Parametric Approximation of Geometric Evolution Equations

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ABSTRACT

Geometric flows, in which hypersurfaces move such that an energy, involving surface and bending terms, decreases appear in many situations in the natural sciences and in geometry. Classic examples are mean curvature, surface diffusion and Willmore flows. Computational methods to approximate such flows are based on one of three approaches (i) parametric methods, (ii) phase field methods or (iii) level set methods. The first tracks the hypersurface, whilst the other two implicitly capture the hypersurface. A key problem with the first approach, apart from the fact that it is does not naturally deal with changes of topology, is that in most cases the mesh has to be redistributed after every few time steps to avoid coalescence of mesh points.

In this talk we present a variational formulation of the parametric approach, which leads to an unconditionally stable, fully discrete finite element approximation. In addition, the scheme has very good properties with respect to the distribution of mesh points, and if applicable volume conservation. We illustrate this for (anisotropic) mean curvature and (anisotropic) surface diffusion flows of closed curves in $\mathbb{R}^2$ and closed hypersurfaces in $\mathbb{R}^3$. We extend these flows to curve networks in $\mathbb{R}^2$, and surface clusters in $\mathbb{R}^3$. Here the triple junction conditions, that have to hold where three curves/surfaces meet at a point/line, are naturally approximated in the discretization of our variational formulation. Finally, we extend these approximations to the case when the hypersurface motion depends also on an underlying background equation, such as the Stefan problem with kinetic undercooling and the Mullins–Sekerka/Hele–Shaw flow with surface tension.