

Freely Cooling Granular Gases

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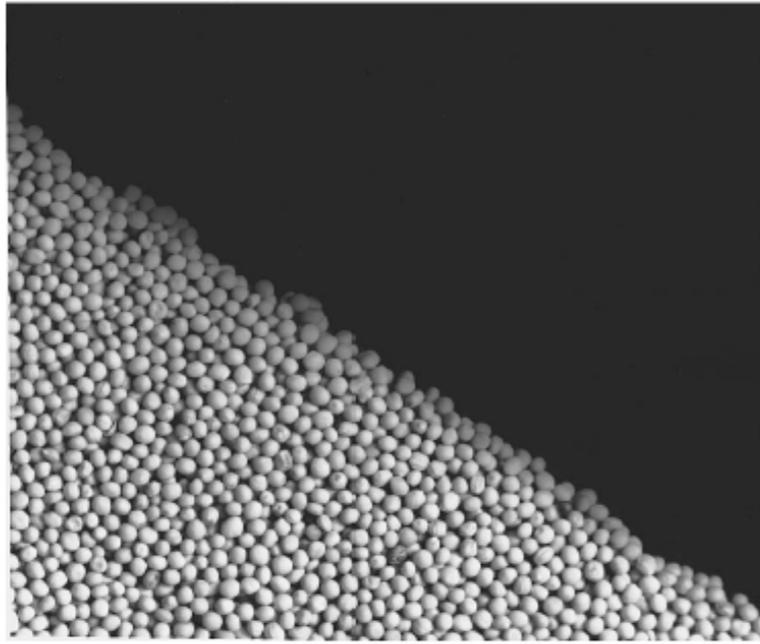
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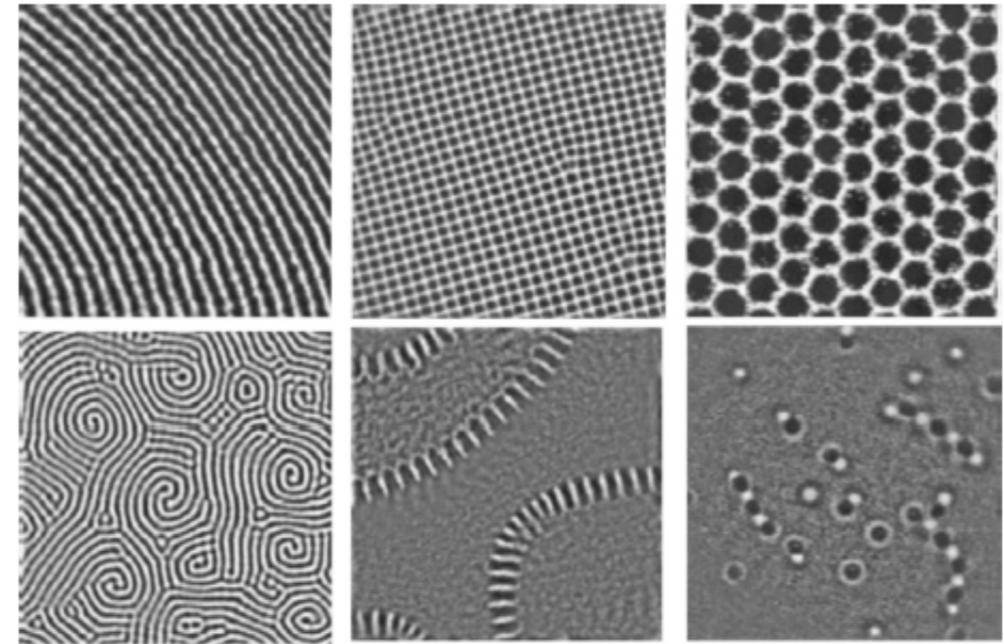
Outline

- Granular systems
- The model
 - Definition
 - Some earlier results
- New results
- Conclusions

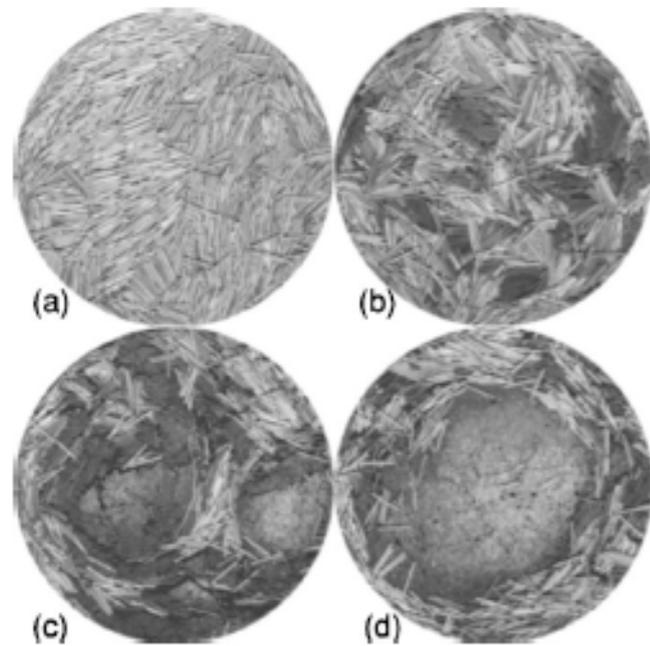
Granular Matter



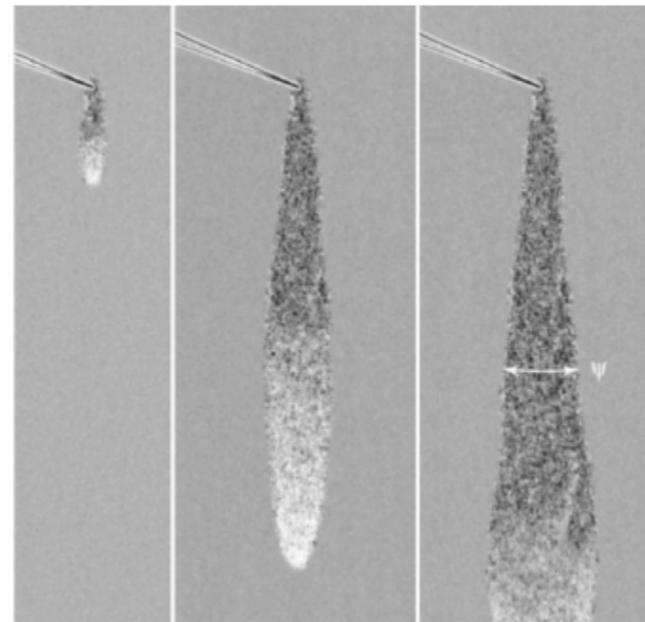
Jaeger et al, 1996



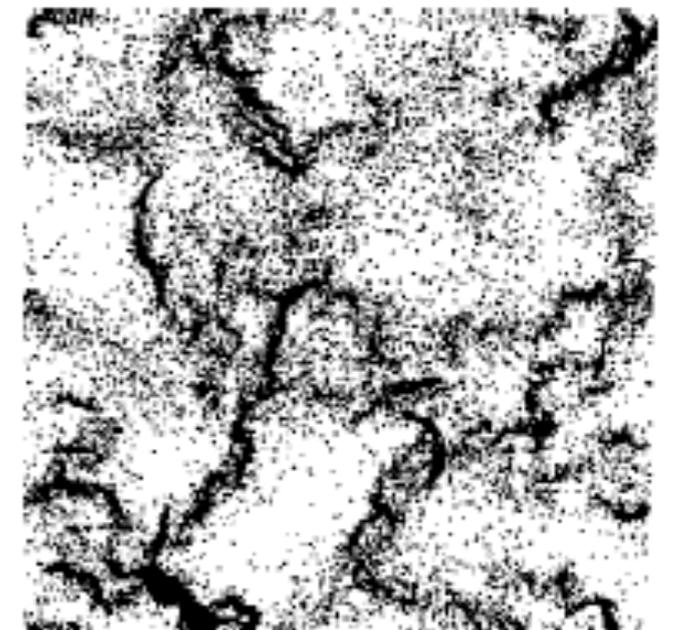
Aranson et al, 2006



Blair et al, 2003



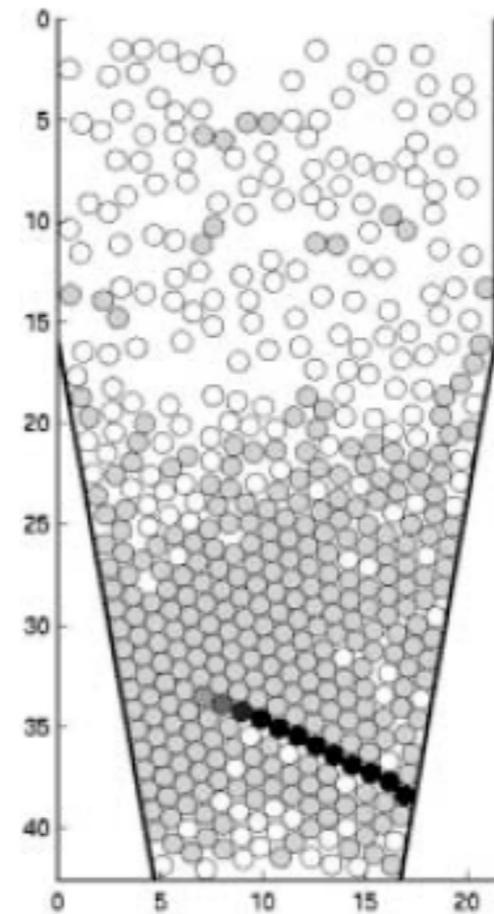
Daerr et al, 1999



Goldhirsch et al, 1993

Freely cooling (motivation)

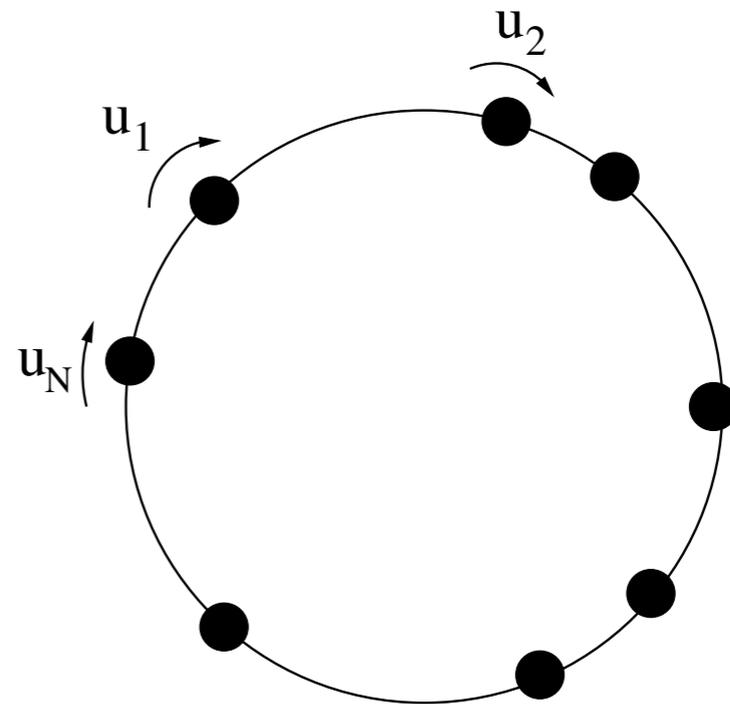
- Isolates effect of dissipation
- Direct experiments
- As parts of larger driven systems
- Interacting particle systems



Ferguson et al, 2004

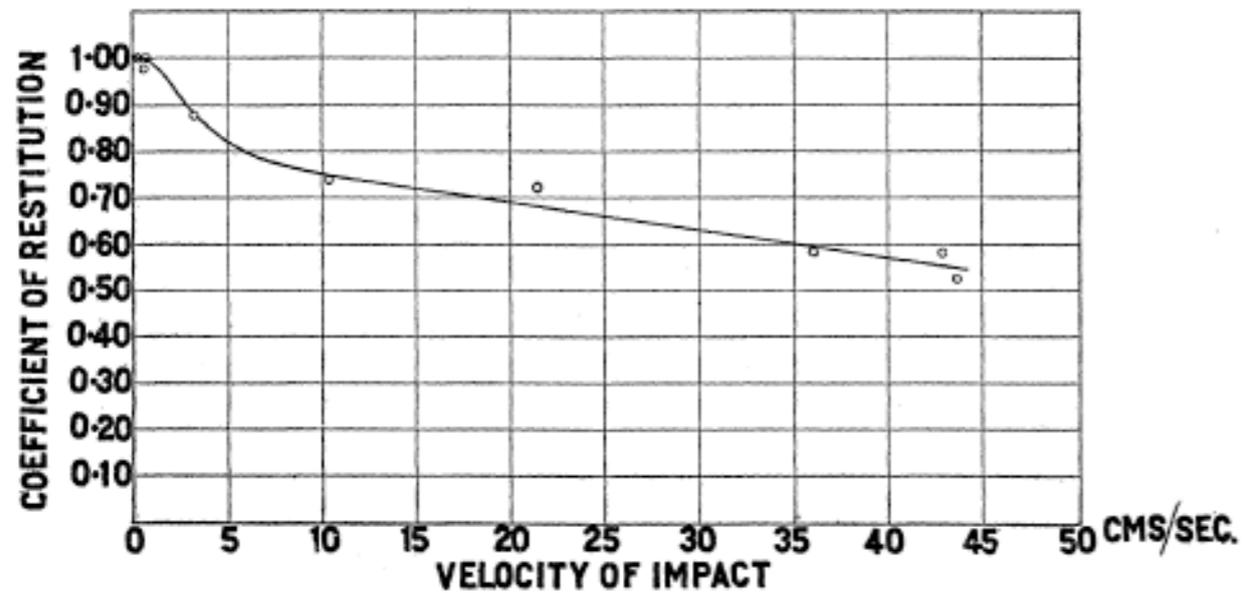
Model

- Particles on a ring
- Initial conditions
- Momentum conserving
- Energy dissipating

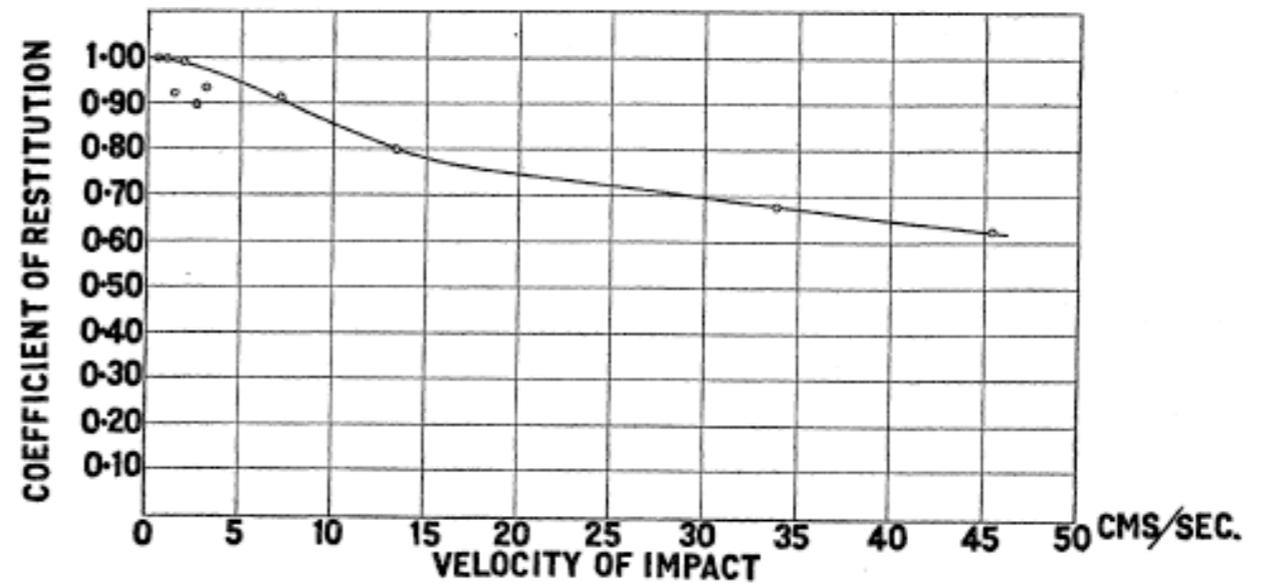


$$u'_1 = u_1 \left(\frac{1-r}{2} \right) + u_2 \left(\frac{1+r}{2} \right)$$
$$u'_2 = u_2 \left(\frac{1-r}{2} \right) + u_1 \left(\frac{1+r}{2} \right)$$

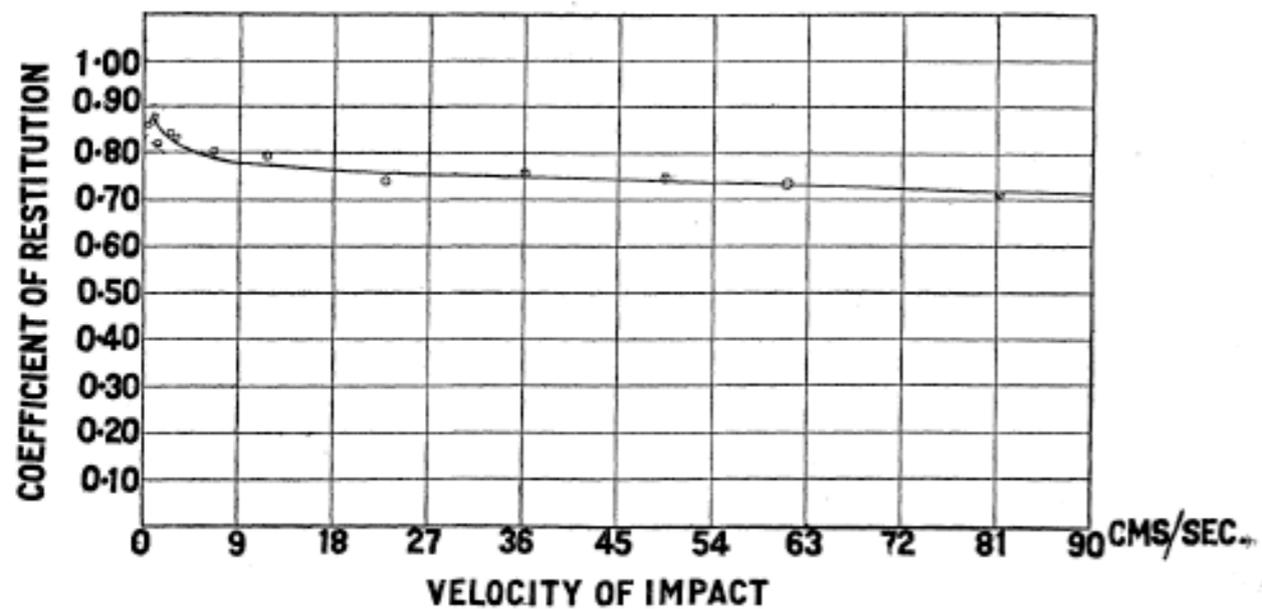
Coefficient of Restitution: r



Brass



Aluminium



Marble

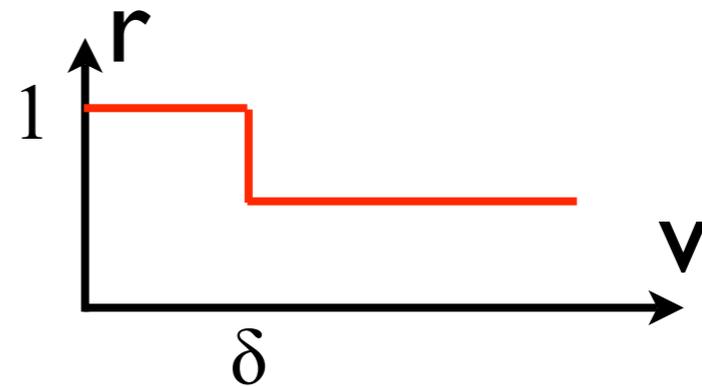
$$r \rightarrow 1 \text{ when } v \rightarrow 0$$
$$r \rightarrow r_0 \text{ when } v \rightarrow \infty$$

C.V. Raman, 1918

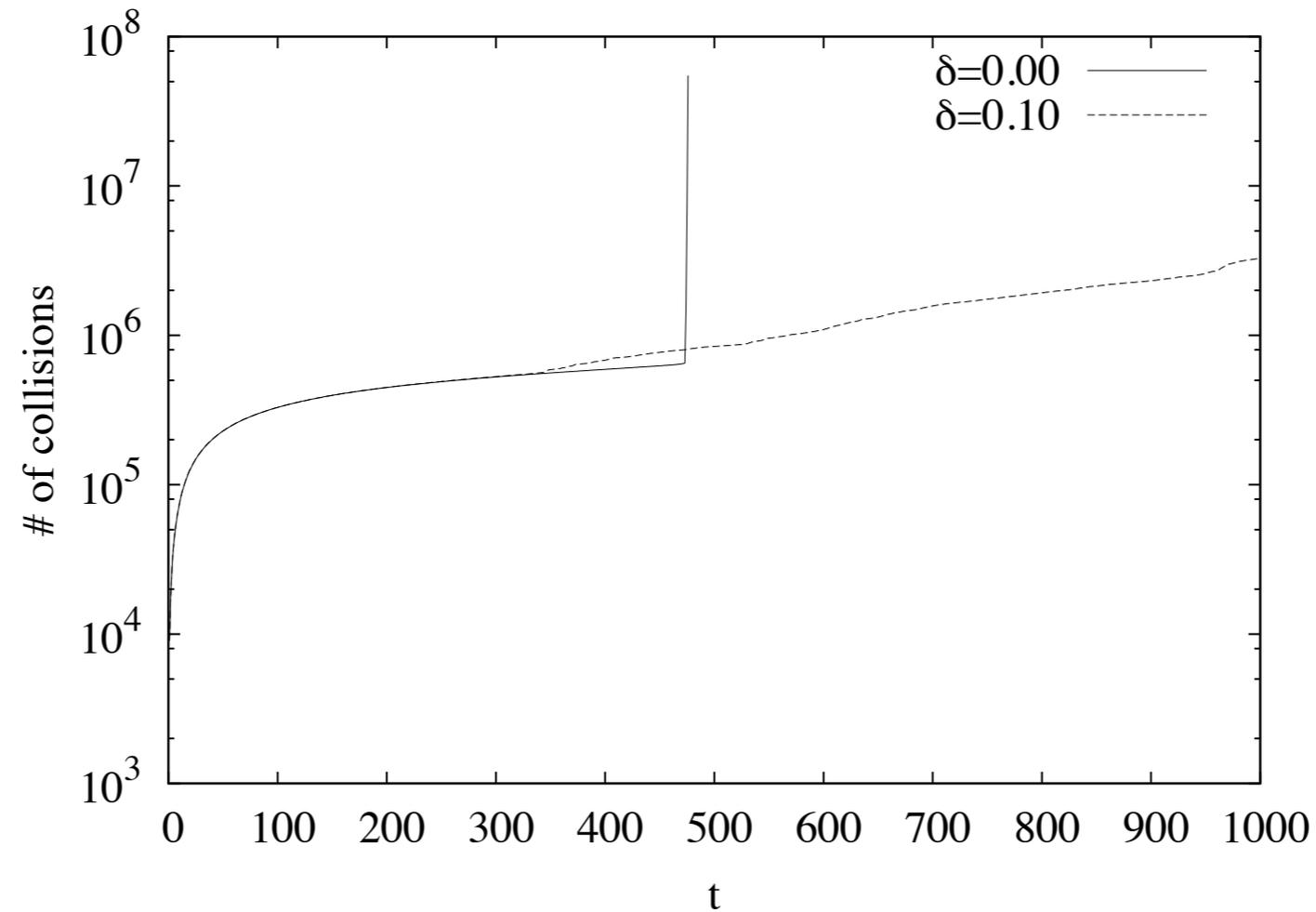
Coefficient of Restitution

$$r(v) = r_0 + (1 - r_0) \exp[-(v/\delta)^\sigma]$$

- A new velocity scale δ
- An exponent σ
- $\sigma=1/5$ [viscoelastic theory]
- σ takes many values [experiment]
- $\sigma=\infty$ convenient



Do we need δ ?



Inelastic collapse

What role does δ play?

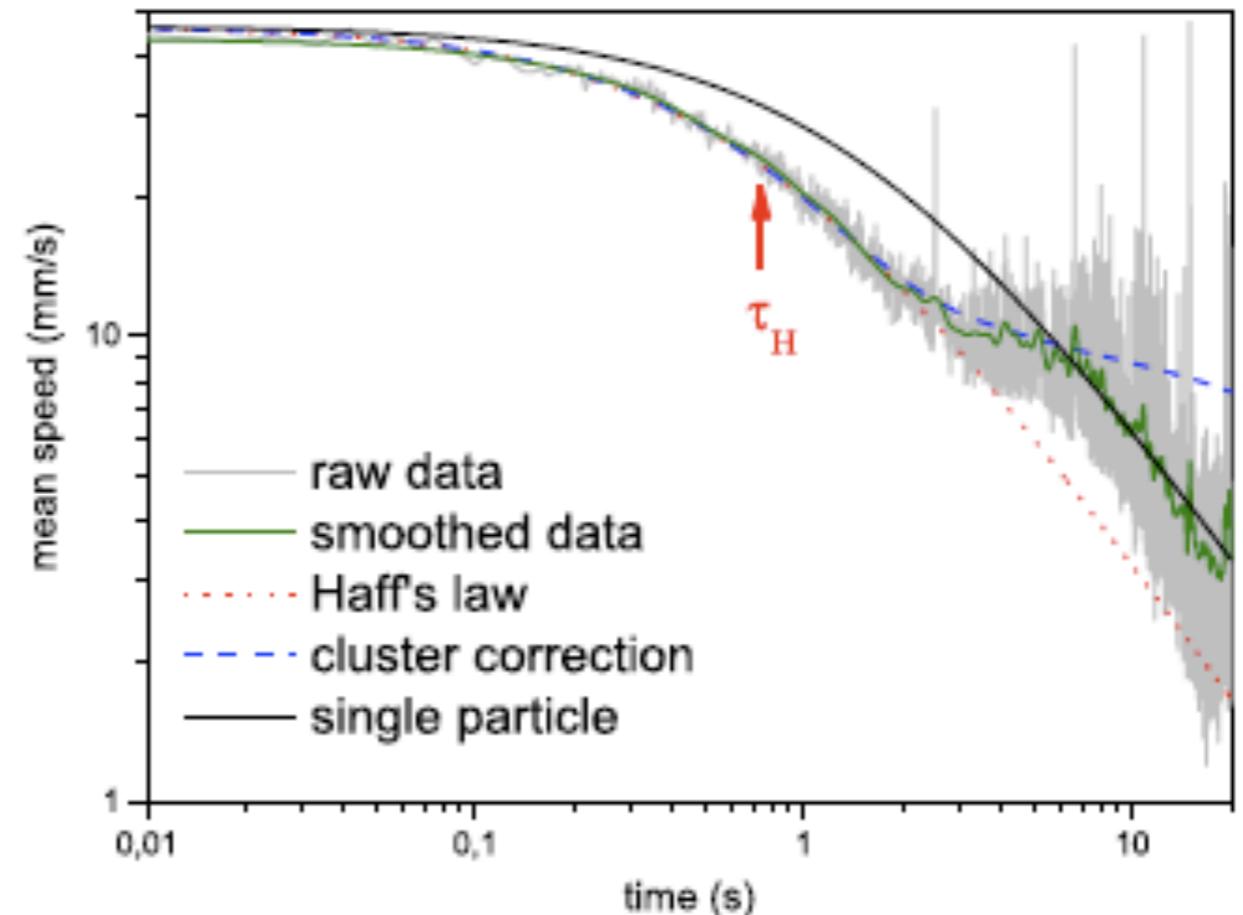
Homogeneous Cooling

$$\frac{dE}{dt} = -\frac{\Delta E}{\tau}$$

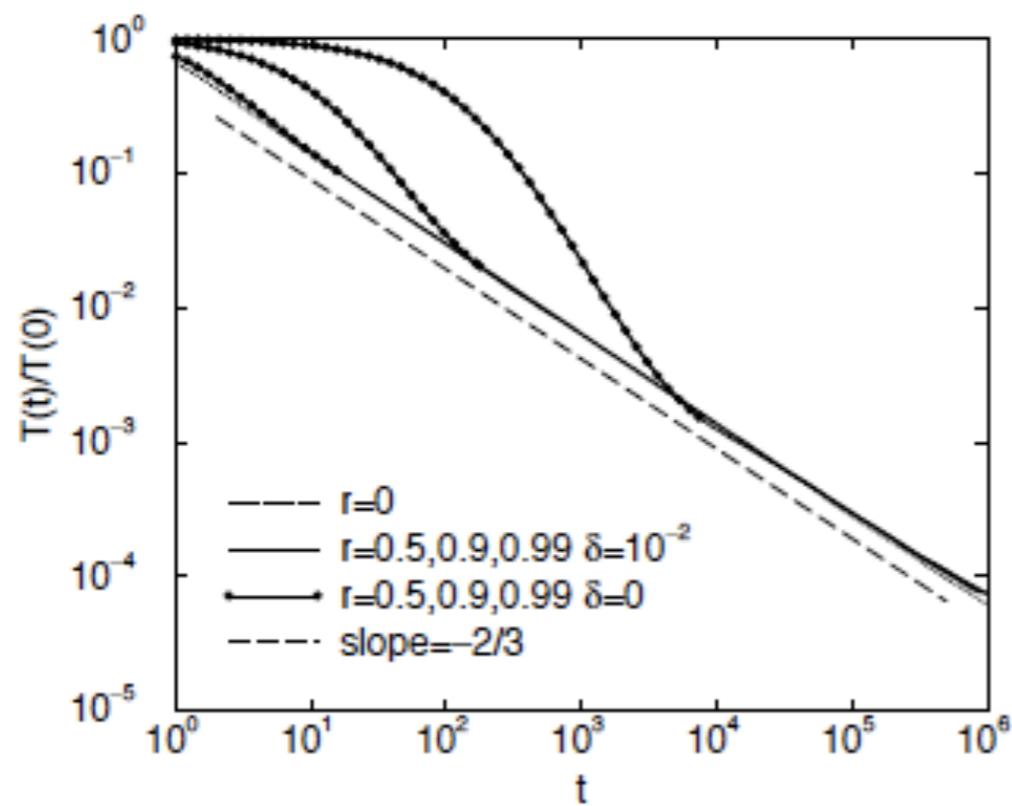
$$\frac{dE}{dt} \sim \frac{(1 - r^2)E}{a/\sqrt{E}}$$

$$E \sim \frac{1}{(1 - r^2)t^2 + c^2}$$

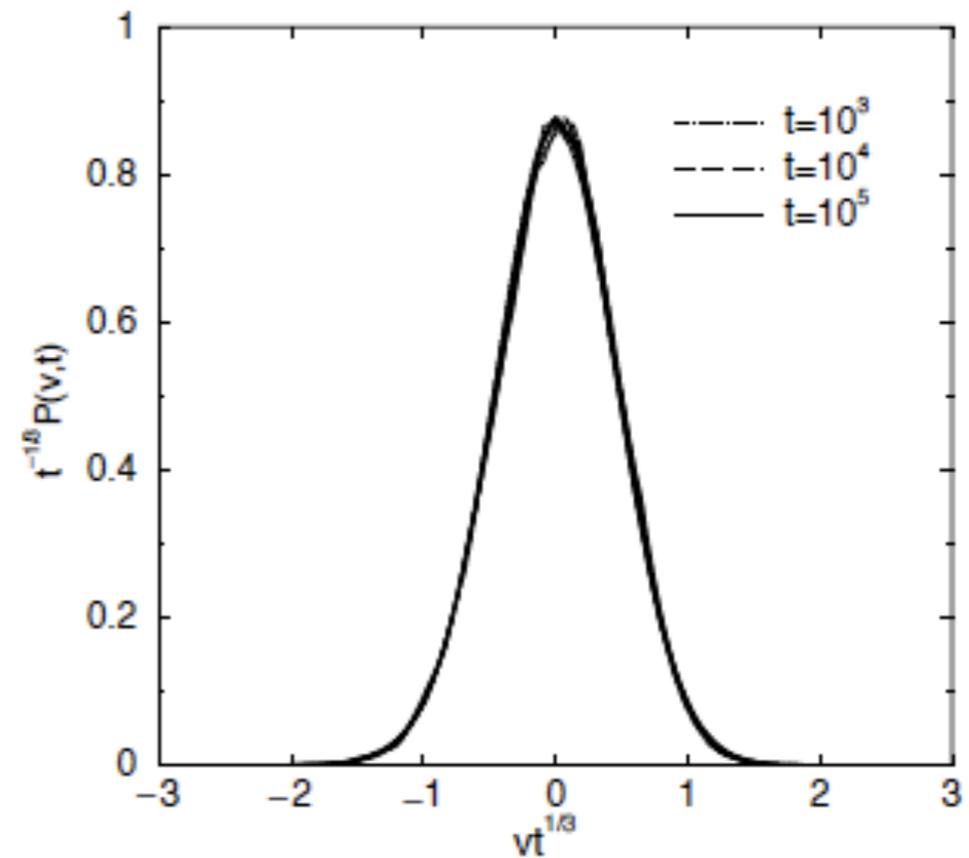
Haff's law Haff, 1982



Maaß et al, 2008

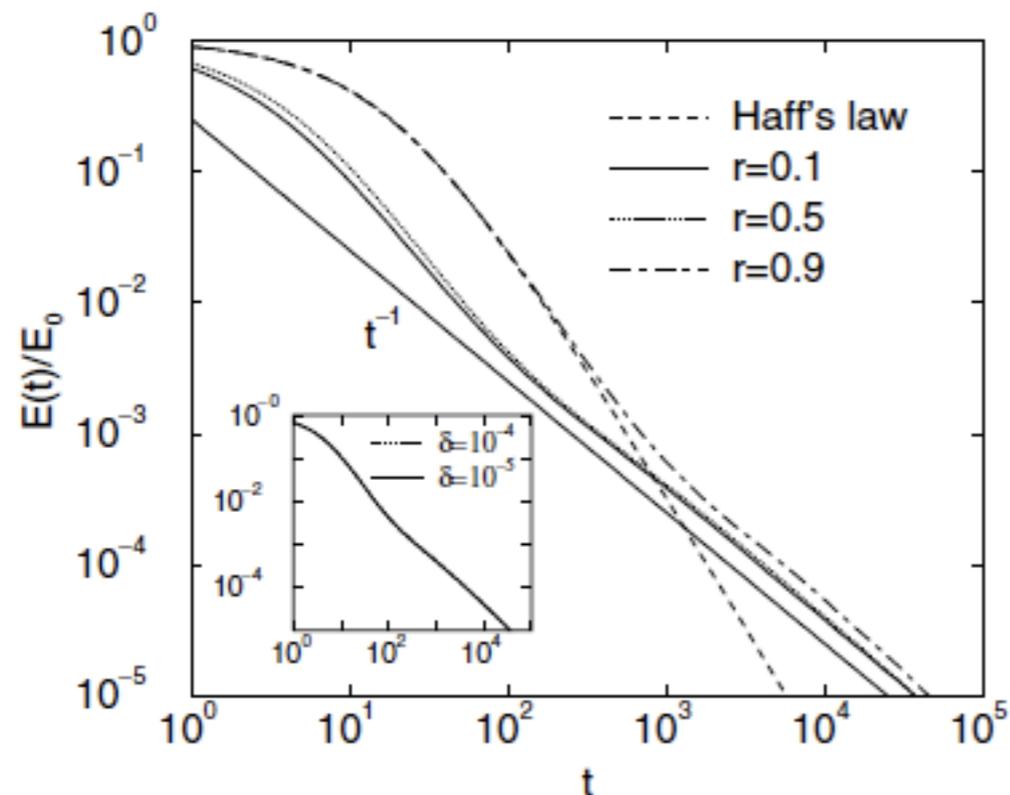


1d-energy



1d- $P(v,t)$

Ben-Naim et al, 1999, Nie et al, 2002

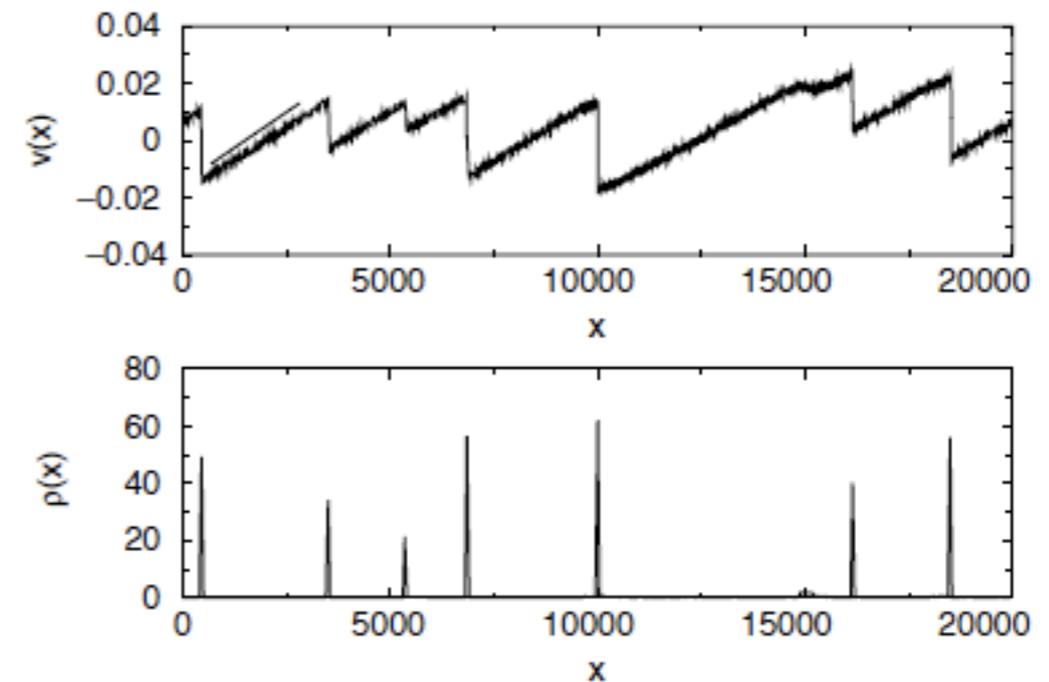


2d-energy

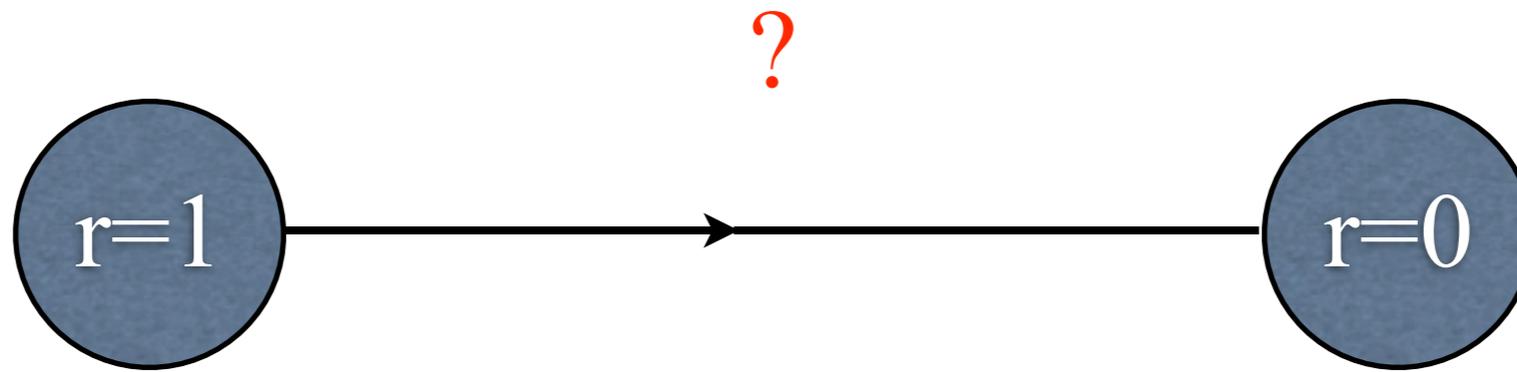
$r < 1$: As $t \rightarrow \infty$, $r \rightarrow 0$

Sticky Gas ($r=0$)

- Exactly solvable [Frachebourg, 1999](#)
- Mapping to Burgers equation
[Kida, 1979](#)
- $E(t) \sim t^{-2/3}, t \gg t_c$

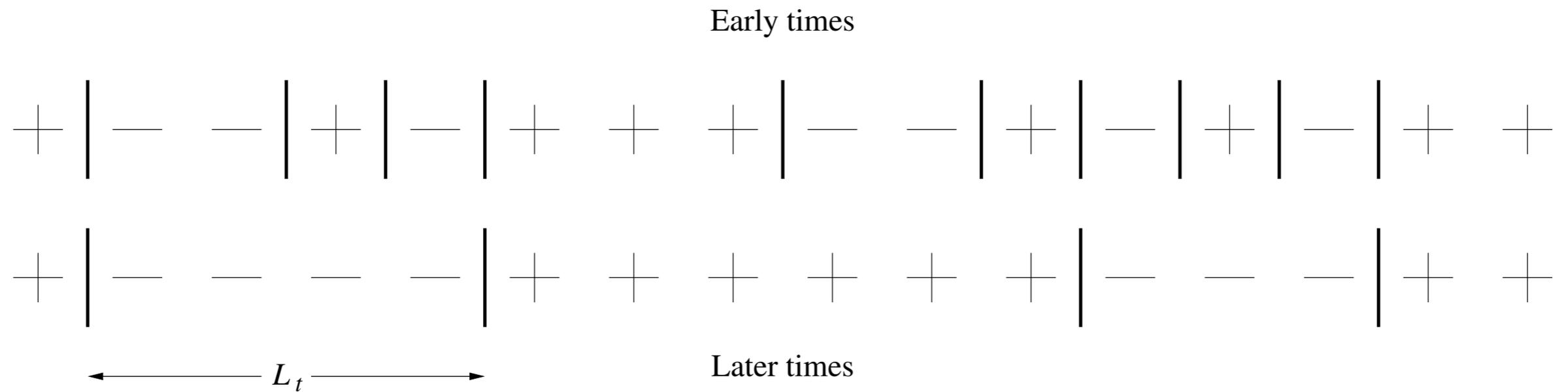


[Ben-Naim et al, 1999](#)



Spatial correlations?

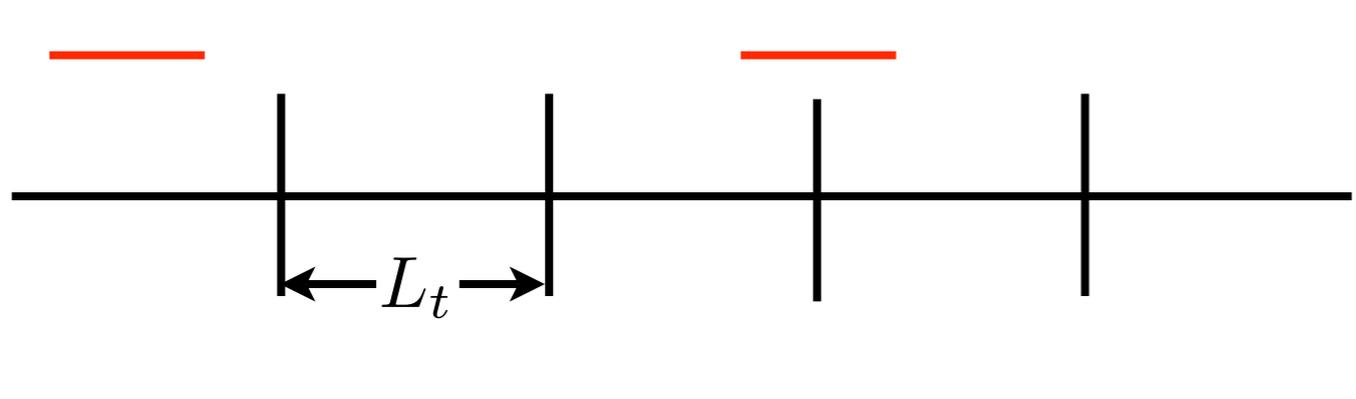
Phase Ordering



$$C(r, t) = \langle s_i(t) s_{i+r}(t) \rangle = f(r/L(t))$$

Phase Ordering

Well defined domain sizes



The diagram shows a horizontal line representing a 1D lattice. Two vertical lines are drawn on the line, with a double-headed arrow between them labeled L_t . Above the line, there are two red horizontal bars representing domains. The first bar is on the left, and the second bar is on the right, with a gap between them. The distance between the right edge of the first bar and the left edge of the second bar is labeled r .

$$C(r, t) \approx \frac{1 \times (L_t - r) - 1 \times r}{L_t}$$
$$= 1 - \frac{2r}{L_t}$$

$$C(r, t) = \langle s_i(t) s_{i+r}(t) \rangle = f(r/L(t))$$

$$f(x) \sim b(1 - c|x|) \text{ for } x \ll 1$$

$$S(k, t) \sim \frac{L^d}{(Lk)^{d+1}} \text{ for } kL \gg 1$$

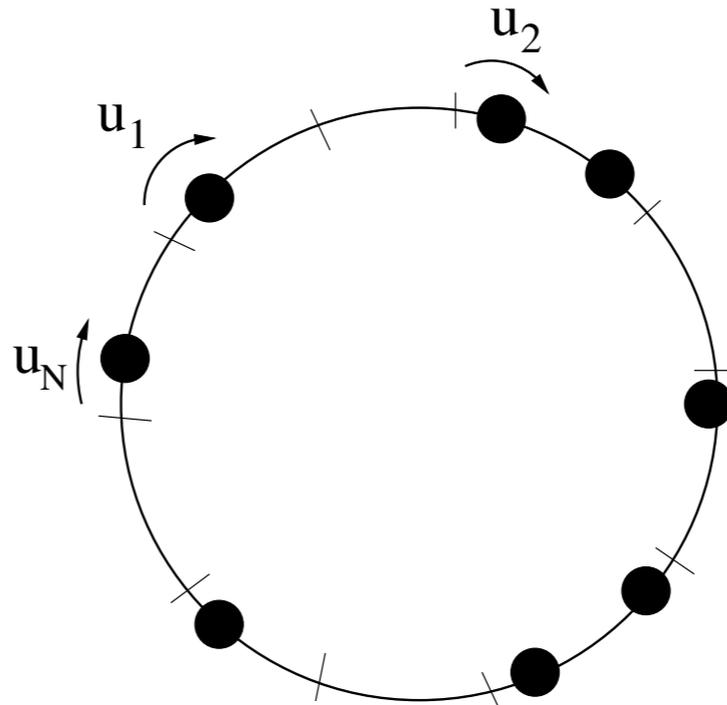
Porod law

$$S(k, t) \sim \frac{L^d}{(Lk)^\alpha}, \alpha \neq d + 1$$

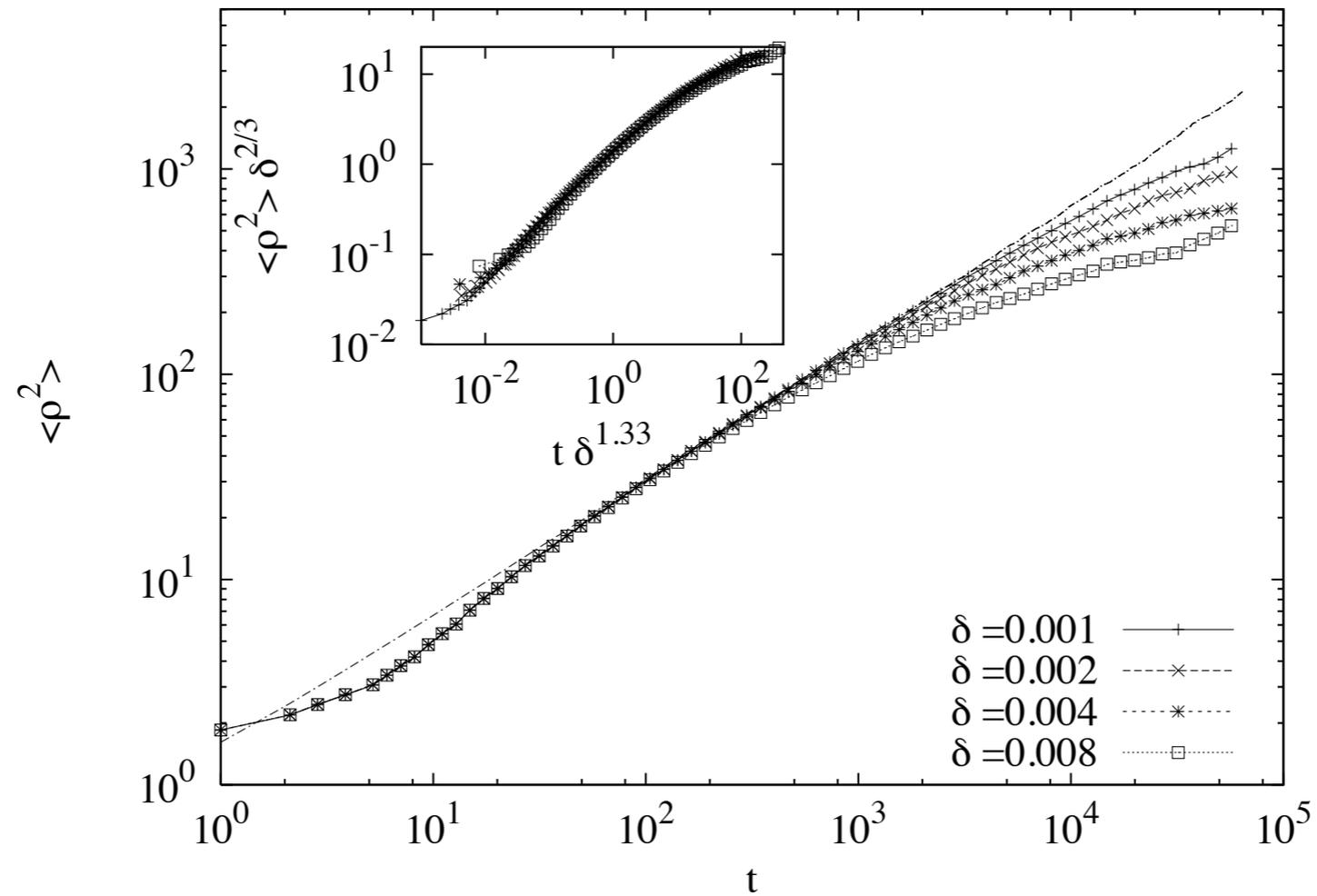
Non-Porod

Coarse Graining

- Density
- Velocity

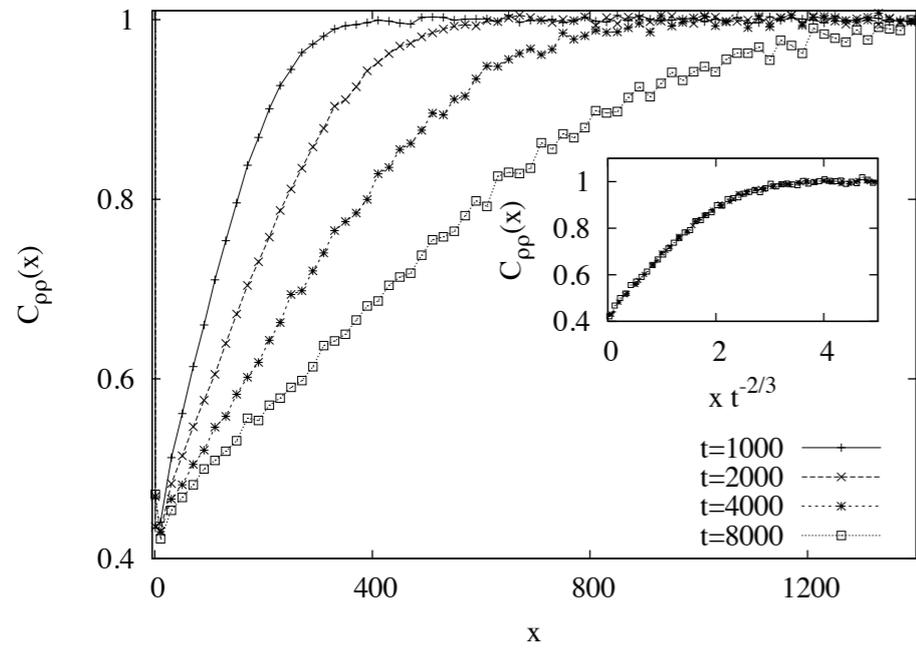


A new time scale

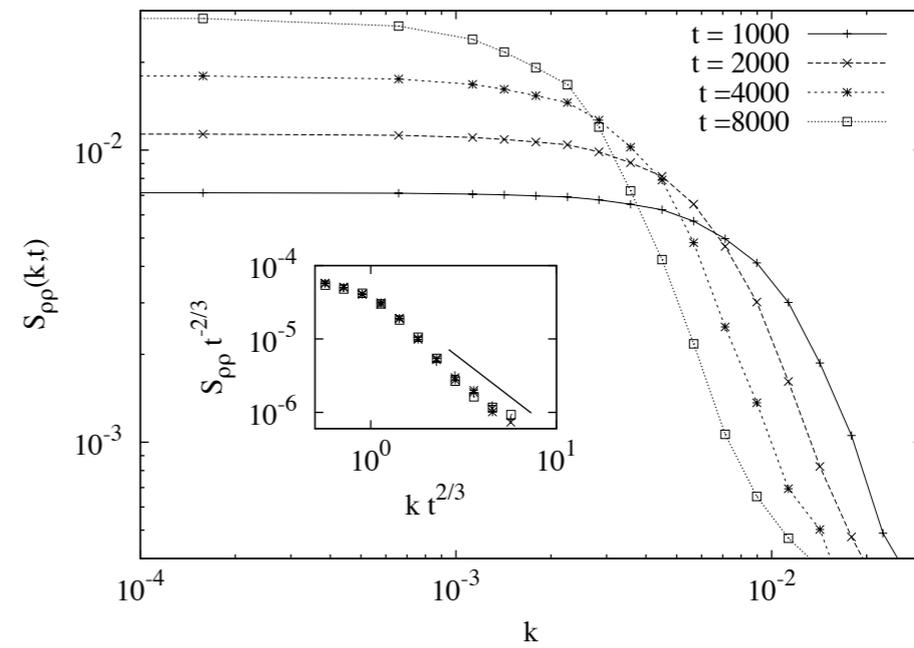


$$t_c \sim \delta^{-1.33}$$

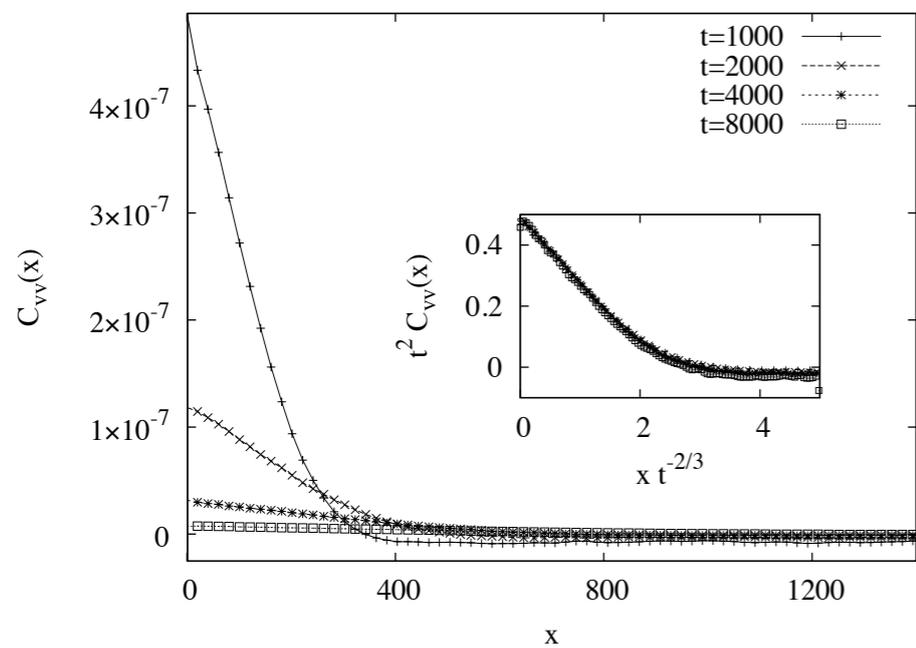
Note: $E(t) \sim t^{-2/3} \implies t^* \sim \delta^3$



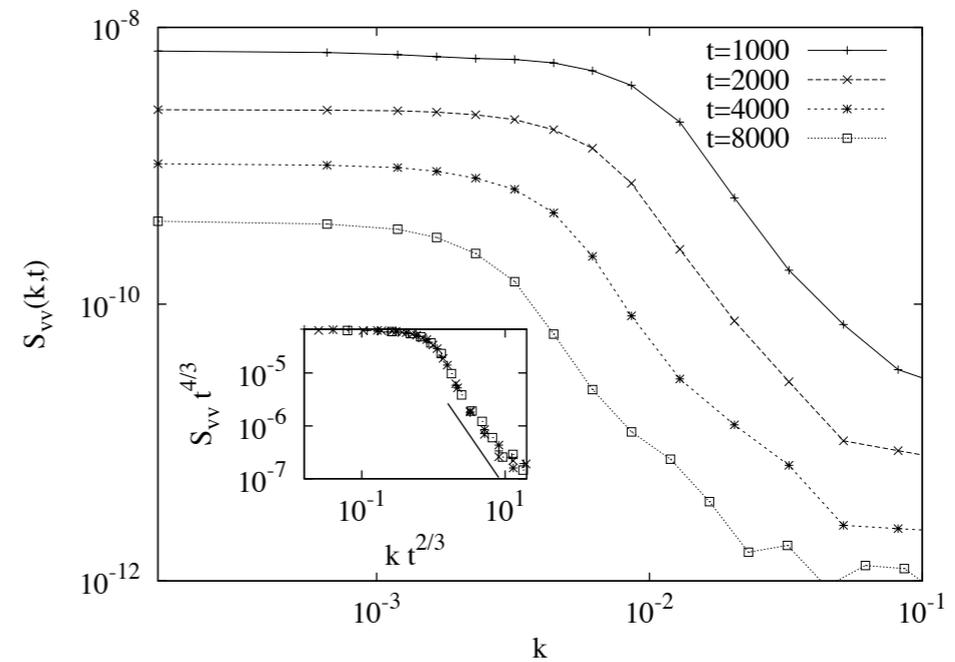
Sticky ρ - ρ (real)



Sticky ρ - ρ (-2.0)

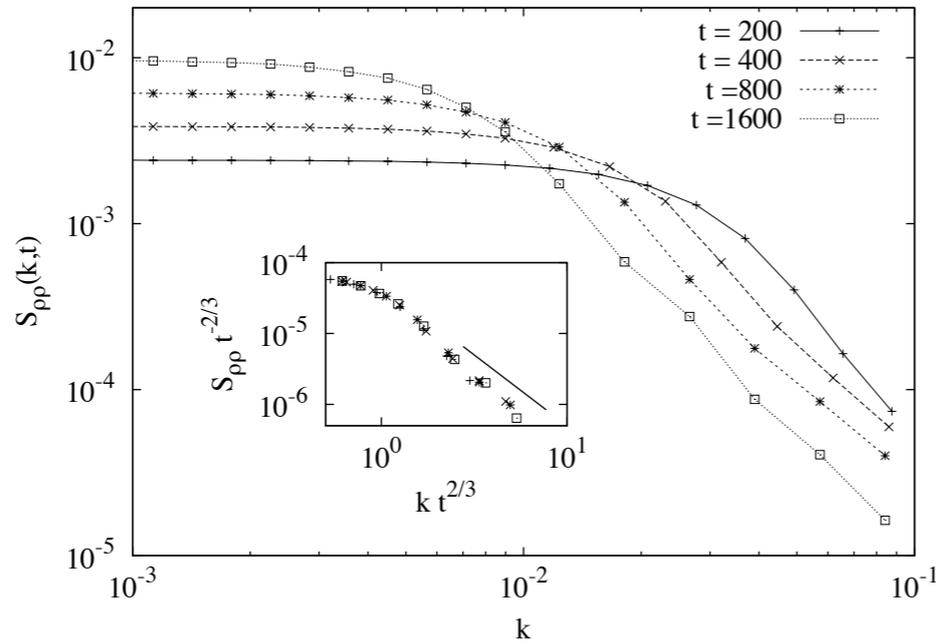


Sticky v - v (real)

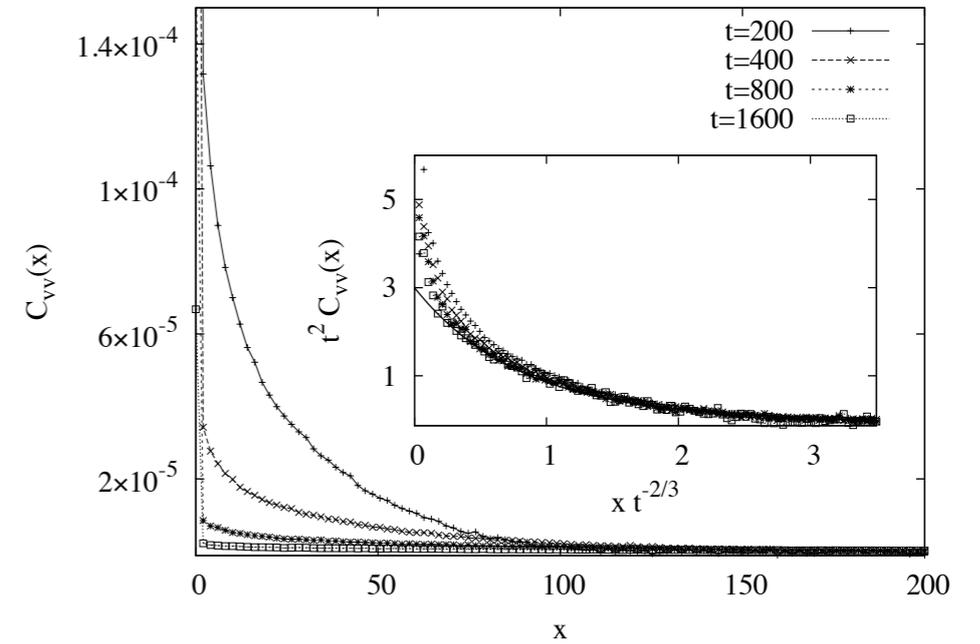


Sticky v - v (-2.0)

$t < t_c$

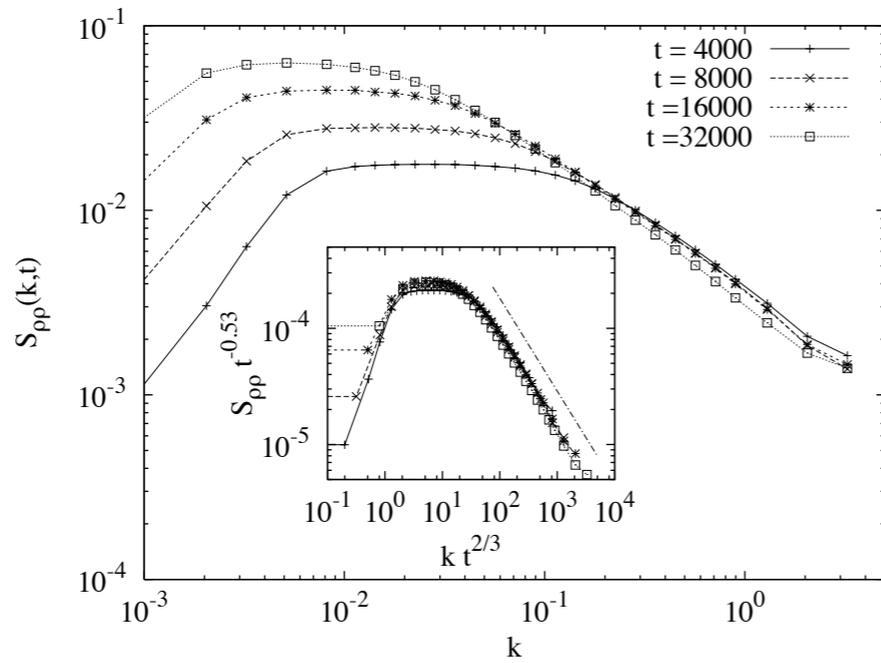


Granular ρ - ρ (-2.0)

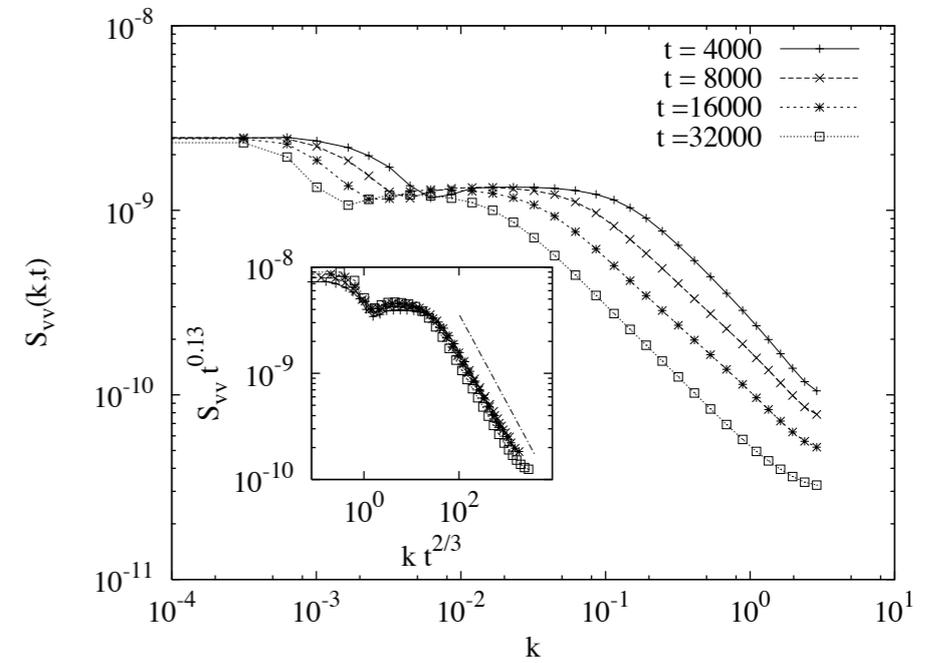


Granular v - v (real)

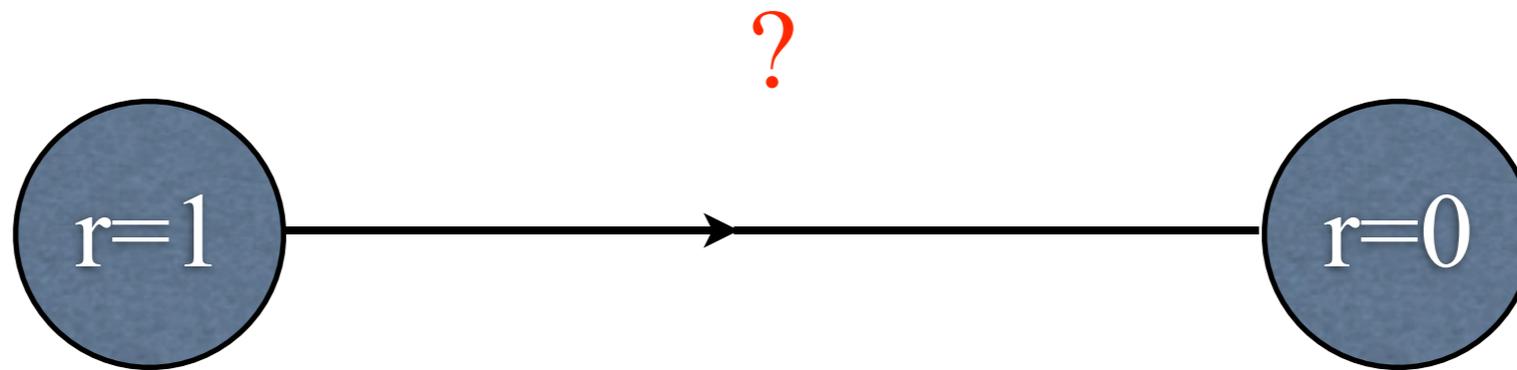
$t > t_c$



Granular ρ - ρ (-0.8)

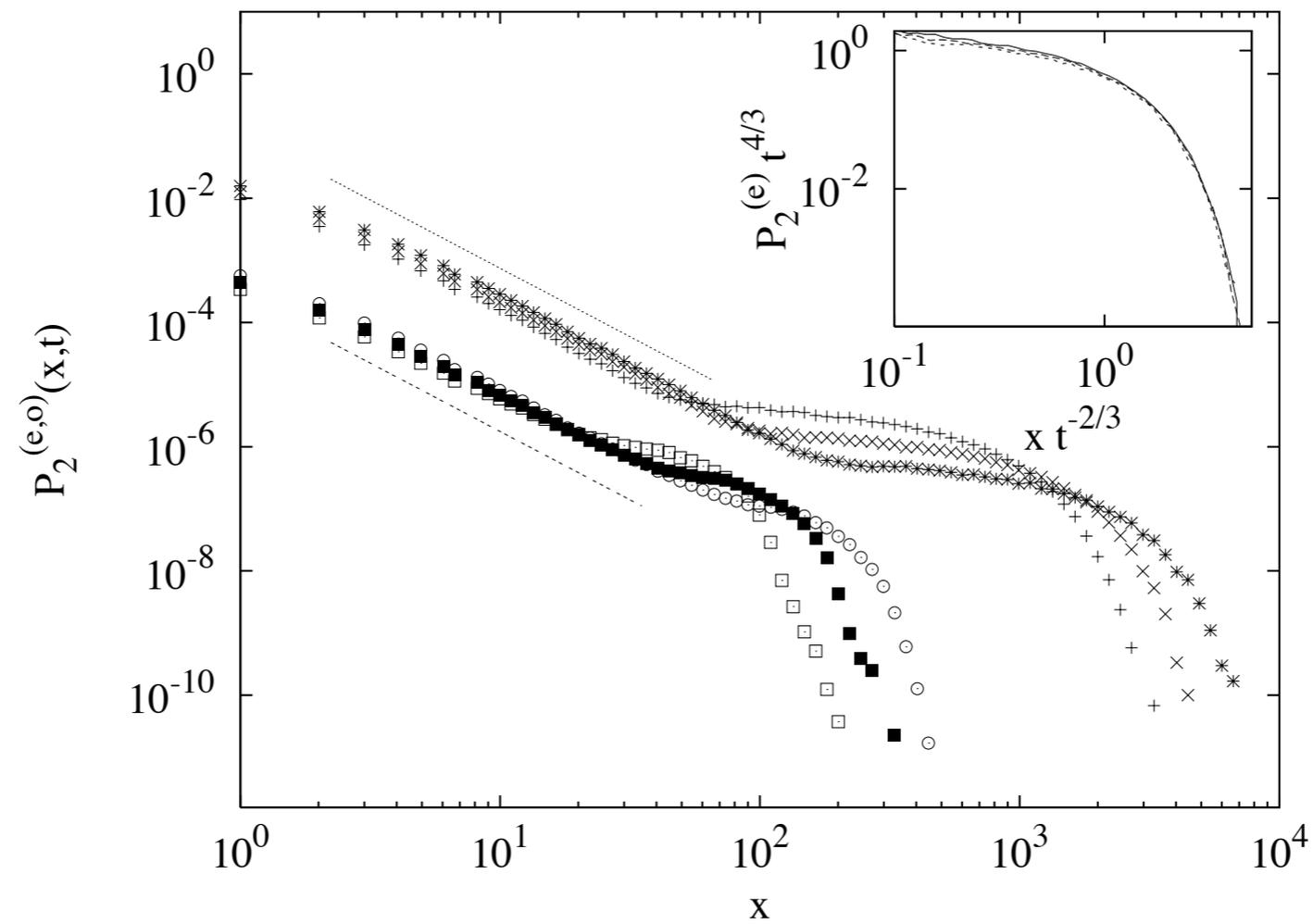


Granular v - v (-0.8)

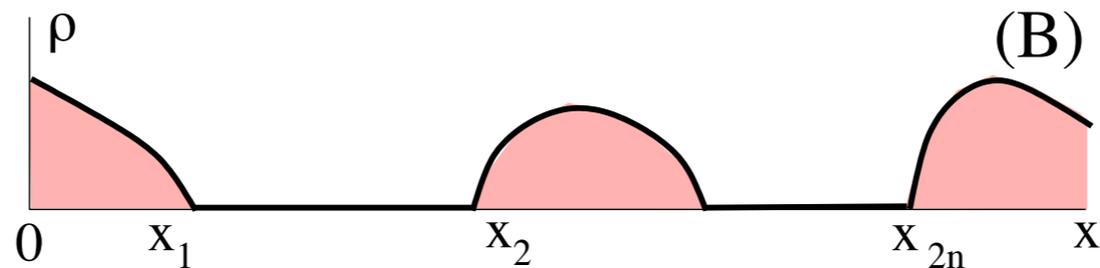
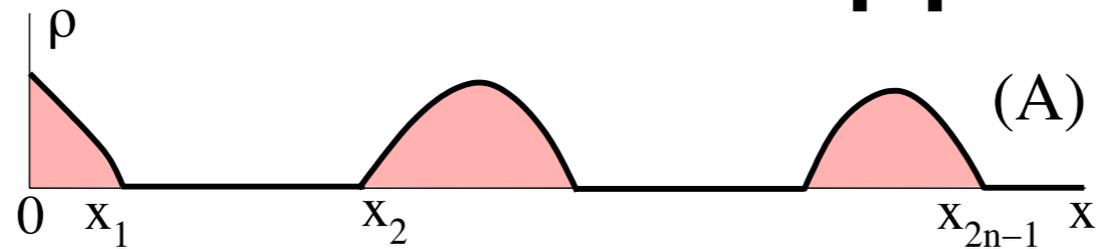


No. For large times, the nature of coarsening changes completely

Why does Porod law fail?



Independent Interval Approximation



$$C_{\rho\rho}(x) \approx \sum_{n=0}^{\infty} p_{2n}(x)$$

$$p_{2n}(x) = \alpha \int dx_1 \dots dx_{2n} Q(x_1) E(x_{2n} - x_{2n-1}) Q(x - x_{2n}) \prod_{j=1,3,\dots}^{2n-3} [E(x_{j+1} - x_j) O(x_{j+2} - x_{j+1})], \quad n = 1, 2$$

$$\theta = 3 - \gamma$$

$$\theta \approx 0.8; \quad \gamma \approx 2.2$$

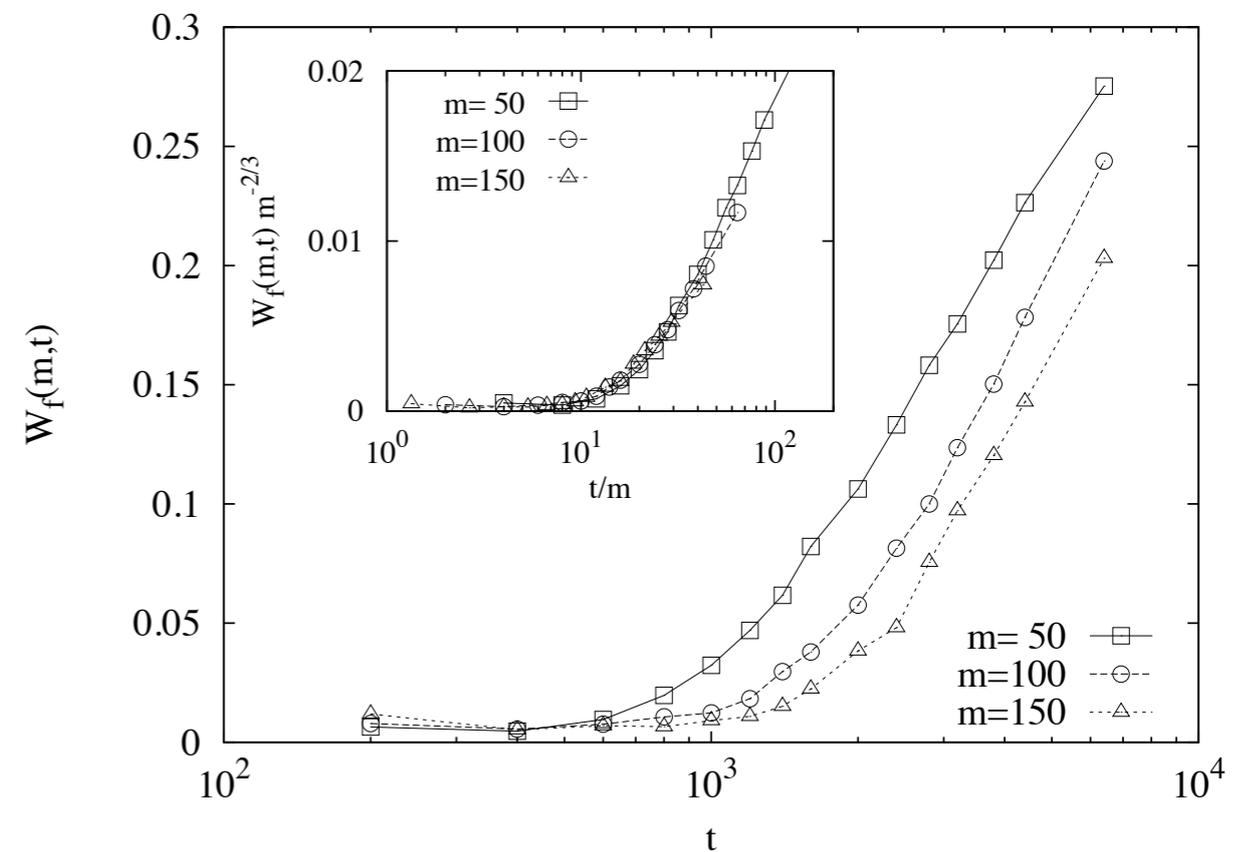
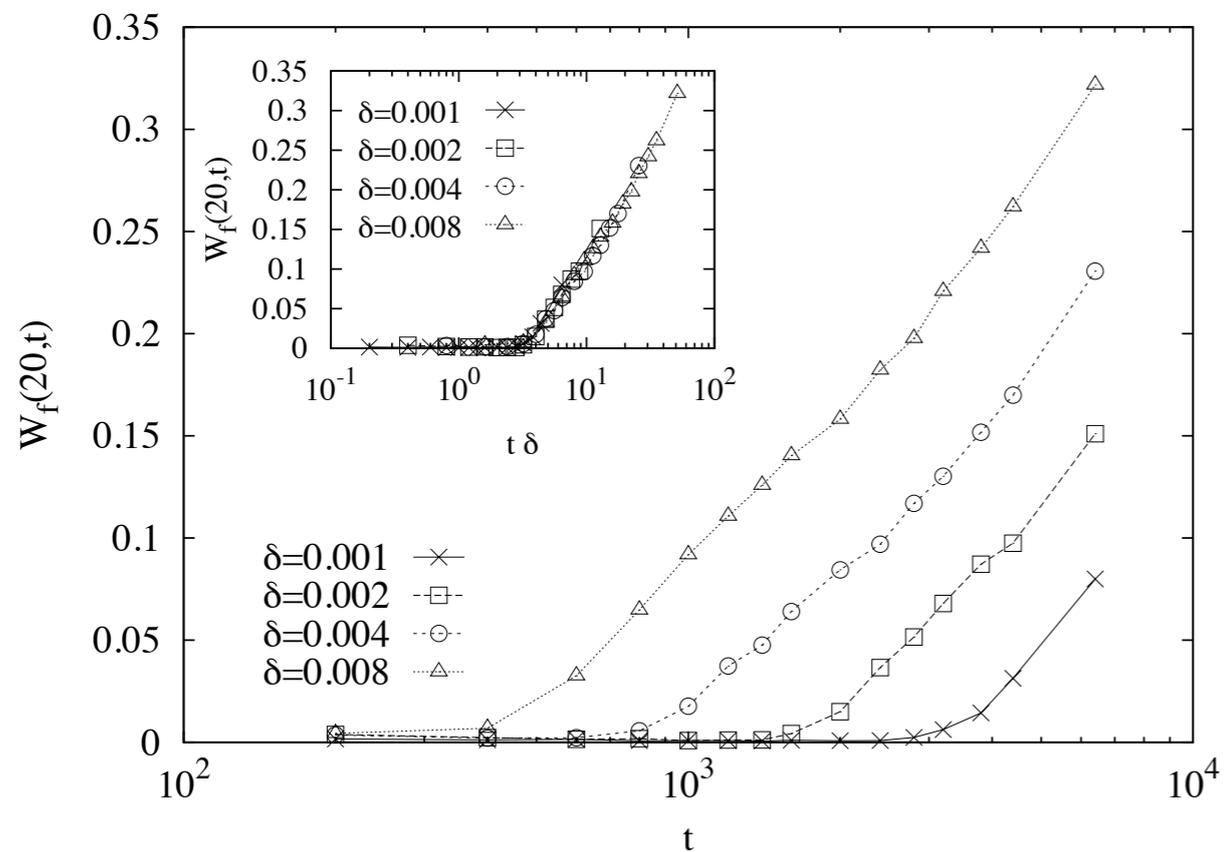
Summary and Outlook

- A new time scale in freely cooling gas
- Porod law violated (different from the sticky gas limit)

Effective model

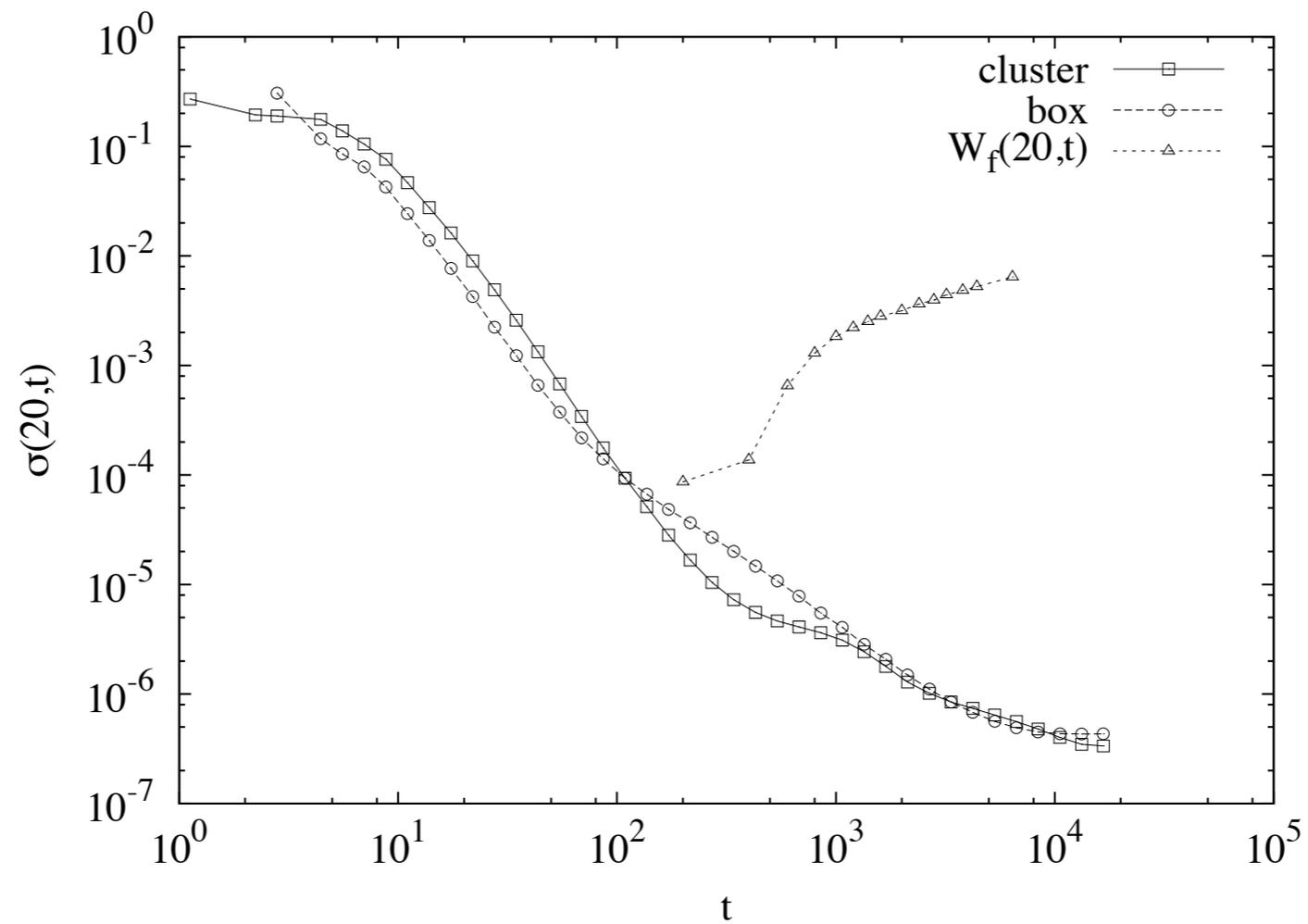
Sticky gas \rightarrow a model of aggregation

Intermediate times: aggregation + fragmentation



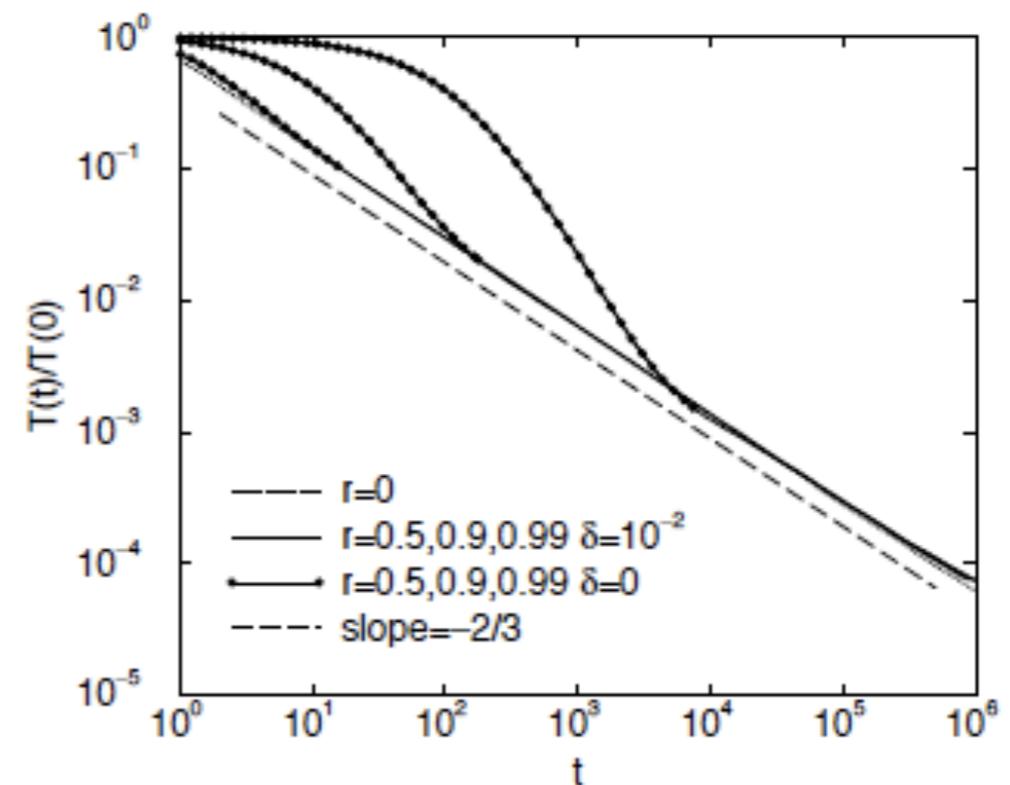
Effective model

Velocity fluctuations



Summary and Outlook (cont)

- Continuum description?
- r -independence?



Ben-Naim et al, 1999

Thank You

References

Phys. Rev. Lett., **99**, 234505 (2007)

Phys. Rev. E, **79**, 021303 (2009)

ongoing work