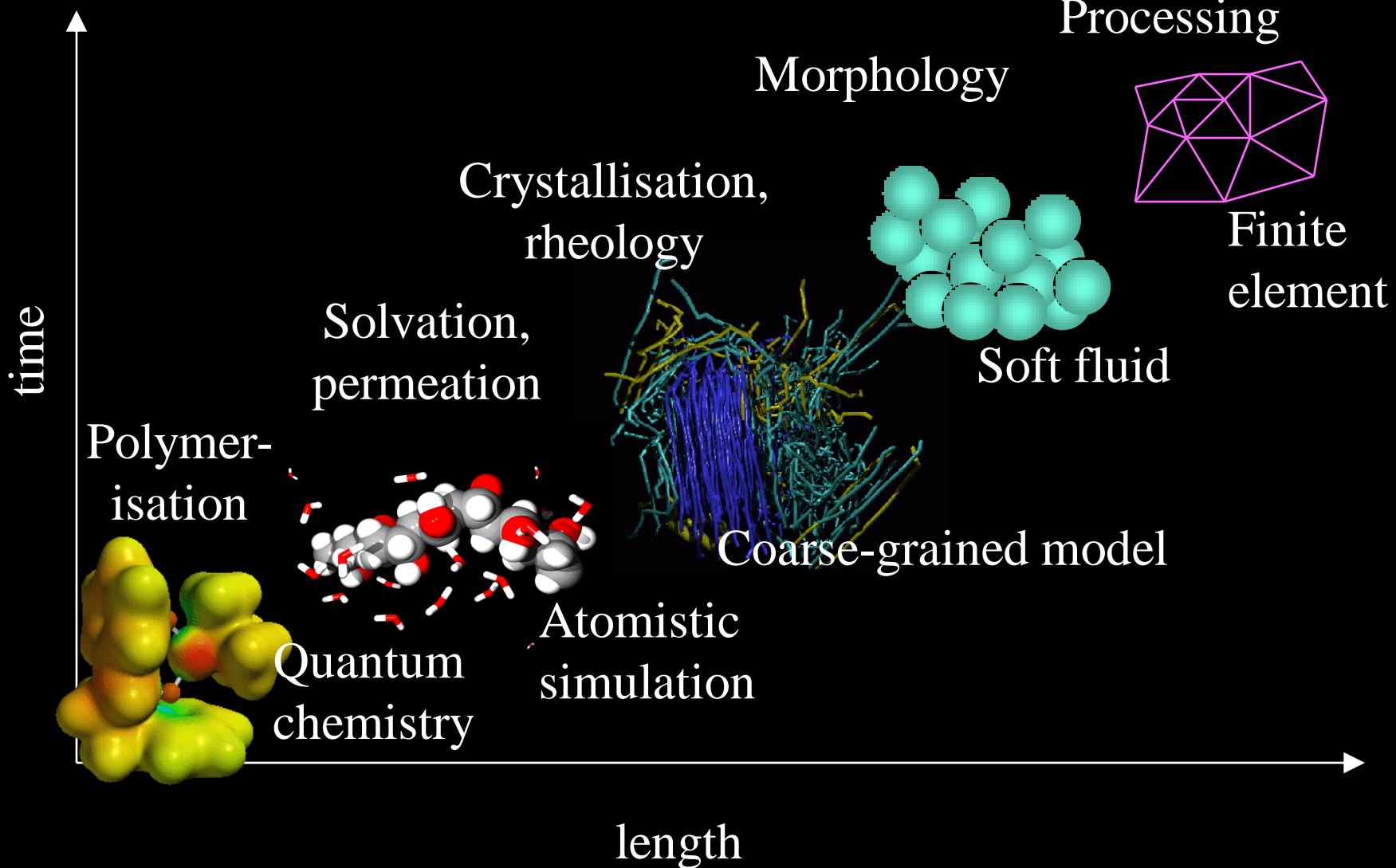
# Atomistic Model – Bridge Incomplete





# Polymers: Scales & Methods

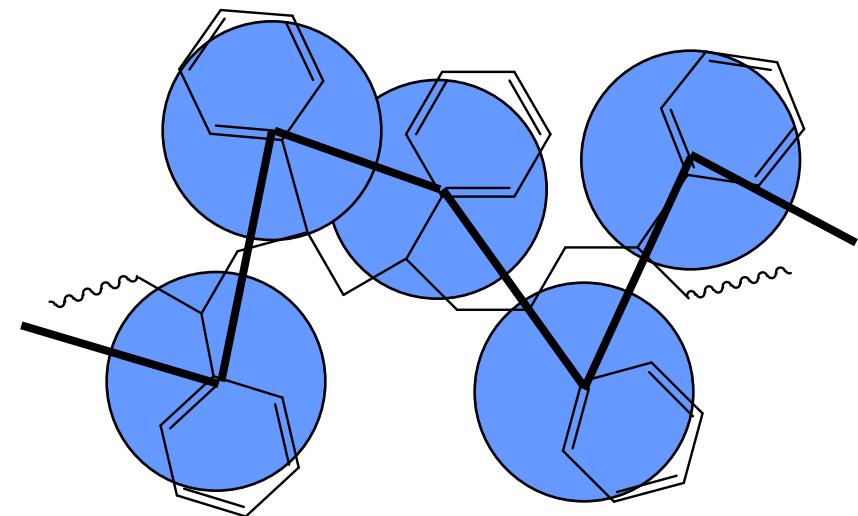
Review: FMP, Soft Materials 1, 1 (2003)





# Coarse-grained Force Field

~10 atoms → 1 superatom  
1000-100000 times cheaper



Effective interactions:  
“bonds”, “angles”, non-bonded

*Systematic* coarse-graining:

- interactions as realistic as possible
- model is material-specific
- no generic “bead-and-spring” model

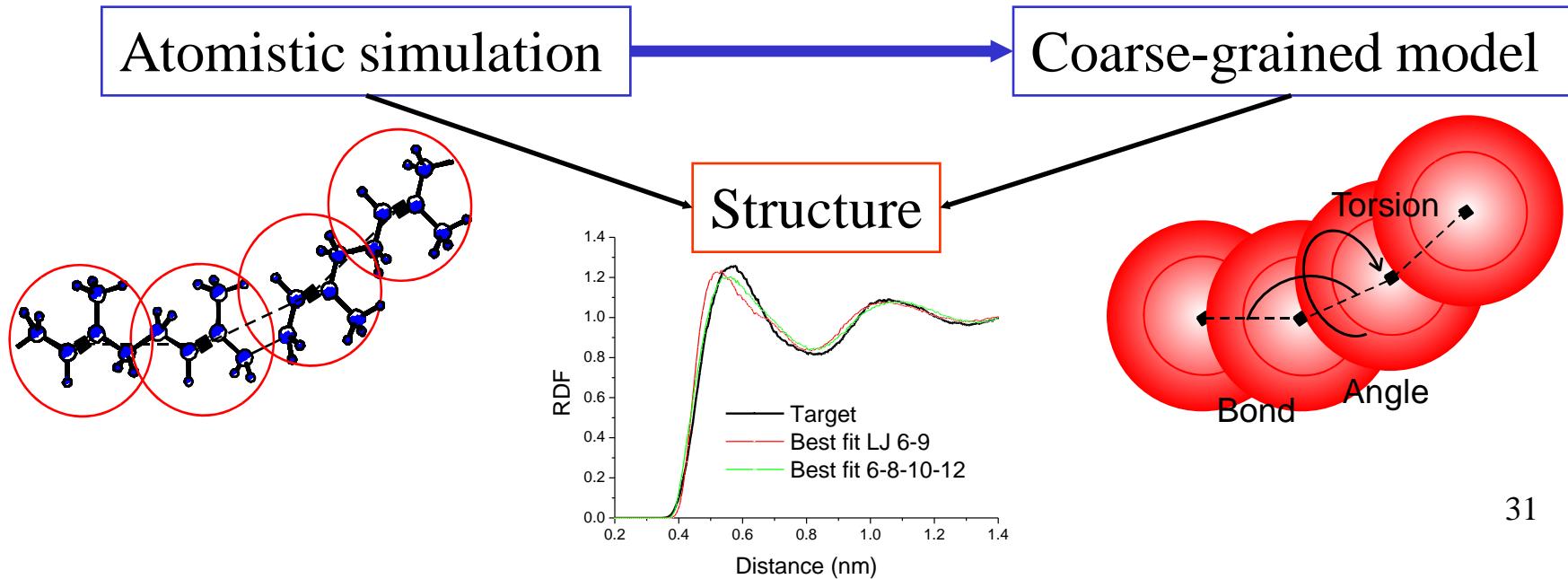
Reviews: F. Müller-Plathe, ChemPhysChem **3**, 754 (2002); Soft Materials **1**, 1 (2003).

# Coarse Graining



Objectives for coarse-grained model:

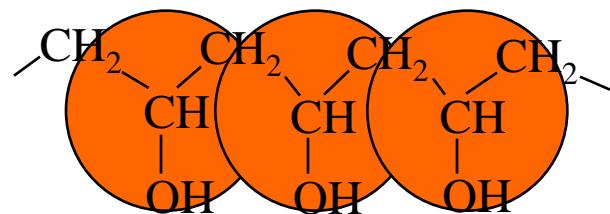
- Simpler than atomistic model:  $\sim 10$  real atoms  $\rightarrow$  1 “superatom”
- Material-specific
- Reproduce structure of atomistic model





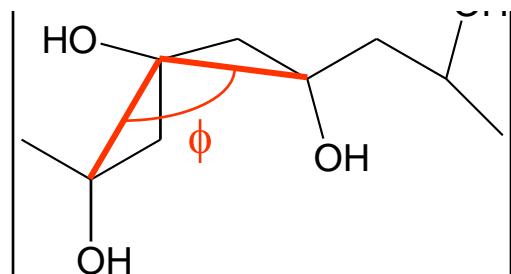
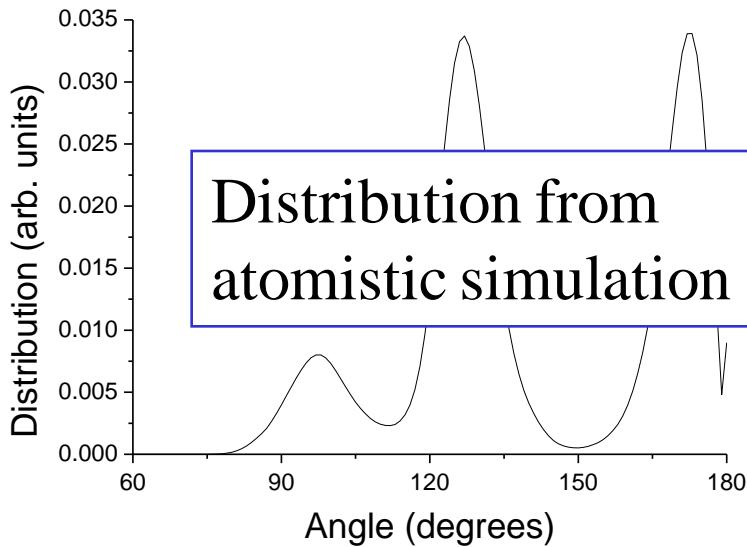
## 1<sup>st</sup> Example: Poly(vinyl alcohol)

- Polymer in the melt
- 48 decamers
- Use:
  - Packaging: O<sub>2</sub> barrier
  - Fishing nets: same refractive index as water
  - Pervaporation membranes:  
remove water from organic solvents





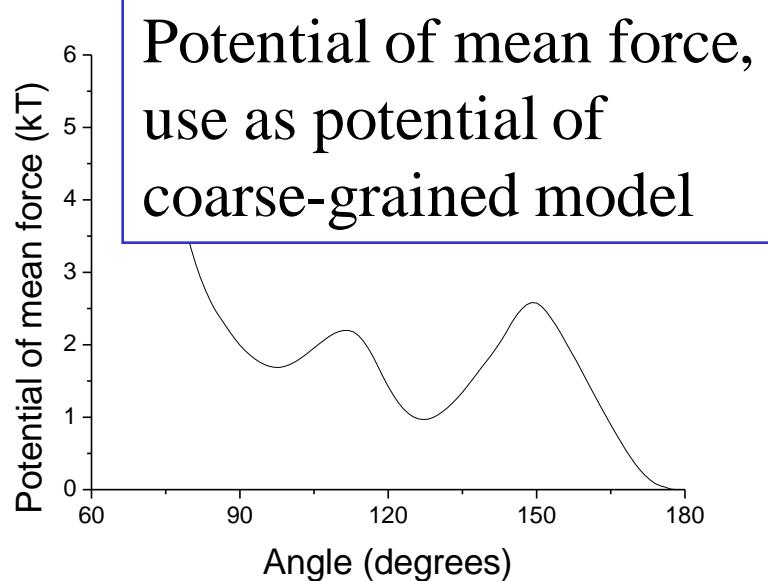
# Coarse-Graining PVA: Angles



The easy part:

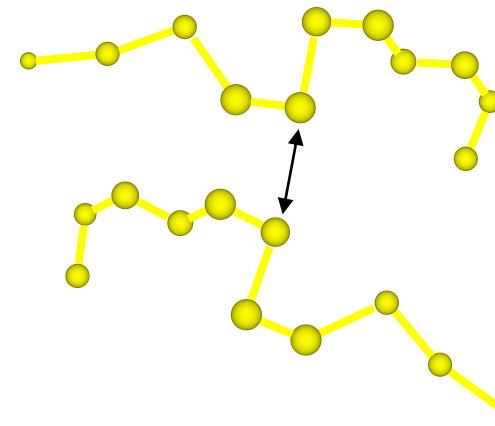
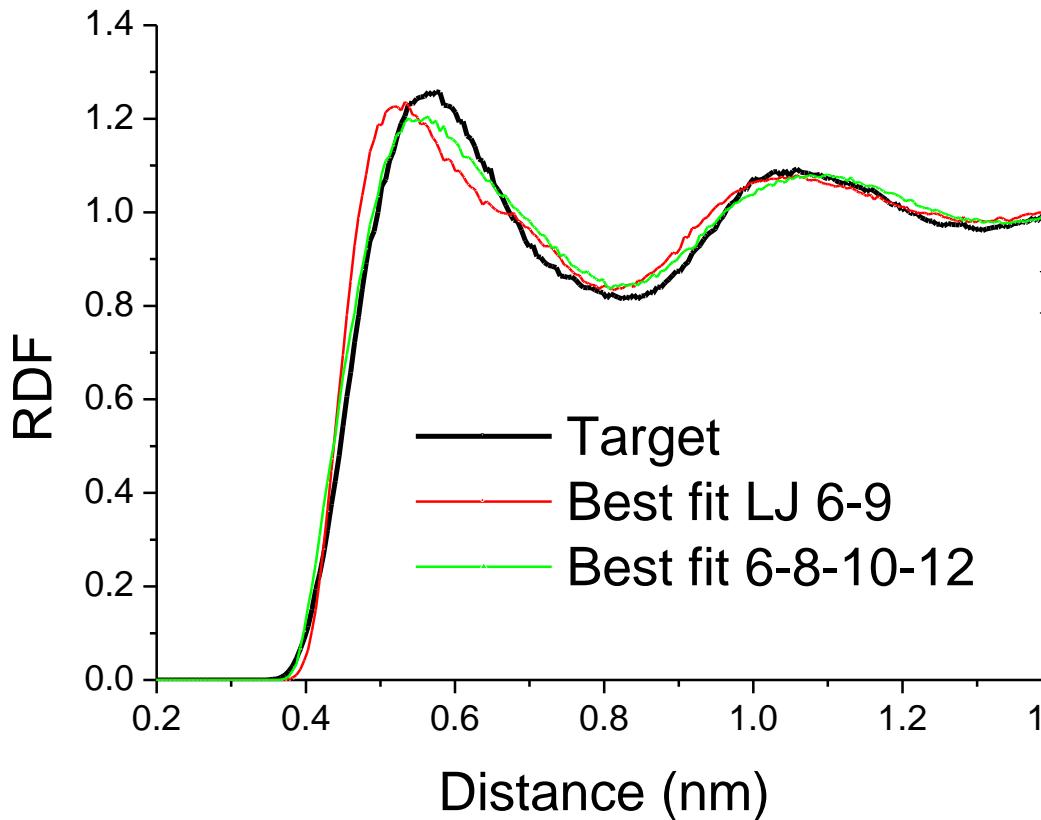
- Stiff degrees of freedom
- Direct Boltzmann inversion

$$-\frac{V(\phi)}{kT} = \ln P(\phi)$$





# Coarse-Graining PVA: Nonbonded



Less easy:

- soft degrees of freedom
- potential of mean force  
     $\neq$  potential energy
- cannot Boltzmann-invert directly

$S = k \cdot \log W$



## Iterative Boltzmann Inversion

- Tabulated numerical potential  $V(r)$
- Starting guess  $V_0(r) \rightarrow \text{RDF}_0(r) \neq \text{RDF}_{\text{target}}(r)$
- Potential correction

$$V_1(r) = V_0(r) + kT \ln \frac{\text{RDF}_0(r)}{\text{RDF}_{\text{target}}(r)}$$

- Iterate

$$V_{n+1}(r) = V_n(r) + kT \ln \frac{\text{RDF}_n(r)}{\text{RDF}_{\text{target}}(r)}$$

until  $V_n(r) \rightarrow \text{RDF}_n(r) = \text{RDF}_{\text{target}}(r)$

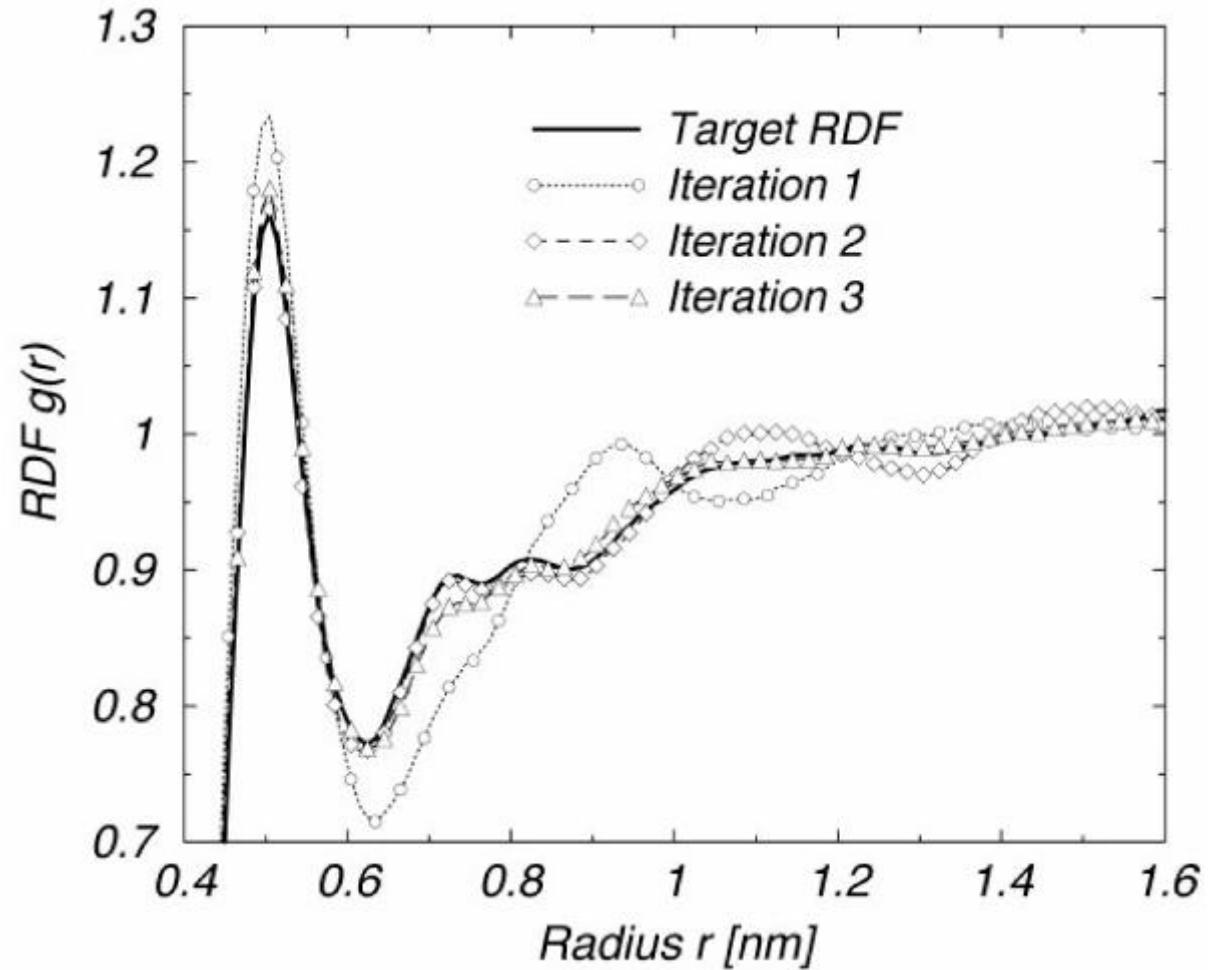
- Converges in few iterations
- Density/pressure correction can be added

D. Reith, H. Meyer, FMP, Comput. Phys. Commun. **148**, 299 (2002).  
35

LUDWIG  
BOLTZMANN  
1844–1906



# Iterative Boltzmann Inversion (2)

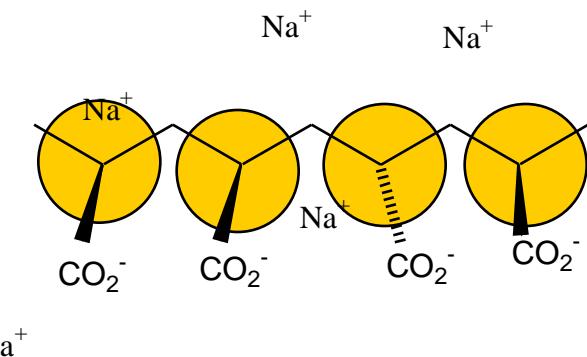


[D. Reith, M. Pütz, F. Müller-Plathe, J. Comp.Chem. **24**, 1624 (2003)]



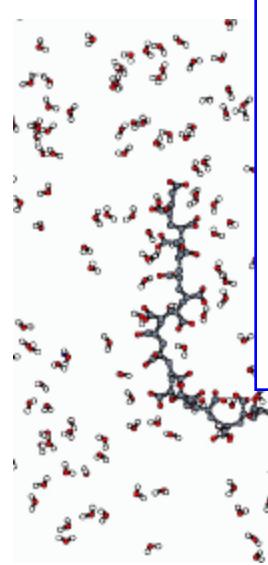
## 2<sup>nd</sup> Example: Poly(acrylic acid)

- Polymer in solution
- 23-mer, Na salt, 2 wt.% in water
- Use:
  - Additive in washing powders, detergents, etc.
  - Flocculation agent in water treatment
  - Disposable nappies

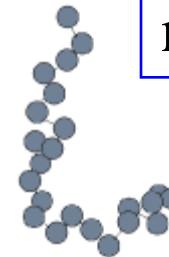




$\text{Na}^+$



Atomistic:  
209 atoms  
3000  $\text{H}_2\text{O}$   
counterions  
~ 10000 atoms



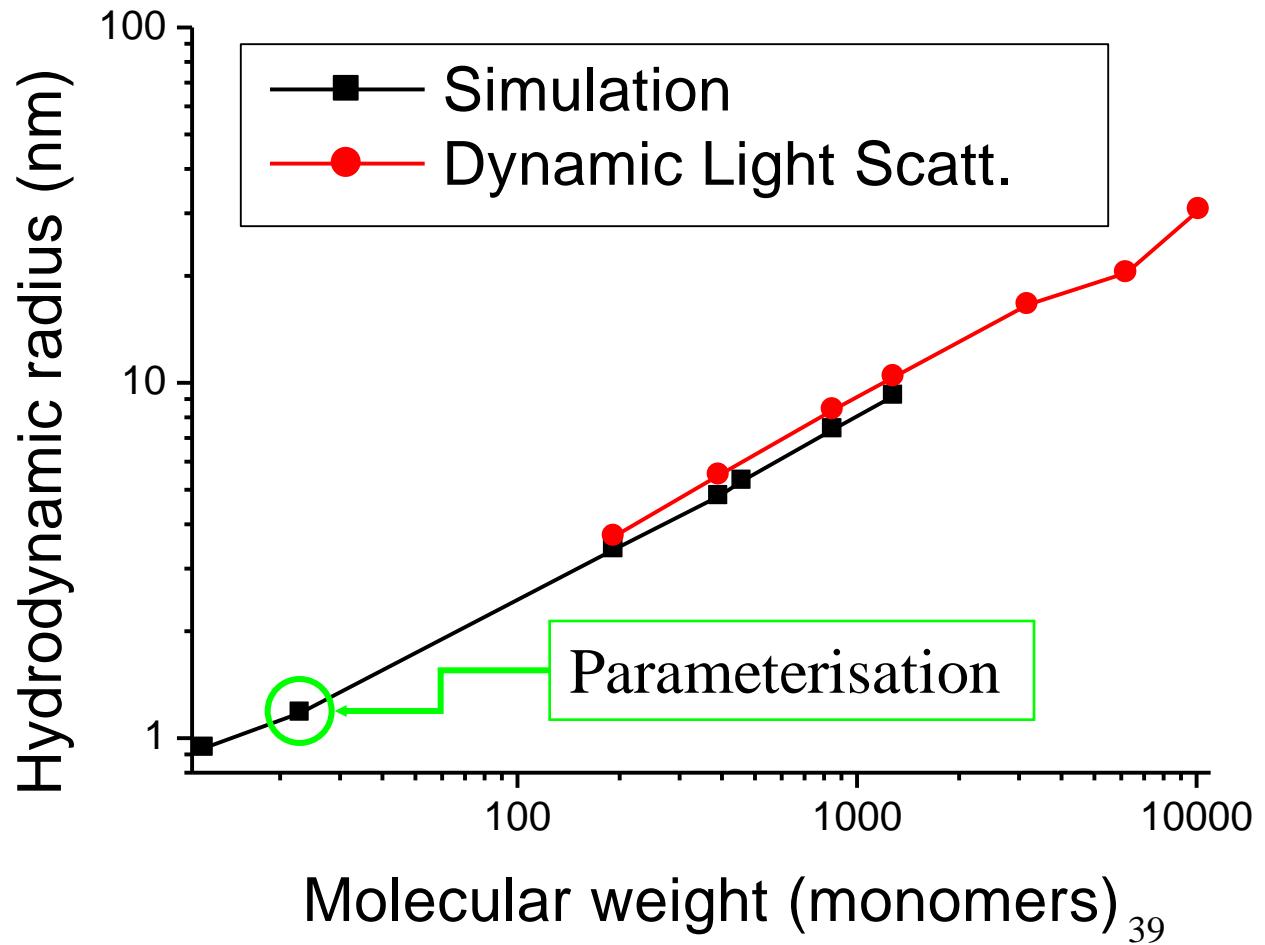
Coarse-grained:  
23 particles  
no  $\text{H}_2\text{O}$

Mapping: coarse-grained superatom  $\leftrightarrow$   
centre of mass of atomistic monomer  
Water: effective interaction, viscous medium



# Comparison with Experiment

$$\frac{1}{R_H} = \left\langle \frac{1}{N^2} \sum_{i,j} \frac{1}{R_{ij}} \right\rangle$$





# Systems coarse-grained

## Polymer melts:

- Poly(vinyl alcohol)  
D. Reith, H. Meyer, FMP, Macromolecules **34**, 2235 (2001).
- Polyisoprene  
*trans*: R. Faller, FMP, Polymer **43**, 621 (2002); *cis*: T. Spyriouni, C. Tzoumanekas, D. Theodorou, FMP, G. Milano, Macromolecules **40**, 3876 (2007).
- Amorphous cellulose  
S. Queyroy, S. Neyertz, D. Brown, FMP, Macromolecules **37**, 7338 (2004).
- Atactic polystyrene  
G. Milano, FMP, J. Phys. Chem. B **109**, 18609 (2005).
- Amorphous polyamide-6,6  
P. Carbone, H. A. Karimi Varzaneh, X.Y. Chen, FMP, J. Chem. Phys. **128**, 064904 (2008)

## Polymer Solutions

- Poly(acrylic acid)  
D. Reith, B. Müller, F. Müller-Plathe, S. Wiegand, J. Chem. Phys. **116**, 9100 (2002).
- Poly(ethylene oxide)  
R. Cordeiro, FMP, in preparation.



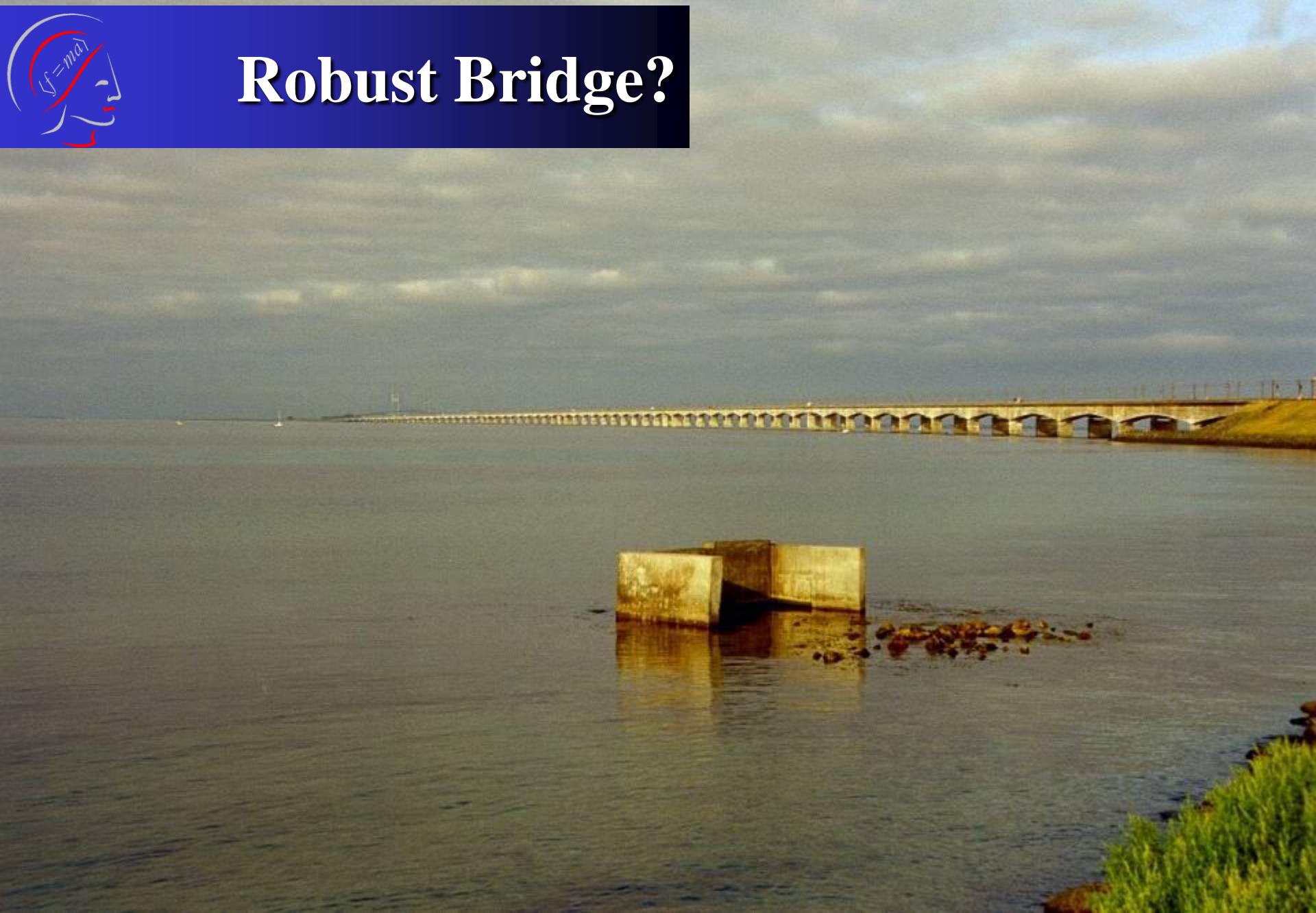
# Systems coarse-grained

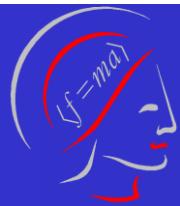
## Non-polymers

- Liquid: Diphenyl carbonate  
H. Meyer, O. Biermann, R. Faller, D. Reith, FMP, J. Chem. Phys. **113**, 6265 (2000).
- Liquid: Ethylbenzene  
H.-J. Qian, P. Carbone, X. Chen, H. A. Karimi-Varzaneh, C. C. Liew, FMP, Macromolecules **41**, 9919 (2008).
- PAMAM dendrimers  
P. Carbone, F. Negri, FMP, Macromolecules **40**, 7044 (2007).
- Ionic liquid: [bmim][PF<sub>6</sub>]  
W. Zhao H.A. Karimi Varzaneh, FMP, in preparation.
- Carbon nanotubes  
G. Illya, H.A. Karimi Varzaneh, FMP, in preparation.
- Phospholipid membranes  
G. Illya, T.J. Müller, H.A. Karimi Varzaneh, FMP, in preparation.

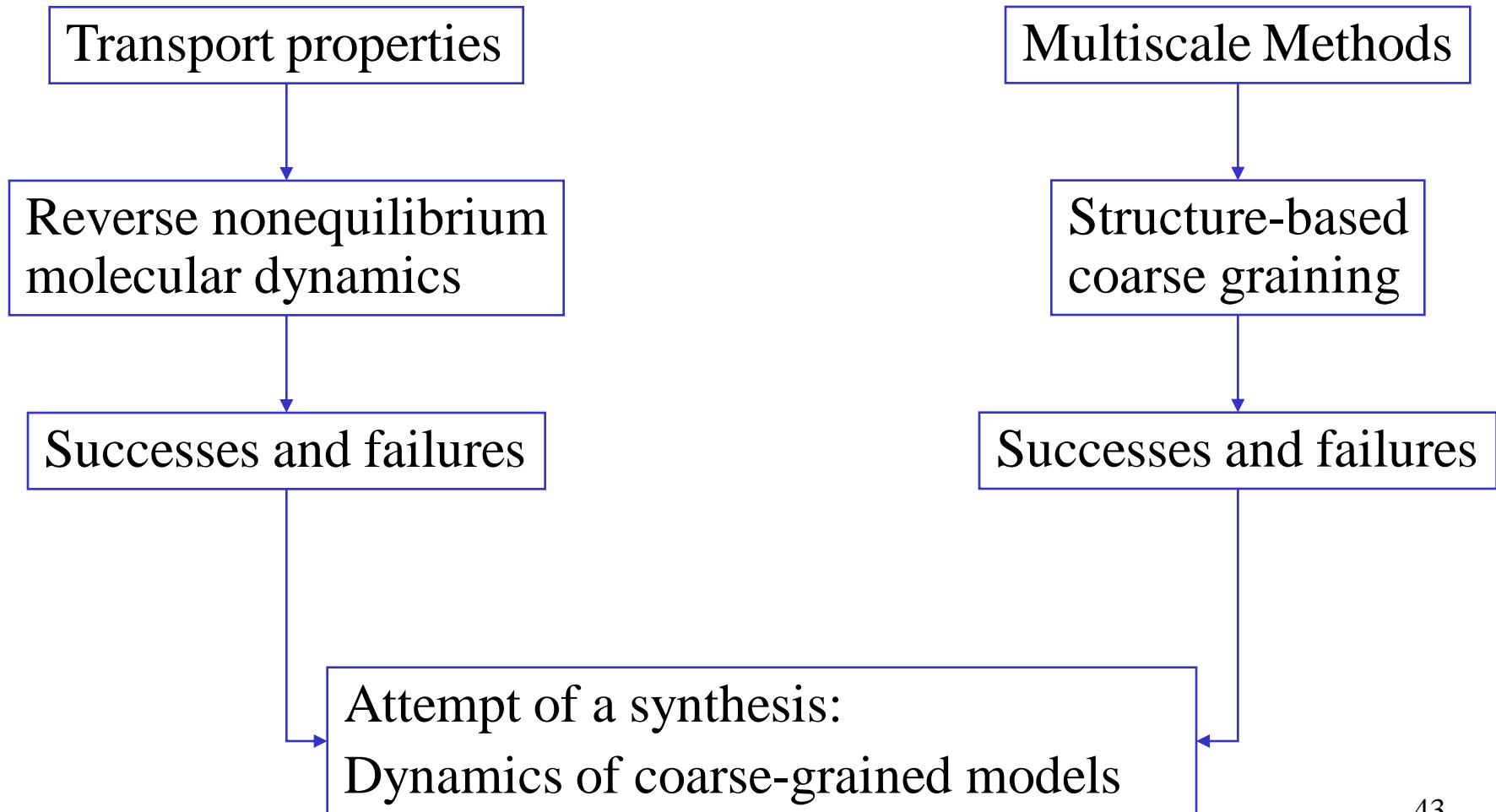


# Robust Bridge?





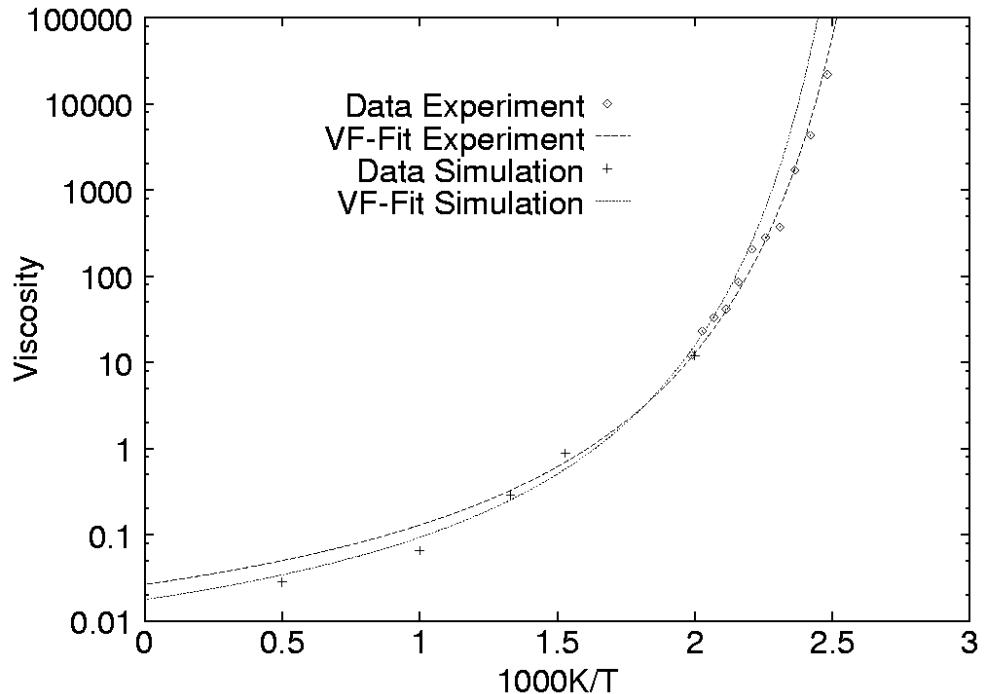
# Multiscaling and Dynamics





# Challenge #1: Dynamics

- One known case:  
polycarbonate
- Temperature dependence  
(Vogel-Fulcher)  
reproduced after  
empirical curve shift



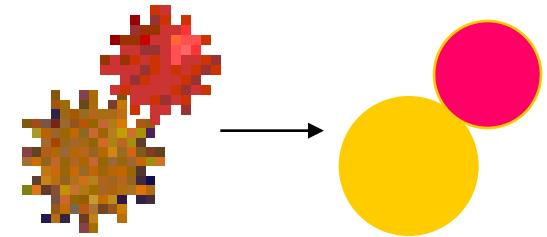
[W. Tschöp, K. Kremer, J. Batoulis, T. Bürger, O. Hahn, Acta Polym. **49**, 61 (1998); ibid. **49**, 75].



# Dynamics

Known feature of coarse-grained potentials

- repulsion (excluded volume) softer than atomistic
- less friction
- faster dynamics: higher  $D$ , lower  $\eta$

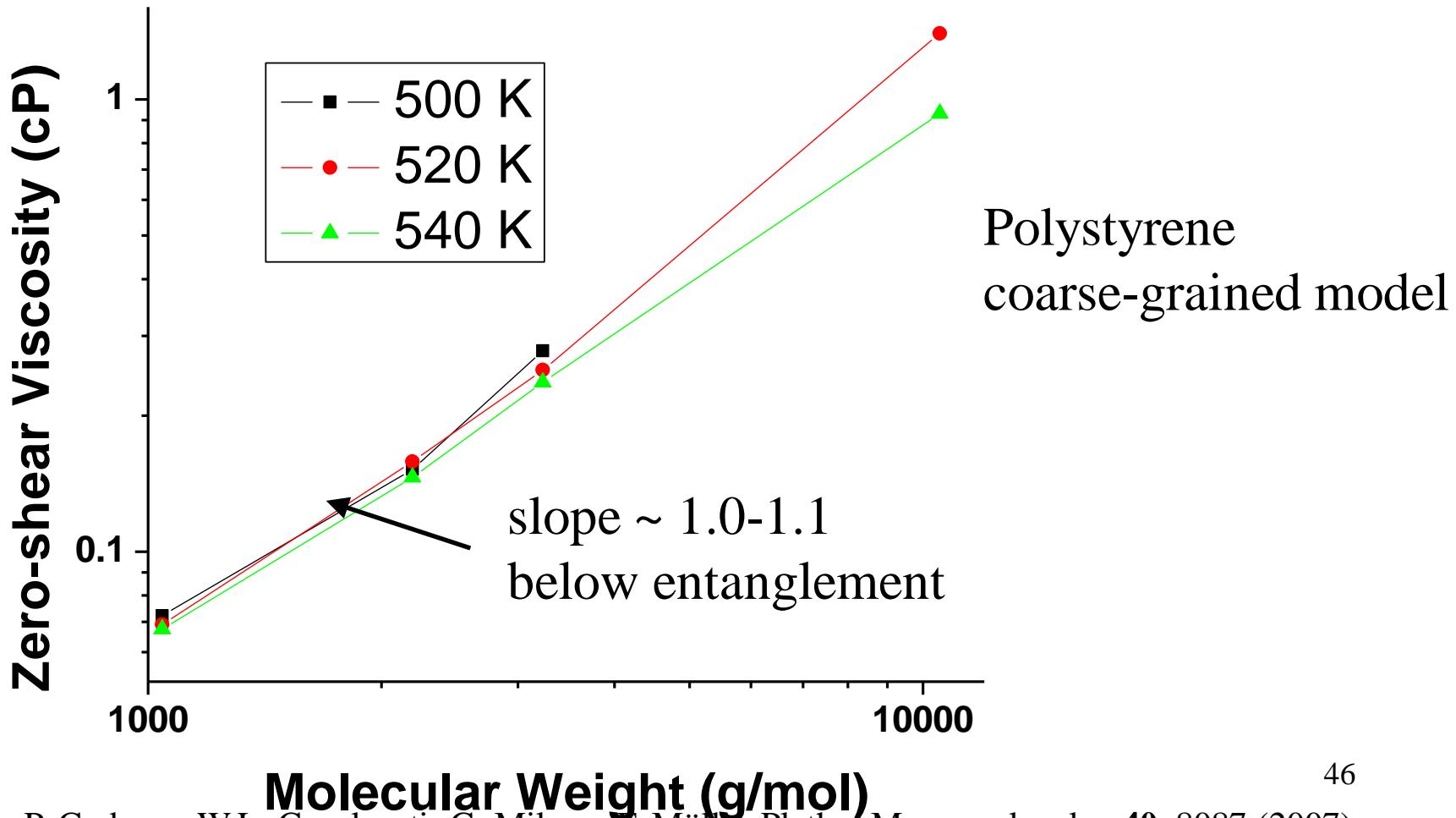


Questions

- Can coarse-grained models be constructed with correct dynamics?
- Is there a constant scale factor?

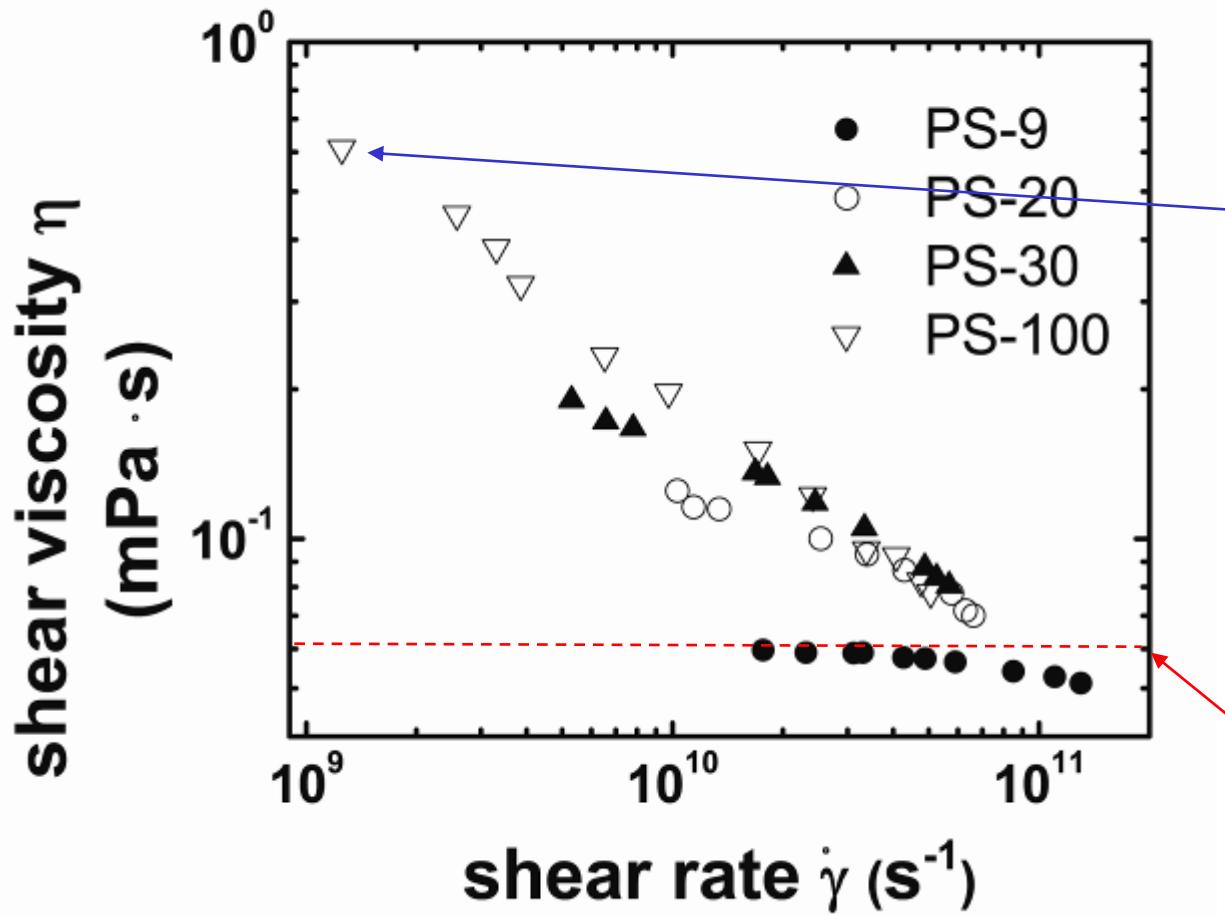


# Viscosity: MW dependence





# Viscosity: shear-rate dependence



100 Monomers:

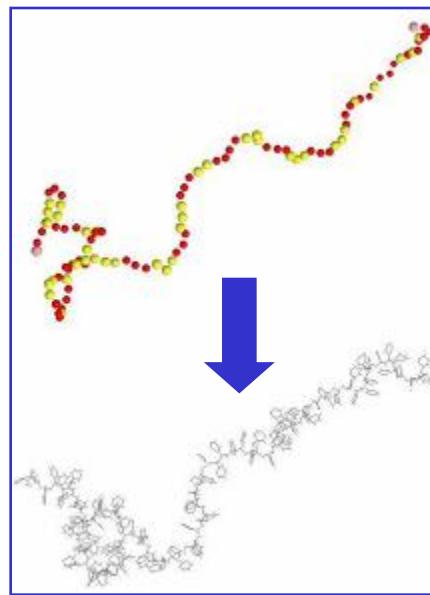
- zero-shear limit not yet reached
- melt is not Newtonian

9 Monomers:  
zero-shear limit



# Chain structure under shear

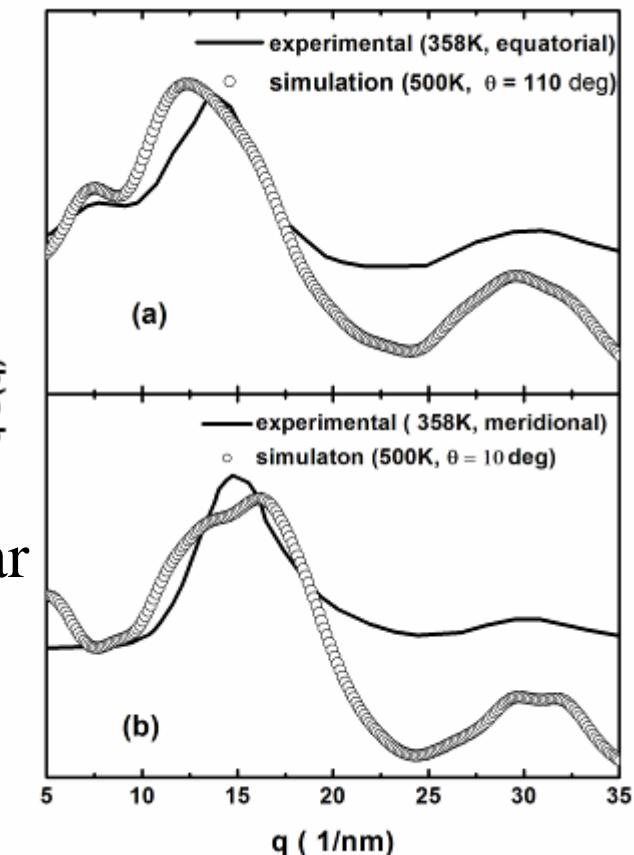
Atomistic structure  
after re-insertion of  
the atoms:  
“backmapping”



parallel to  
stretching

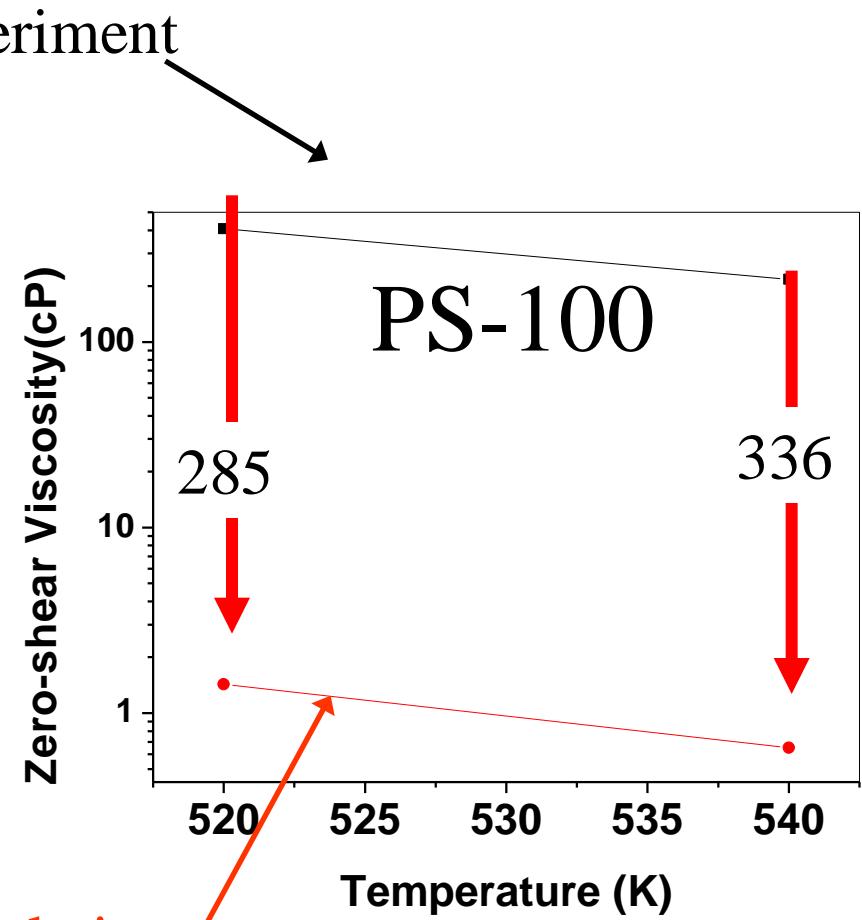
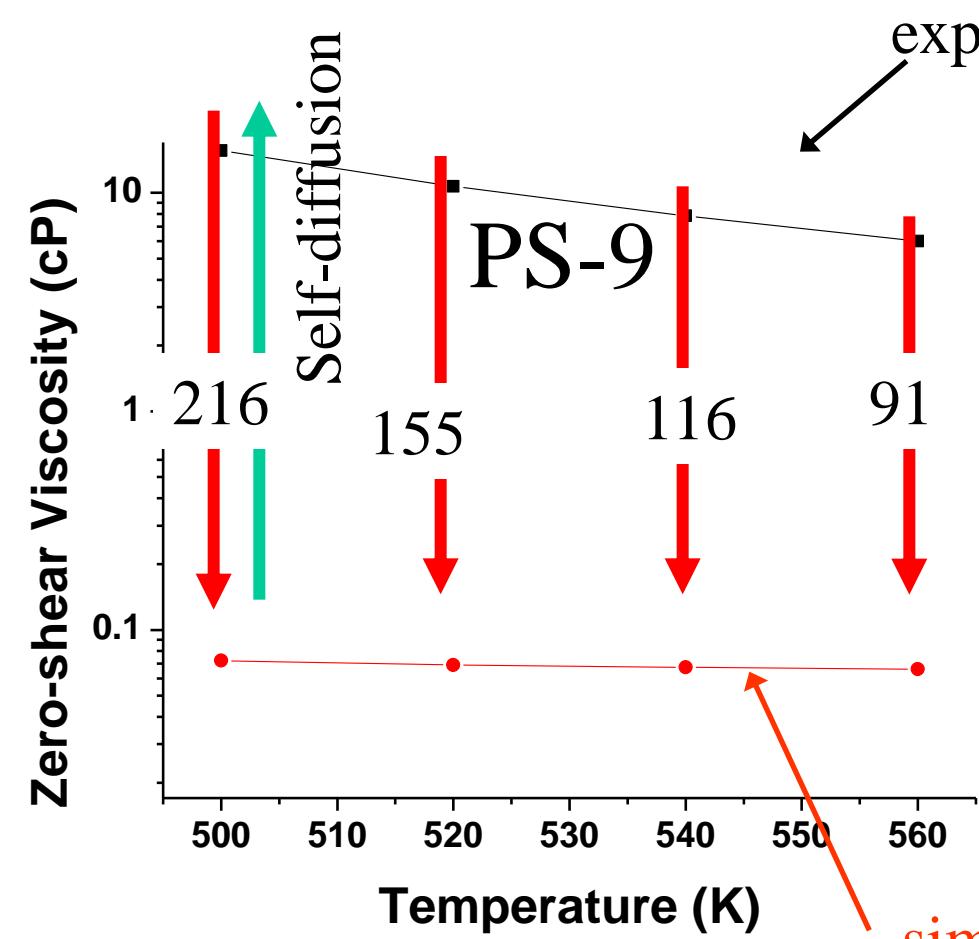
perpendicular  
to stretching

$n$  scattering, PS-30





# Viscosity: Comparison with Experiment



But...

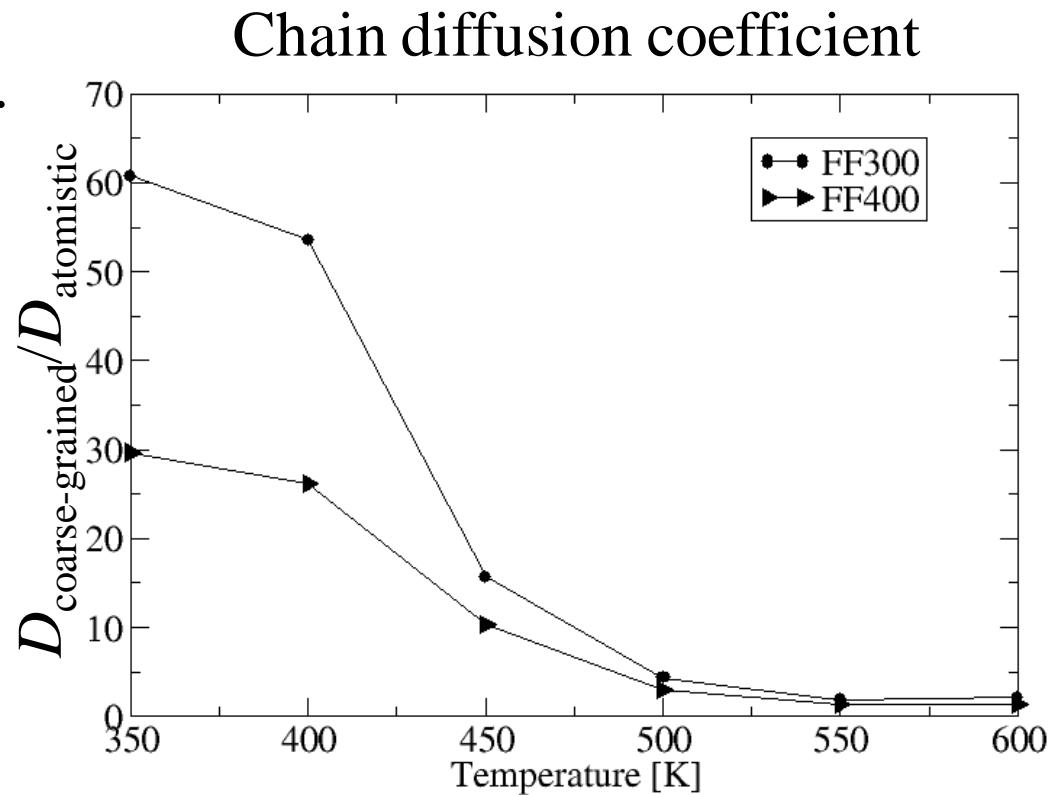
**Polyamide-6,6** above 500 K:  
coarse-grained dynamics  
→ atomistic dynamics

“finer coarse-graining”  
6, 7, or 9 atoms/superatom  
(polystyrene: 18)

Explanation???

Interaction details unimportant at high  $T$ ?

Only excluded volume matters?





# Challenge #1: Dynamics

- RNEMD works technically also for coarse-grained models
- Reproduce qualitative features: MW dependence, microstructure, ...
- Deviation from experiment: 1-2 orders
- Coarse-grained model reproduces simultaneously:
  - structure
  - density
  - ~~absolute dynamics~~
  - ~~temperature dependence~~
- Instead: try the equations of motion



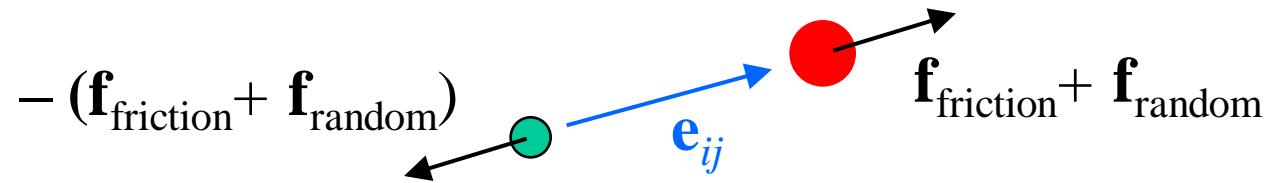
# Dynamics: Cure the Symptoms (2)

[H.-J. Qian, C. C. Liew, FMP, Phys. Chem. Chem. Phys. **11**, 1962 (2009).]

Newtonian dynamics → Dissipative particle dynamics

$$\mathbf{f}_{ij} = \mathbf{f}_{ij} \xrightarrow{\text{interaction}} \underbrace{\zeta \mathbf{e}_{ij} \cdot \hat{\mathbf{e}}_{ij}}_{\text{friction}} + \sigma \underbrace{\frac{\text{rand}}{\Delta t^{1/2}} \hat{\mathbf{e}}_{ij}}_{\text{random}}$$

friction forces and random forces act between *pairs* of atoms



no net momentum is added/lost

DPD is momentum-conserving → can be used with RNEMD



# Dynamics: Cure the Symptoms (3)

[H.-J. Qian, C. C. Liew, FMP, Phys. Chem. Chem. Phys. **11**, 1962 (2009).]

1. Determine time scaling factor  $\phi$  between coarse-grained dynamics and reference dynamics (atomistic or experiment),

e.g.  $\phi = D^{CG,MD} / D^{ref,MD}$

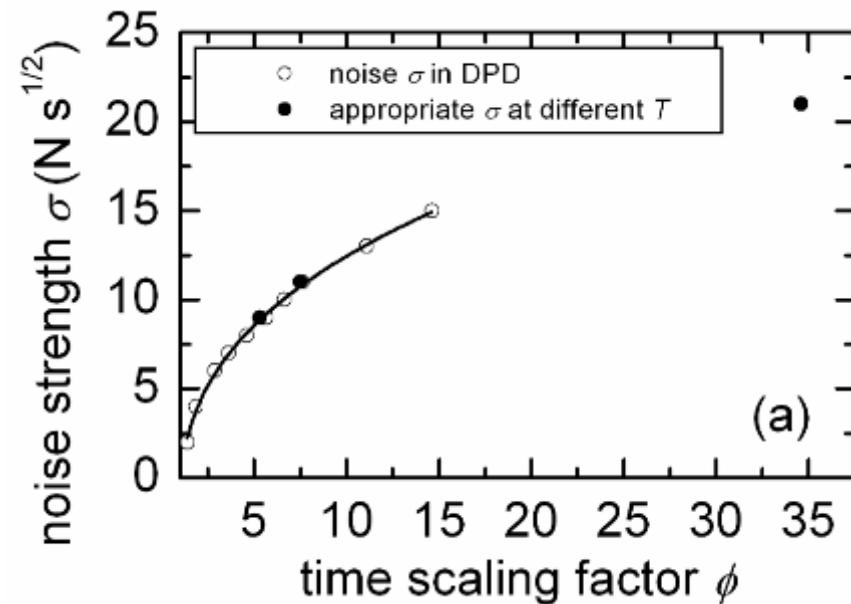
2. Empirical rule for optimum DPD noise strength

$$\sigma [N \cdot s^{1/2}] = 7.2(\phi - 0.9)^{0.35} + 3.4$$

Developed for ethyl benzene (298 K);  
Holds for:

- ethyl benzene (238 K – 380 K)
- polystyrene melts
- ionic liquid [bmim] PF<sub>6</sub>

Analogous rule for Lowe-Anderson dynamics





# Not a Bridge yet!



## Reverse non-equilibrium MD

- Works robustly for thermal conductivity and shear viscosity
- Is as accurate as the force field
- Cannot beat the inherent relaxation times
- Proliferation of RNEMD

## Multiscale simulation

- Iterative Boltzmann Inversion works robustly
- Structure-based coarse graining for many systems
- proliferation of IBI

## Coarse-grained dynamics

- Cannot fix the potential
- EOM with friction works phenomenologically
- Needed: friction contribution of removed degrees of freedom

# Thanks!



< 2002



2002-2005



>2005



BASF, Rhodia, BMBF, EU (Marie-Curie), DFG,  
Max-Planck-Gesellschaft (IMPRS-PMS), Humboldt-  
Stiftung, Fonds der Chemischen Industrie