Challenges in Network Dynamics
Spatio-Temporal Patterns: Direct and Inverse Methods

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Schedule

1. Background: Synchrony & Precise Timing in Biological Network

2. Tractable Model of Spiking Neural Network

3. Direct Modeling (nonlinear coupling):
Network → Dynamics

4. Network Design as Inverse Method
Dynamics → Set of all networks

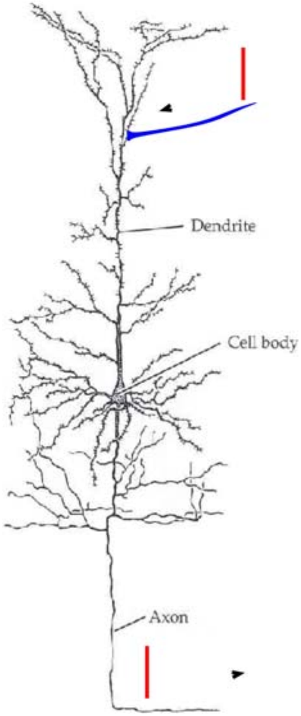
5. Towards **Inference** of Generic Deterministic Networks
Dynamics → The one underlying network

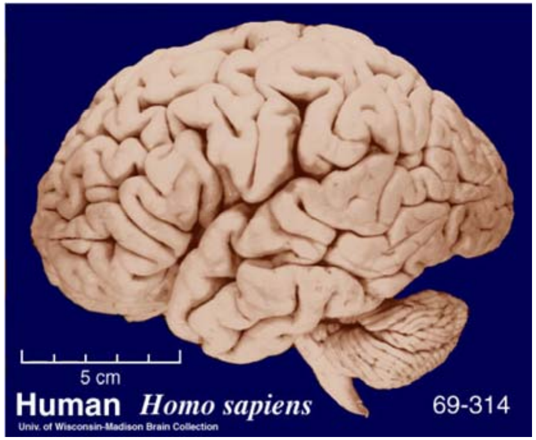
Spatio-Temporal Patterns in Networks of Biology and Physics

Biological Networks

$(10^{-3} - 10^{10}s;\ 10^{-5} - 10^{-1}m)$

- Neurons
  (sensory-/motor processing,
  memory formation…)
- Gene & protein interactions
- „Tree" of life
- Epidemic spreading …





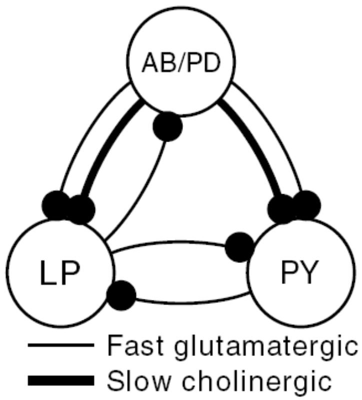
Networks of physical & artificial units

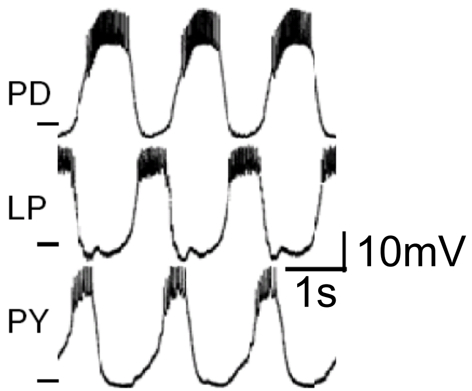
$(10^{-2} - 10^{10}s;\ 10^{-9} - 10^{6}m)$

- Complex disordered media
- Modern power grids (mind the renewables!)
- Autonomous robots

Patterns Generators (exper.)

Biological neural networks generate stereotypic, periodic patterns of activity

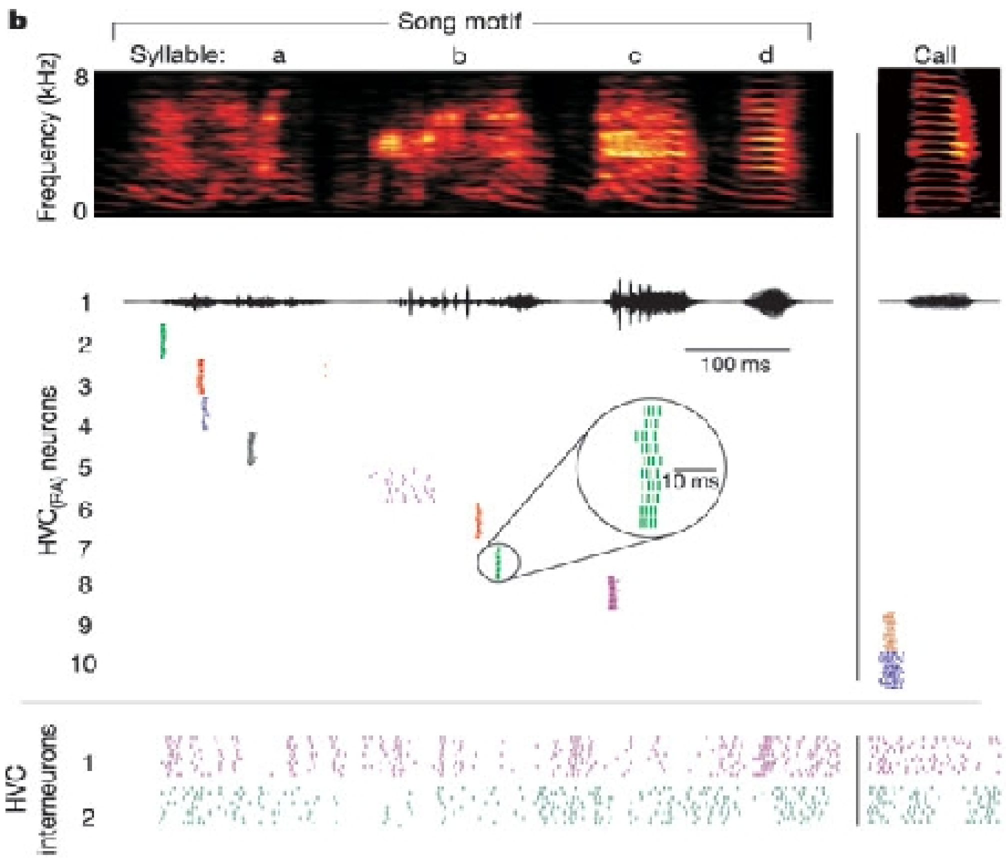




Spatio-temporal patterns control rhythmic movements

Abarbanel et al.; Selverston et al.;...
M.P. Nusbaum, M.P. Beenhakker, *Nature* **417**, 343 (2002)
A.A. Prinz, D. Bucher, E. Marder, *Nat. Neurosci.* **7**, 1345 (2004)

Synchronized activity may propagate



Precision 1:100

First experiments and analysis: R. H. Hahnloser, A. A. Kozhevnikov, M. S. Fee, *Nature* 419:65 (2002)
Summarizing Overview: I. R. Fiete et al, *J. Neurophysiol.* 92:2274 (2004)

Precise Timing and Spatio-Temporal Patterns

→**key elements of neural computation and processing**

e.g., M. Abeles et al.,*J. Neurophysiol.* 70:1629 (1993)

A. Riehle, et al., *Science* 2781:950 (1997)

W. Singer & coworkers; *Neuron* 24:49 (1999); Proc. *NCCD Meeting.* (2007).

Y. Ikegaya et al.; *Science* 304:504 (2004);

A. Mukeichev et al, *Neuron* 53:413 (2007)

R.H. Hahnloser et al, *Nature* 419:65 (2002); *Nature* 421:294 (2003)

S. P. Jadhav et al., *Nature Neurosci*.12:792 (2009)

C. Clopath et al., *Nature Neurosci.* [advanced online] (2010)

→ **but dynamical origin unclear**

(How) can precise spatio-temoral patterns occur in complex heterogeneous networks?

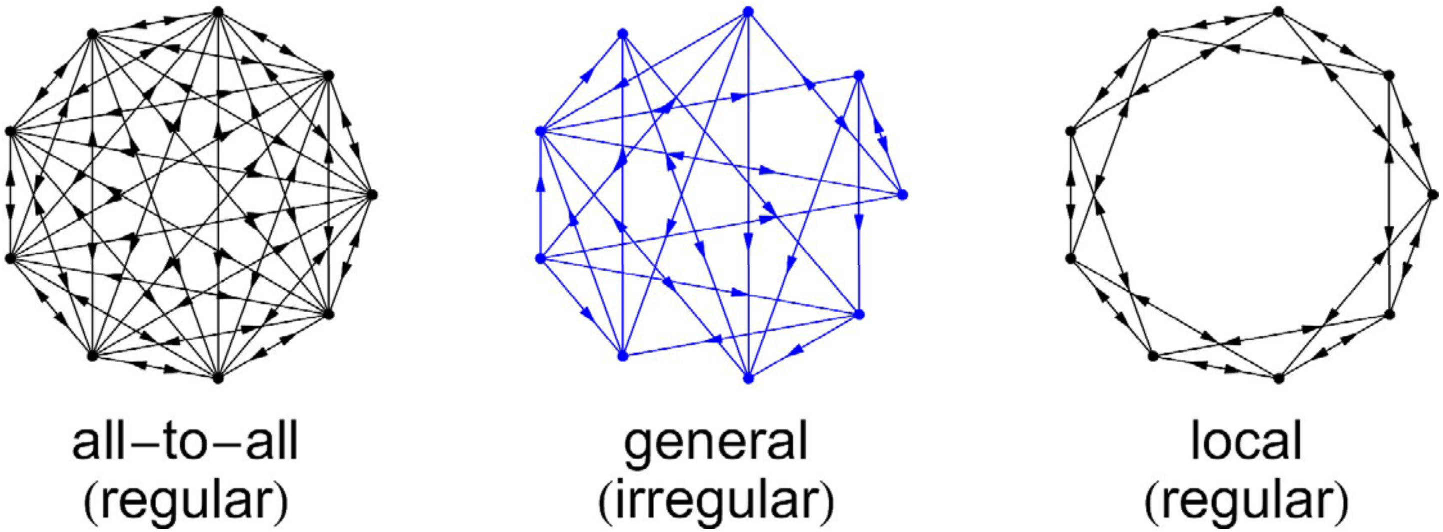
Mathematical Challenges for Theory

Simultaneous occurrence of:

- **Nonlinearity**
- **High dimensionality**
- Complicated Network **Connectivity**
- Interaction **Delays**
- Strong **Heterogeneities**
- **Stochasticity**

<u>common approach:</u>

Mean Field Theories, Statistical Description, e.g. averaging over network



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Neural Model and Phase Description

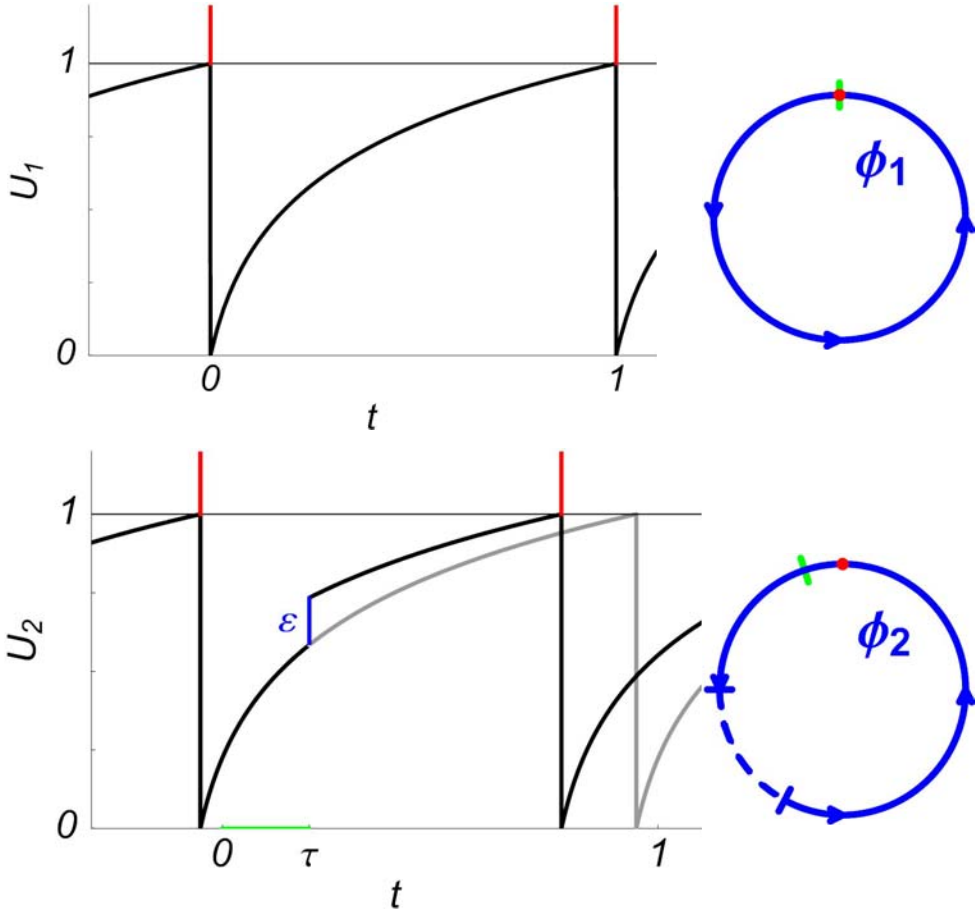
networks of neurons
- oscillatory if driven by current

• uncoupled neurons i
have increasing (concave) potential $U_i(t)$

• spike sent at $\theta = 1$ threshold

• received after delay time τ

• coupling strength $\propto \varepsilon$



original model: R.E. Mirollo, S.H. Strogatz; *SIAM J. Appl. Math*. 50:1645 (1990)
model with delay: U. Ernst, K. Pawelzik, T. Geisel; *Phys. Rev. Lett.* 74:1570 (1995)

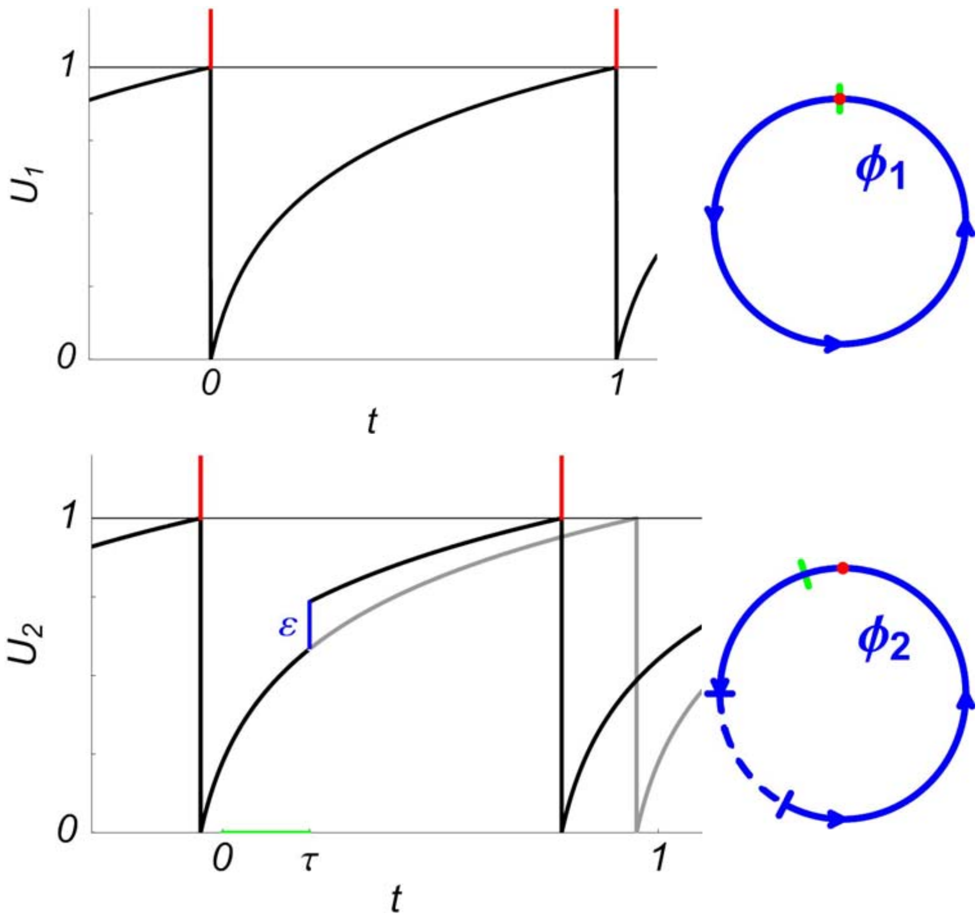
Neural Model and Phase Description

Model class includes:

- Standard integrate & fire neurons
- Normal form of type I neurons
  (θ– neurons, quadratic I&F)
- Conductance based models
- …

Restrictions:

- 1-dimensional units
- Limit of fast response



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Hybrid Model: Events Interrupt Continuous-Time Dynamics

Membrane potential dynamics

$$\frac{d}{dt}V_i = f_i(V_i) + \sum_{j=1}^{N} \sum_{k \in \mathbb{Z}} \varepsilon_{ij} \delta \left(t - t_{jk}^{\mathrm{s}} - \tau_{ij} \right)$$

Threshold crossing

two types of events:

$V_j(t^-) = V_{\Theta,j}$ ➔ spike generated $t =: t_{jk}^{\mathrm{s}}$

spike sending

➔ potential reset $V_j(t_{jk}^{\mathrm{s}}) = 0$

After delay τ_{ij}: $V_i \rightarrow V_i + \varepsilon_{ij}$

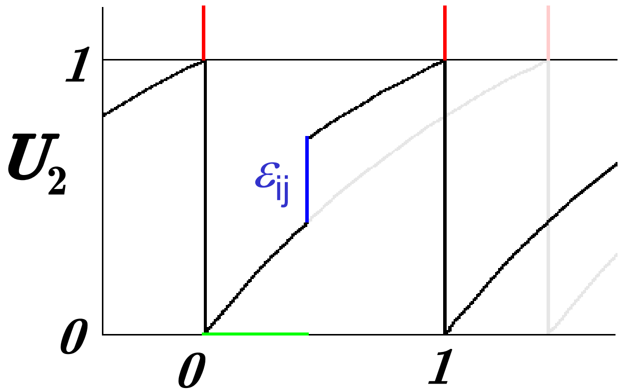
spike reception

transfer of nonlinearity to times of receptions
➔ phase as time-like variable

Phase vs. Potential Description: Interactions

We know action of spike on *potential*: $U_i \rightarrow U_i + \varepsilon_{ij}$

Translate into action on *phase* : $U(\Phi_i) \rightarrow U(\Phi_i) + \varepsilon_{ij}$

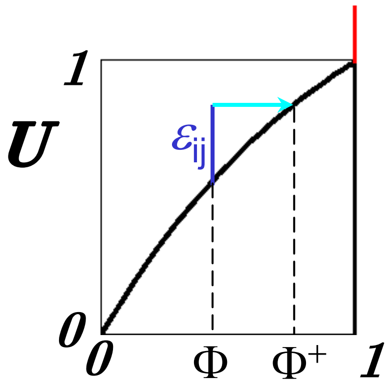


$$\Phi_i^+ = H_{\varepsilon_{ij}}(\Phi_i^-) = U^{-1}(U(\Phi_i) + \varepsilon_{ij})$$

$$\dot{\Phi}_i = \omega_i$$

One-to-one nonlinear mapping

$=$



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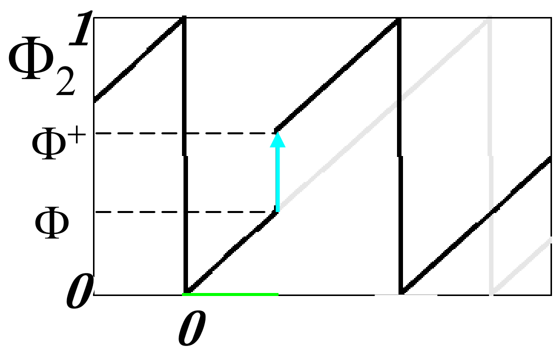

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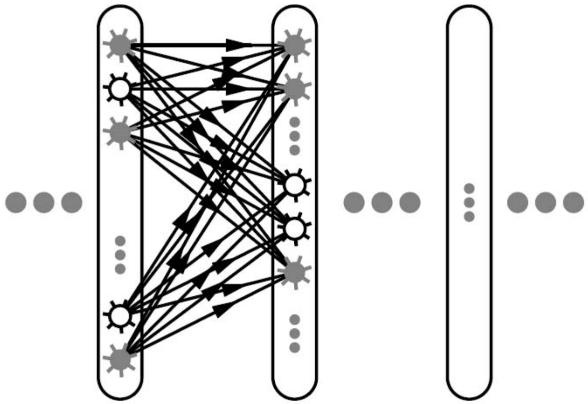
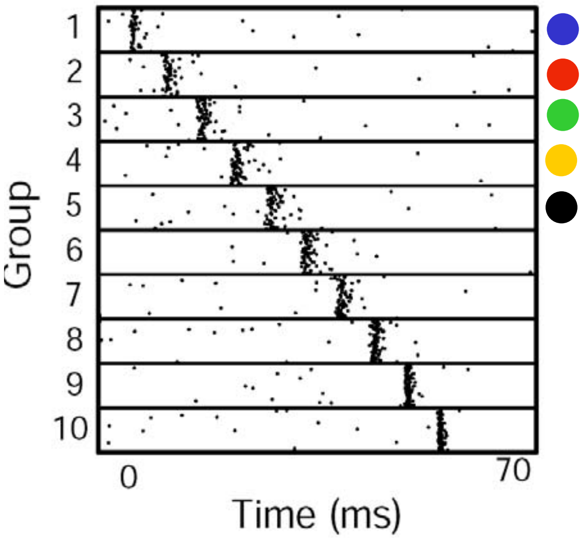
Precise Timing via Propagation of Synchrony

Working hypothesis:
Synfire chain anatomy embedded in recurrent networks



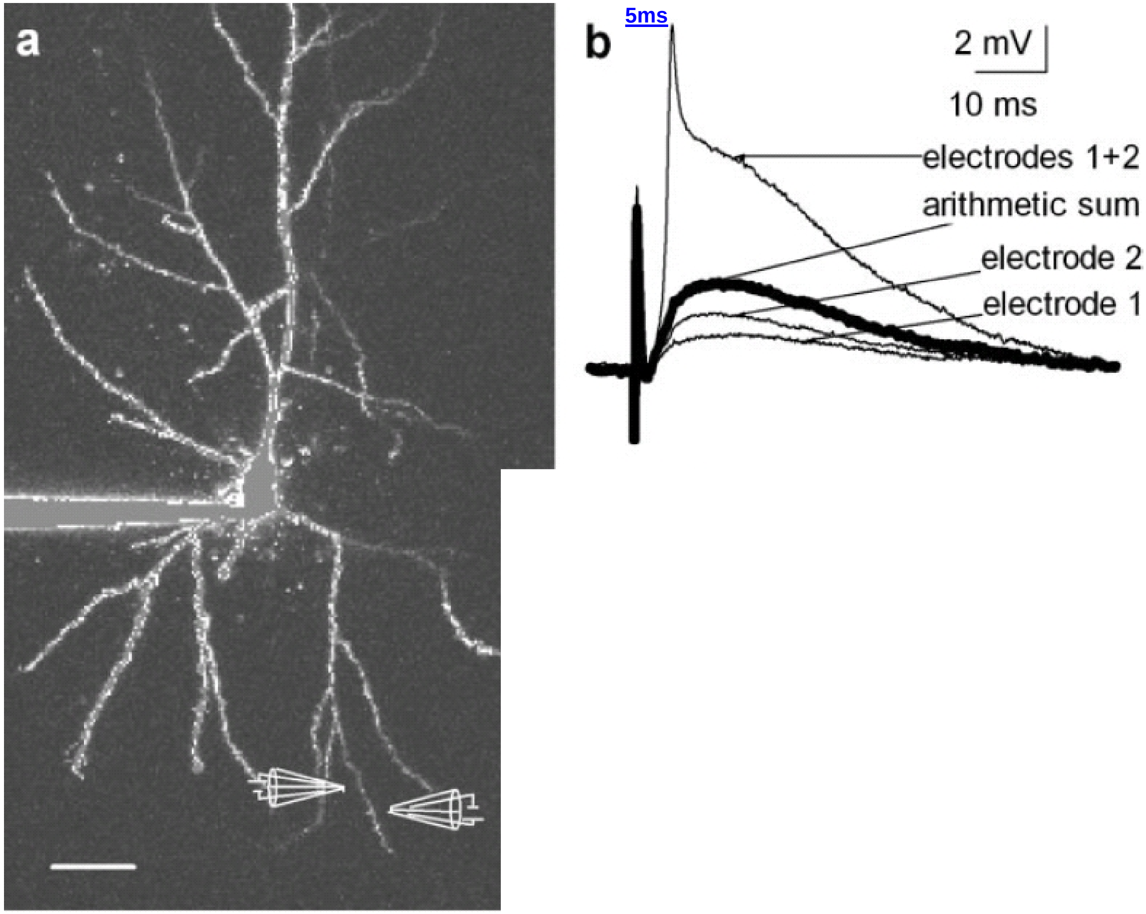


idea: Abeles (1991);
Feed-forward: M. Diesmann, et al., Nature 402:529 (1999);
embedded: Kumar et al., J. Neurosci. (2008)

Problem:
- existence of coarse **feed-forward anatomy not found**

Recurrent connectivity sufficient?

Nonlinear Summation in Hippocampus

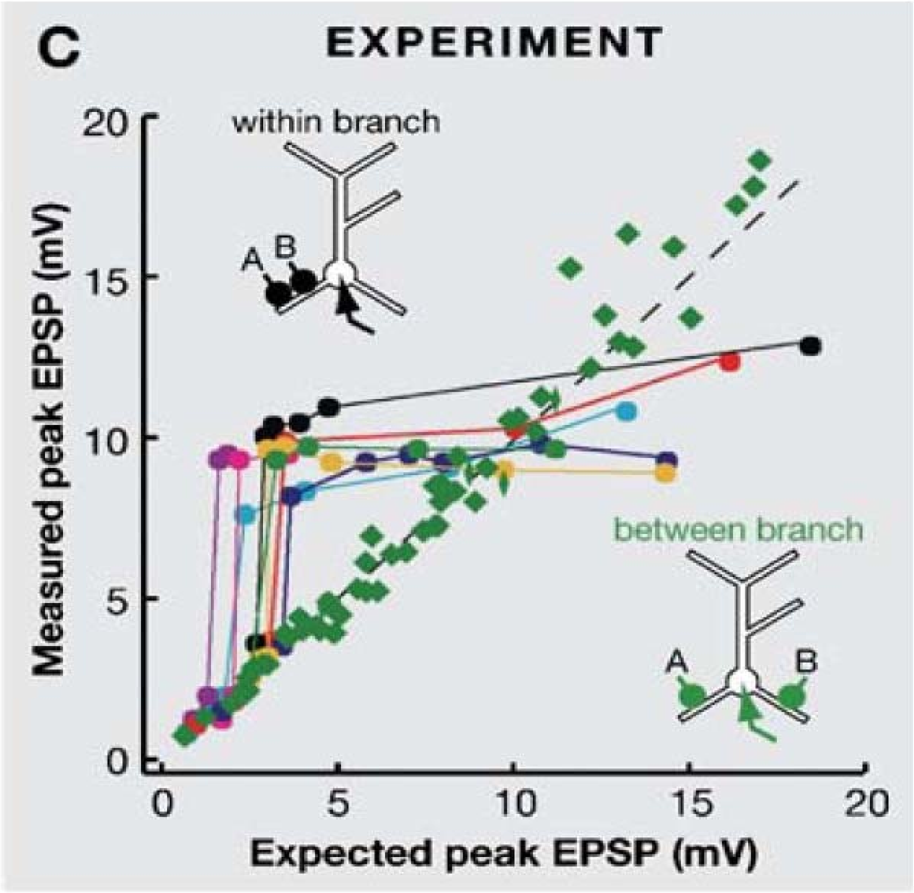


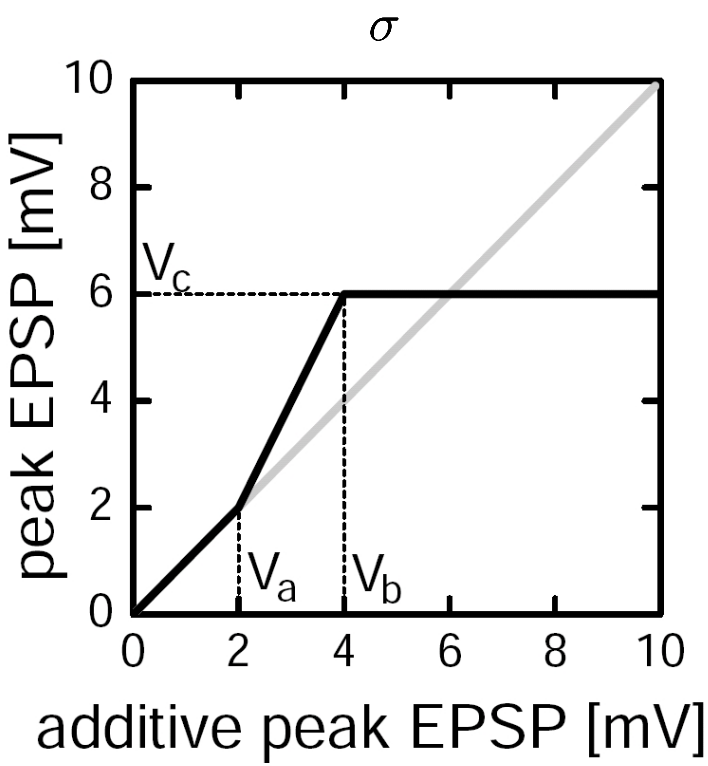
Hippocampus: CA1 pyramidal neurons
Spike initiation after 5.0ms ± 0.14ms

G. Ariav, A. Polsky, J. Schiller, *J. Neurosci.* 23:7750 (2003)

Modeling nonlinear summation

Modulate excitatory input strength: $$\sum_j \varepsilon_{ij} \quad \rightarrow \quad \sigma\Big(\sum_j \varepsilon_{ij}\Big)$$

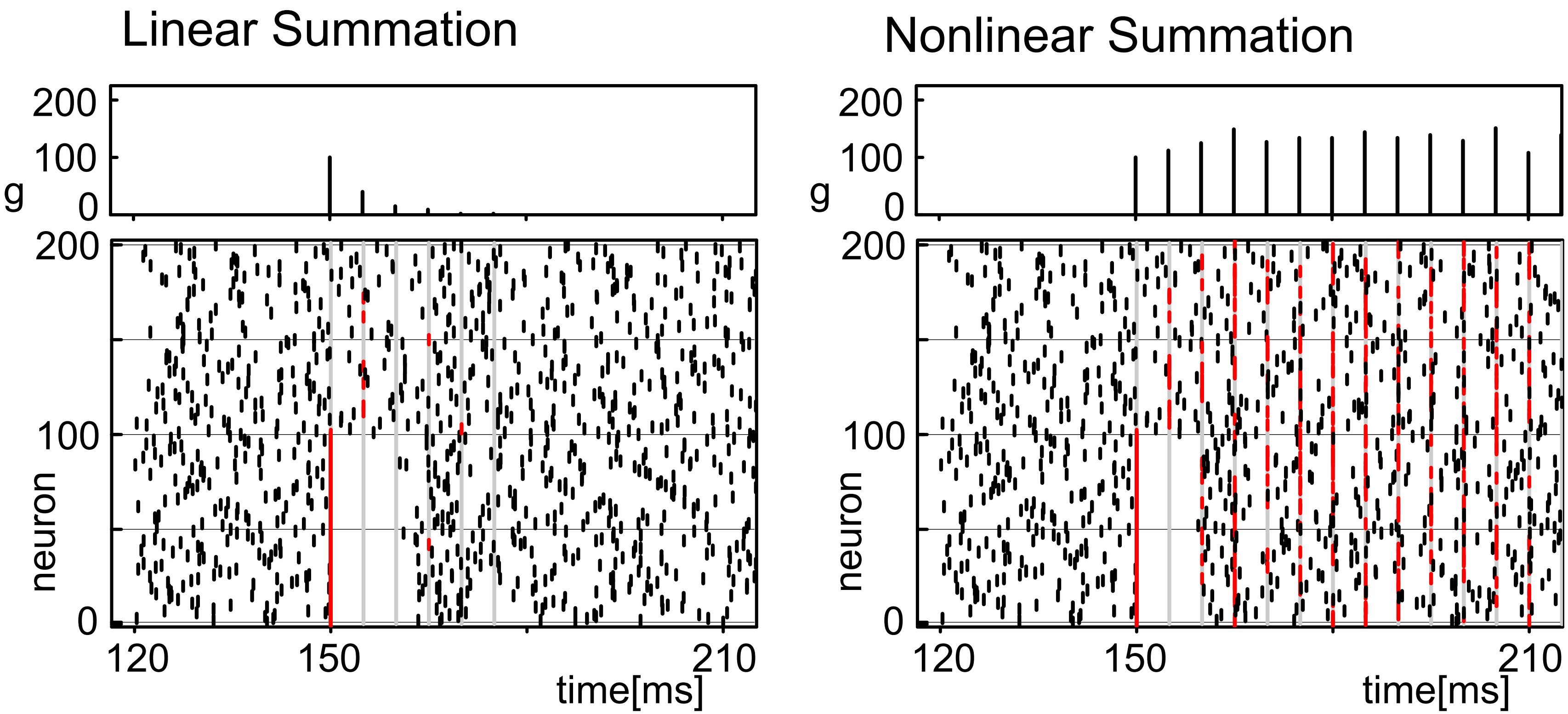




A. Polsky, B. W. Mel, J. Schiller, *Nature Neurosci.* 7:621 (2004)

Nonlinear Summation ➔ Propagation in Random Networks

Randomly coupled sparse neural network
(no embedded feed-forward anatomy)



How do group sizes evolve?

Expected next group size $E(j|i)$

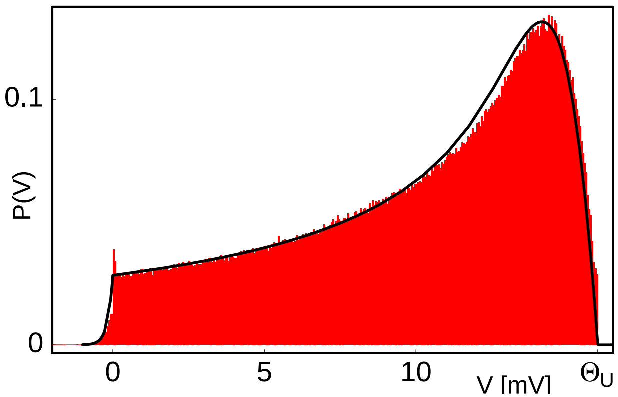
Combinatorics: average fraction of neurons receiving j_1 excitatory and j_2 inhibitory inputs from a cluster of i neurons.

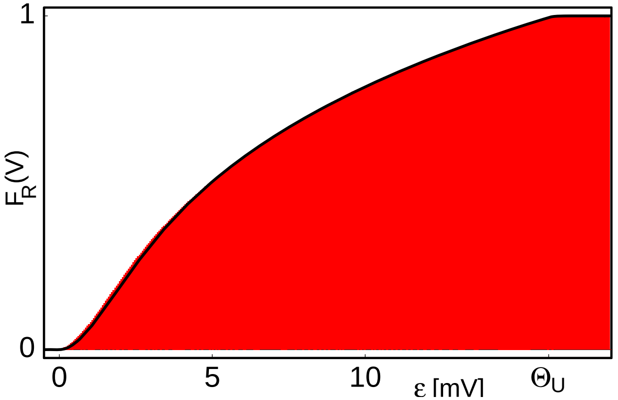
$$\binom{i}{j_1, j_2, i-j_1-j_2} p_{Ex}^{j_1} p_{In}^{j_2} p_0^{j_1+j_2} (1-p_0)^{i-j_1-j_2}$$

Fraction of neurons spiking due to a pulse of strength $\varepsilon = \varepsilon(j_1, j_2)$:

$F_R(\varepsilon) = \int_{\Theta_U - \varepsilon}^{\Theta_U} P(V)dV$

$P(V)$ from diffusion approximation





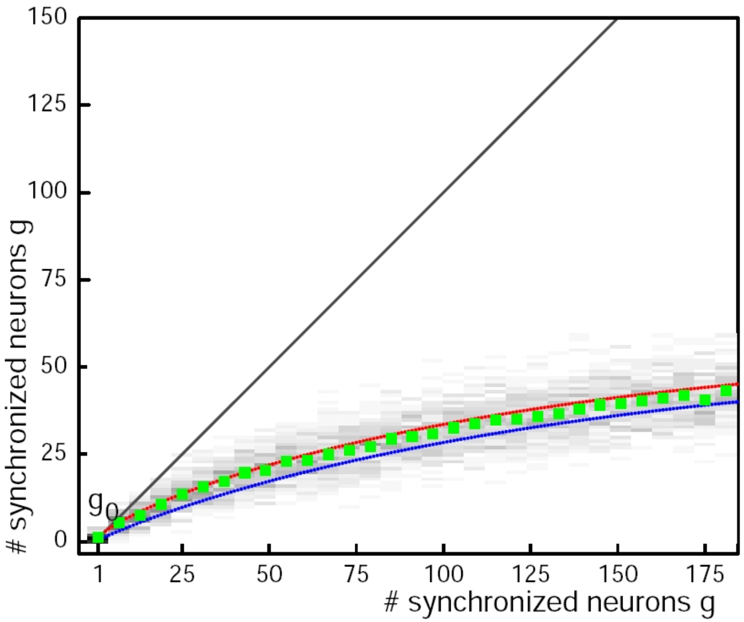
Expected number of neurons firing in response to a group of i neurons

$$E(j|i) = (N-i) \sum_{j_1=0}^{i} \sum_{j_2=0}^{i-j_1} F_R(\varepsilon(j_1, j_2)) \binom{i}{j_1, j_2, i-j_1-j_2} p_{Ex}^{j_1} p_{In}^{j_2} p_0^{j_1+j_2} (1-p_0)^{i-j_1-j_2}$$

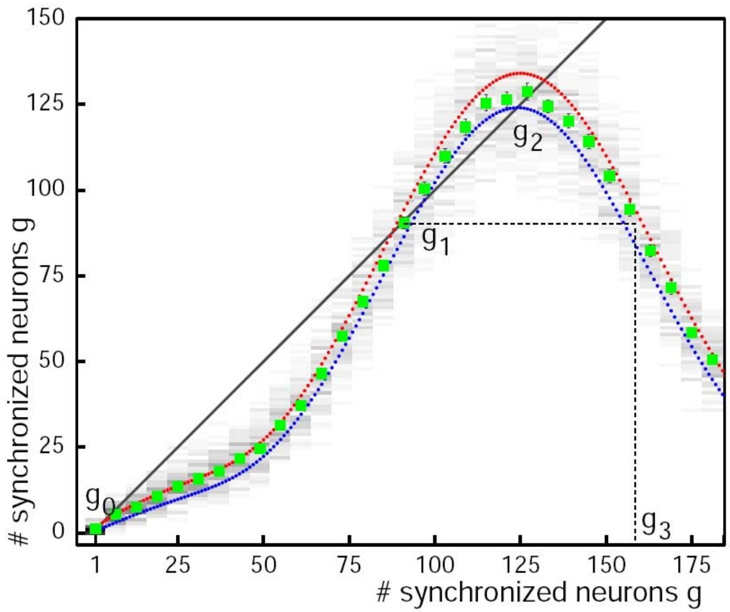
Synchrony Propagates due to Nonlinearity

Analyzing the propagating state: $P(j|i)$ and $E(j|i)$

Linear Summation



Nonlinear Summation



Nonlinear summation ➔
Robust synchrony propagation
without feed-forward structures!

R.-M. Memmesheimer and M.T., *Comput. Neurosci. Abstr.* 158.2 (2007); and (2010, under revision)

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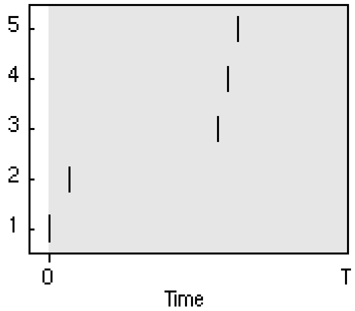
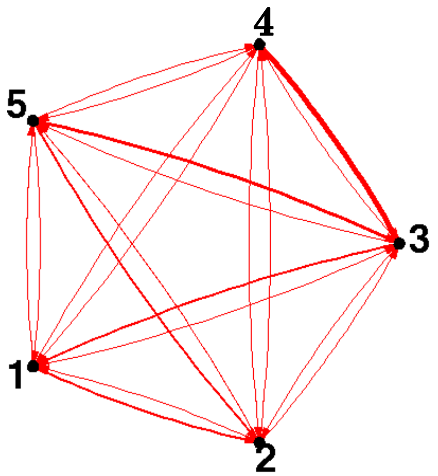
Network Design in Simplest Version

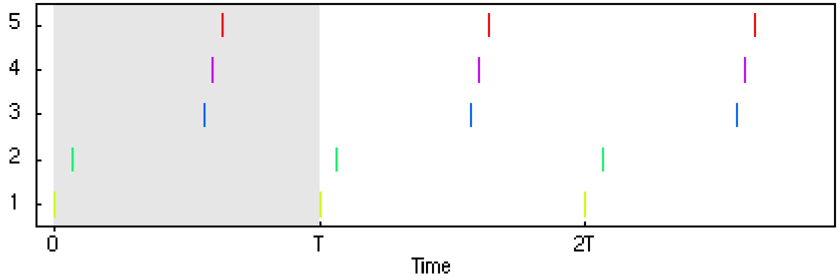
Predefine Pattern



Construct Network(s)



Realize(s) Pattern



First progress: R.-M. Memmesheimer & M.T., Phys. Rev. Lett. 97:188101 (2006).

Patterns of Precisely Timed Spikes

Simple periodic pattern characterized by

Period $T,$

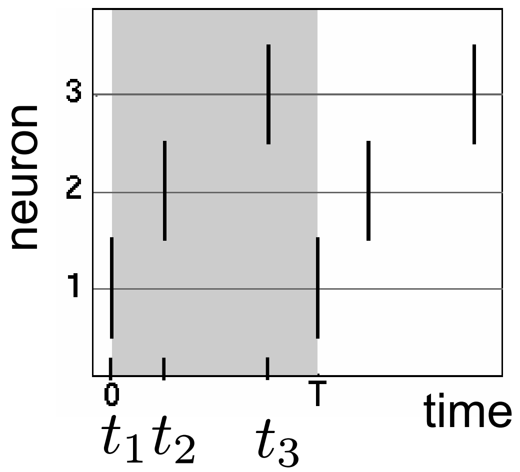
Firing Times $t_1, t_2, ..., t_N$

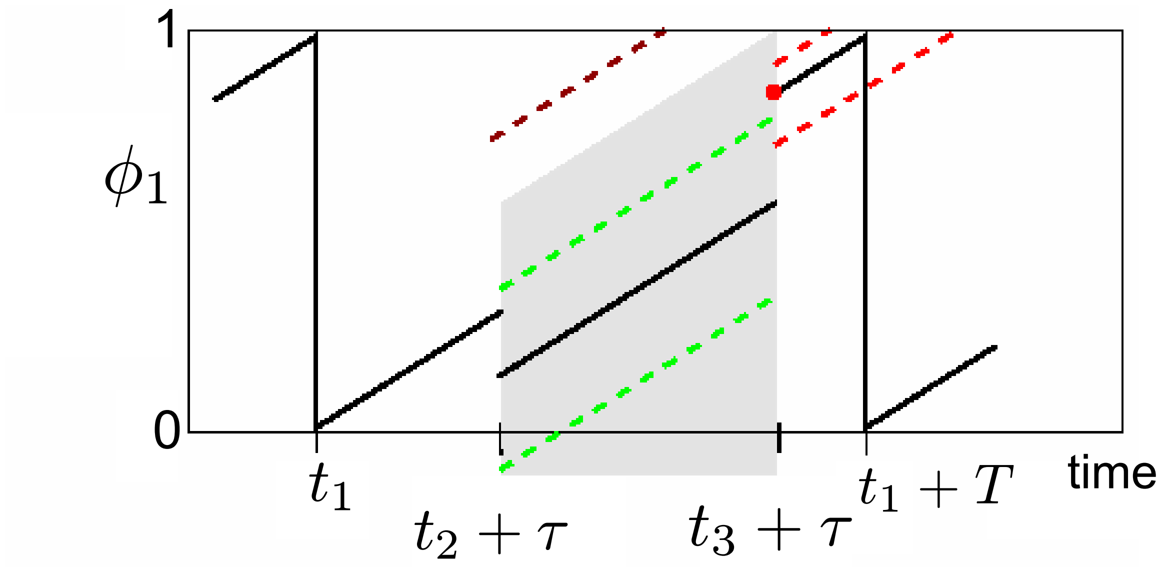
<u>Realization requires:</u>

I) Periodic Firing $\phi_i(t_i^- + nT) = 1, \quad n = 0, \pm 1, ...$

II) Silence $\phi_i(t^-) < 1, \quad t \neq t_i + nT$

Simple Patterns of Precisely Timed Spikes





Firing time $\phi_i(t_3 + \tau) = 1 - (t_i \, [+T] - (t_3 + \tau))$

Silence $\phi_i(t_j + \tau) < 1 - (t_{j+1} - t_j)$

Periodicity $\phi_i(T) = \phi_i(0)$

Solve for interaction network ε_{ij}

Implicit Equations for Network Structure

here $H_{\varepsilon_{ij}}(\Phi_i) = U^{-1}(U(\Phi_i) + \varepsilon_{ij})$

Periodicity

$$
\begin{aligned}
&T - (t_N + \tau) + H_{\varepsilon_{1N}}(...(H_{\varepsilon_{13}}(H_{\varepsilon_{12}}(H_{\varepsilon_{11}}(\tau) + t_2) + t_3 - t_2)...) + t_N - t_{N-1}) &&= 1 \\
&T - (t_N + \tau) + H_{\varepsilon_{2N}}(...(H_{\varepsilon_{23}}(H_{\varepsilon_{22}}(\tau) + t_3 - t_2)...) + t_N - t_{N-1}) &&= \Phi_2(0) \\
&T - (t_N + \tau) + H_{\varepsilon_{3N}}(...(H_{\varepsilon_{33}}(\tau) ...) + t_N - t_{N-1}) &&= \Phi_3(0) \\
&\ldots && \ldots \\
&T - (t_N + \tau) + H_{\varepsilon_{NN}}(\tau) &&= \Phi_N(0),
\end{aligned}
$$

Firing time

$$
\begin{aligned}
& \Phi_1(0) &&= 1 \\
-\tau\ & H_{\varepsilon_{21}}(\Phi_2(0) + \tau) + t_2 &&= 1 \\
-\tau\ & H_{\varepsilon_{32}}(H_{\varepsilon_{31}}(\Phi_3(0) + \tau) + t_2) + t_3 - t_2 &&= 1 \\
\ldots\ & && \ldots \\
-\tau\ & + H_{\varepsilon_{N\,N-1}}(...(H_{\varepsilon_{N3}}(H_{\varepsilon_{N2}}(H_{\varepsilon_{N1}}(\Phi_N(0) + \tau) + t_2) + t_3 - t_2)...) + t_N - t_{N-1} &&= 1,
\end{aligned}
$$

Solution to ‘inverse problem’ : ALL possible networks

Generic space of admissible networks

Set of all networks consistent with given dynamics …

- Is typically **high-dimensional**, N x (N-M) dimensional for M predefined spikes

consequences for networks with predefined dynamics:

- enables **multi-feature** network & dynamics

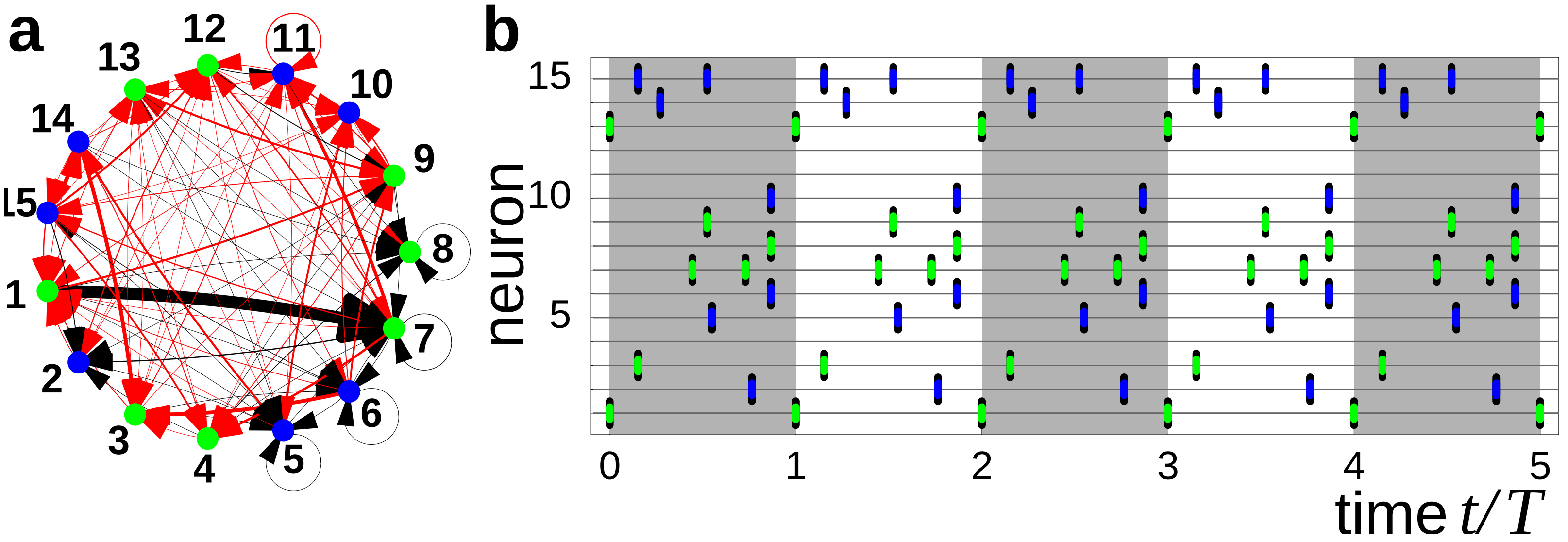
- same pattern may be **stable in one** and **unstable in another** network

- **allows for further restrictions**:
  particular neuron types, coupling type, delays, interaction network

- determining networks that are **optimal** (e.g. structurally)

R.-M. Memmesheimer & M.T., Phys. Rev. Lett. 97:188101 (2006).
R.-M. Memmesheimer & M.T., Physica D **224**:182 (2006).

Multi-feature Networks & Dynamics



Different heterogeneities and design:

R.-M. Memmesheimer & M.T., Physica D **224**:182 (2006).

<u>model features:</u>

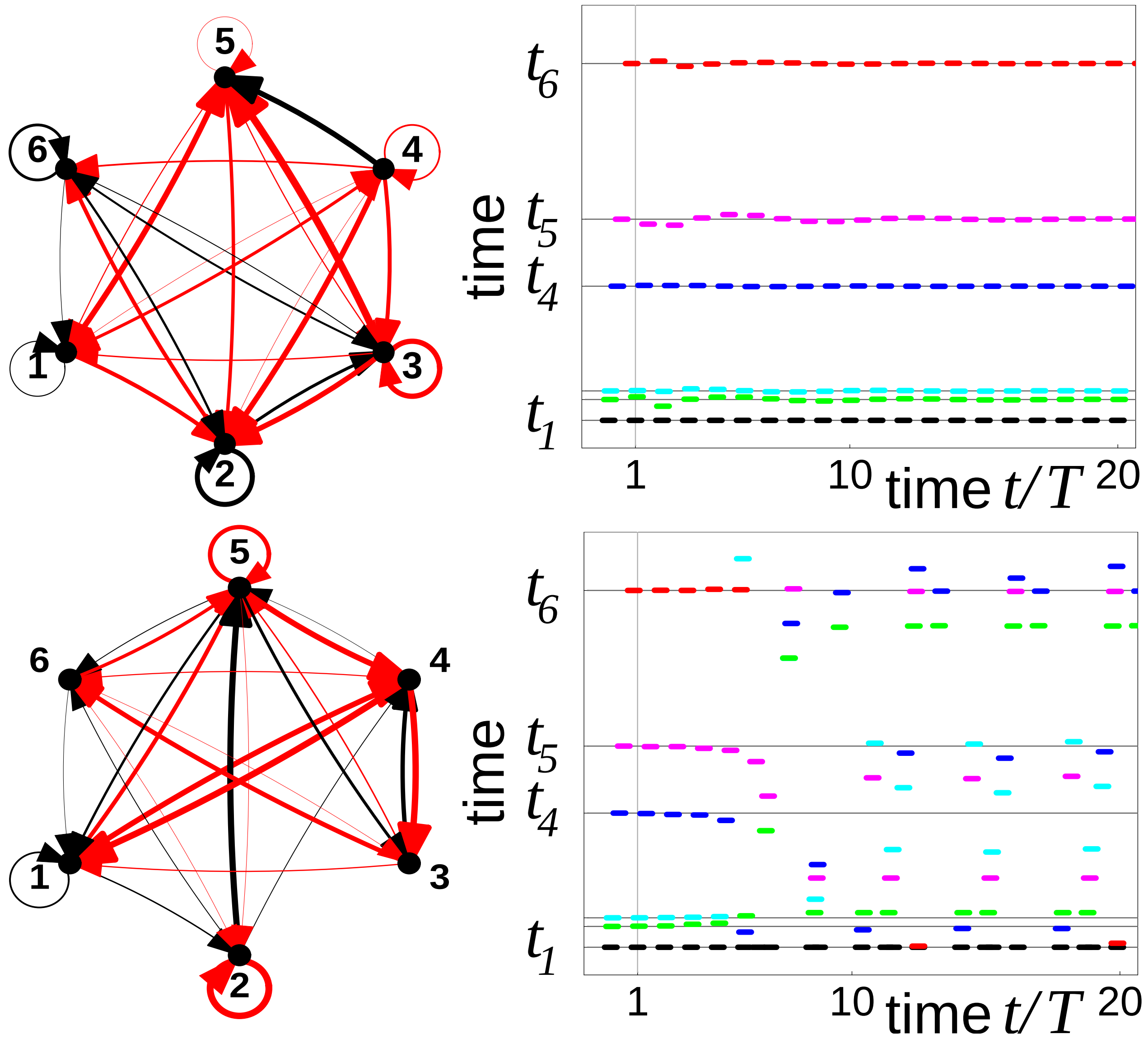
+ significant **delays**

+ complicated **connectivity** +mixed coupling (inh. & exc.)

+ different neuron types + strong **heterogeneities**

+ synchronous, multiply spiking and non-spiking neurons

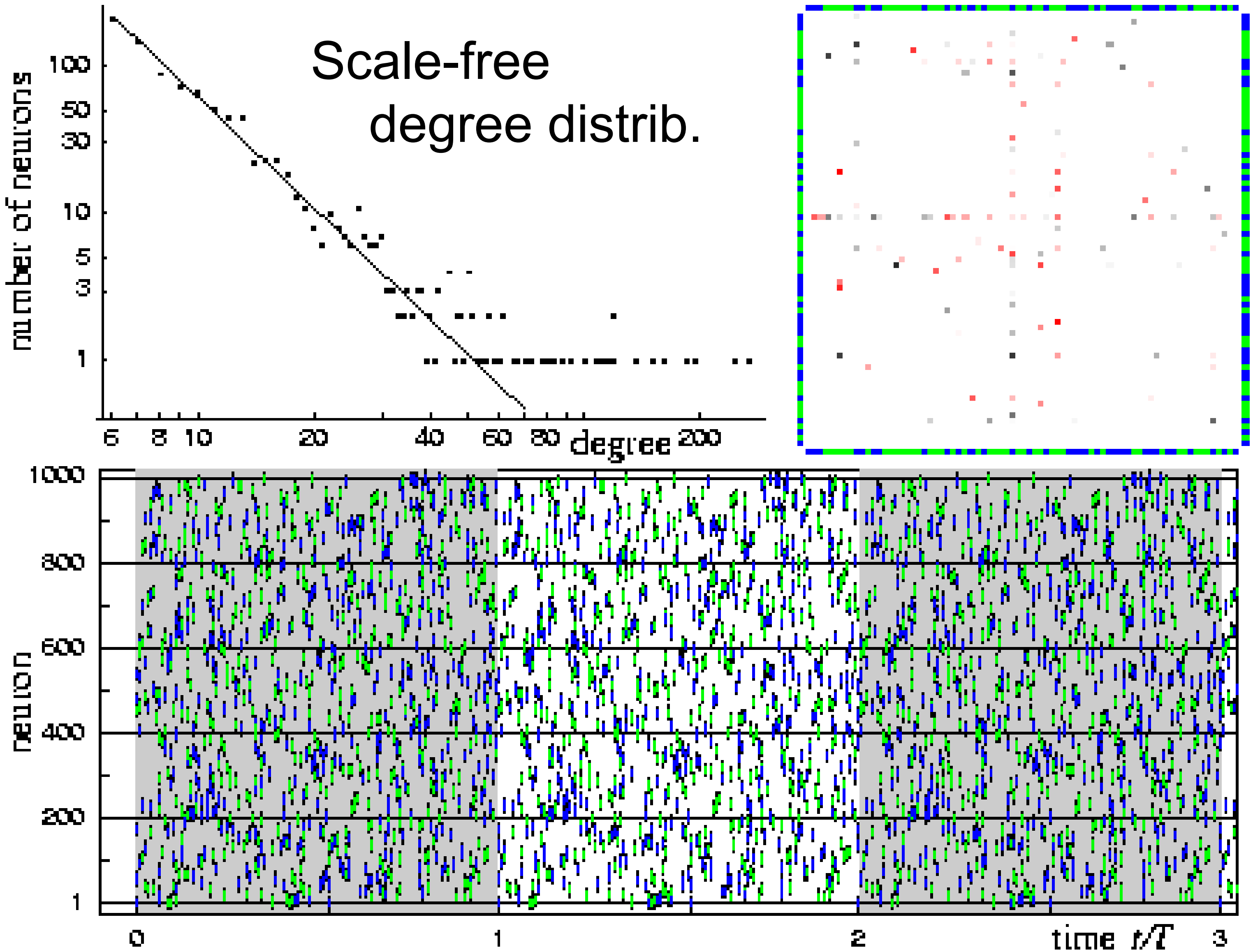
Statistically similar networks: same pattern may be stable or unstable



stable

unstable
(saddle)

Very Different Networks Show Same Pattern



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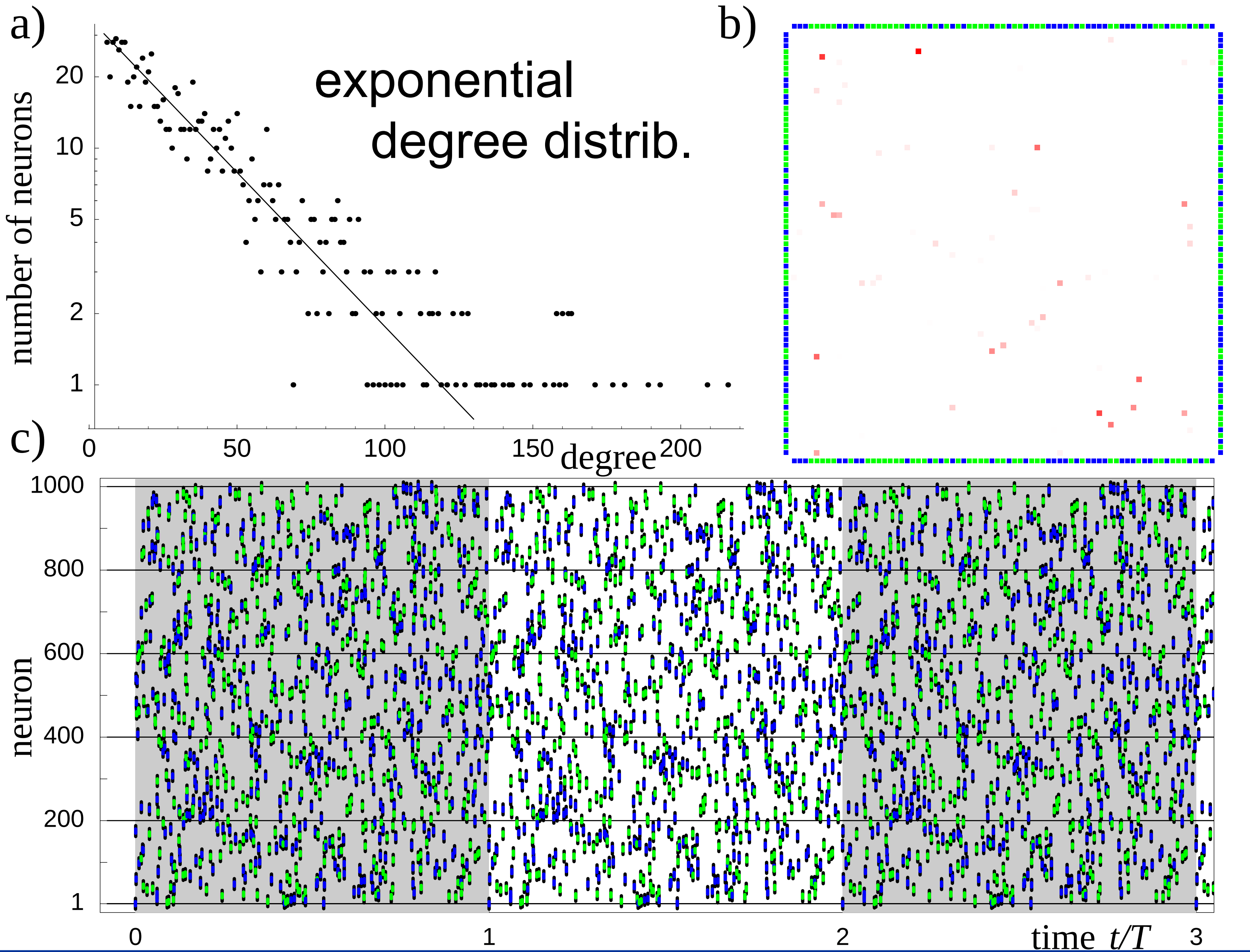
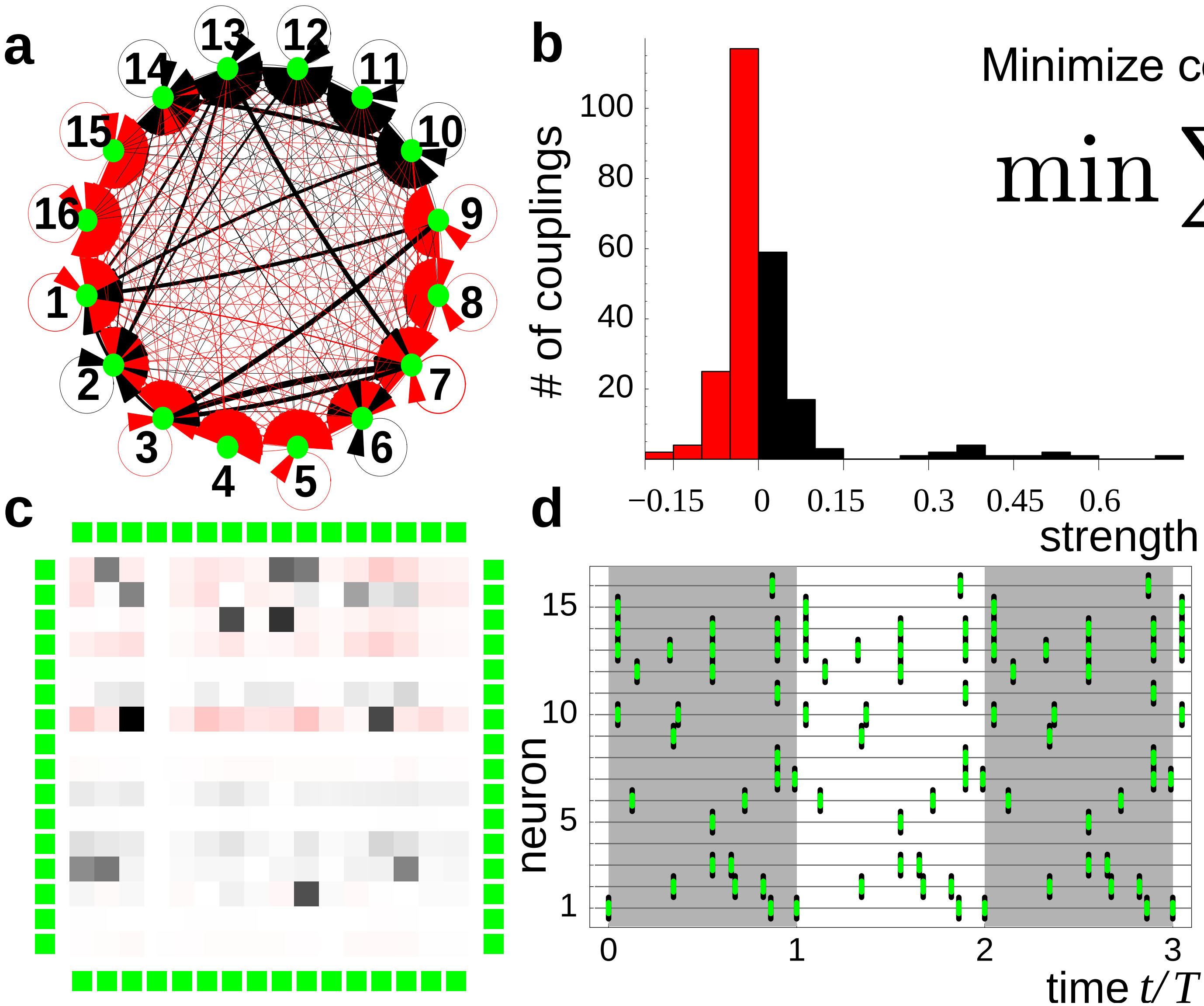


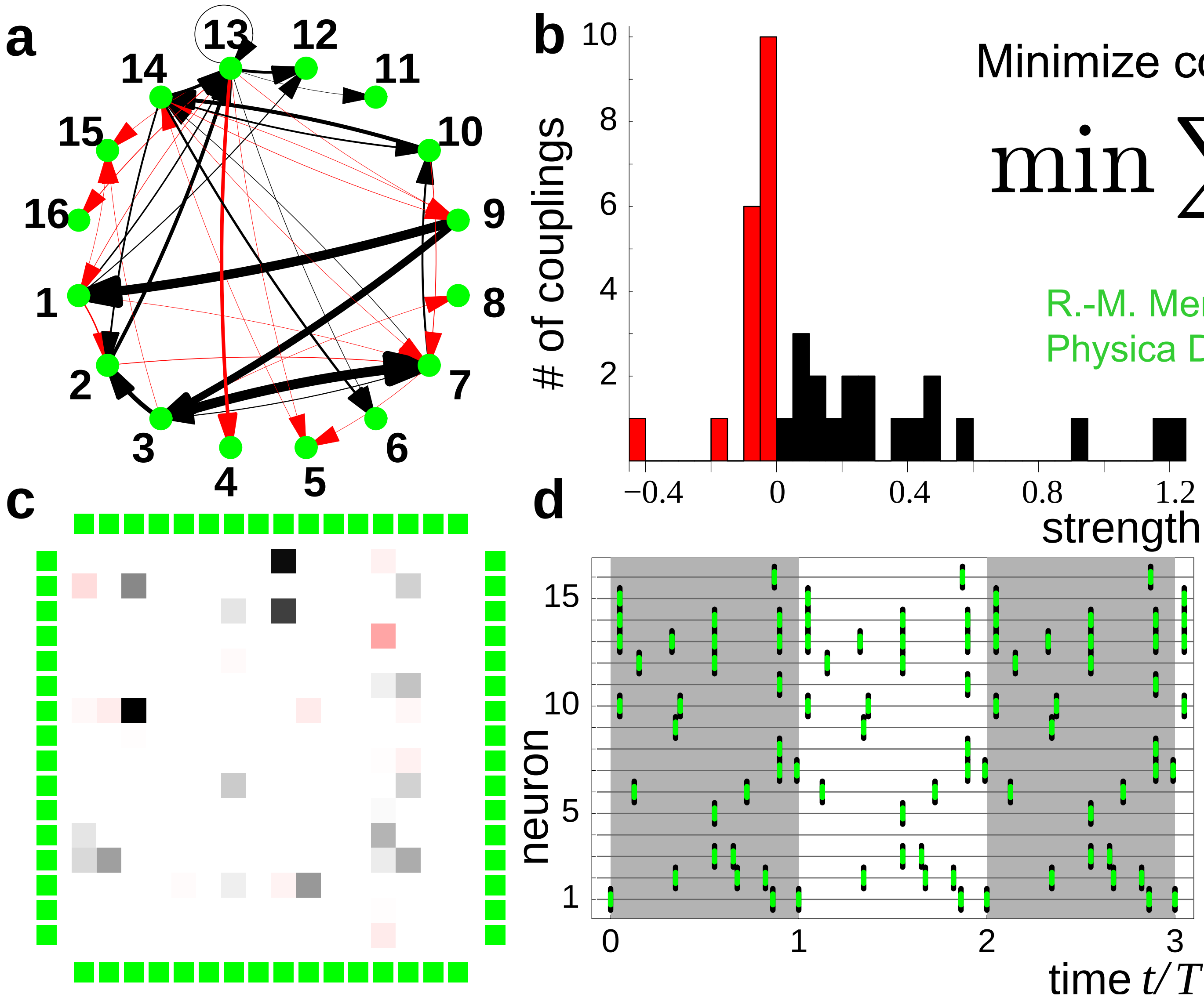
Optimal Design: Minimal Quadratic Coupling



Minimize coupling strengths:

$$\min \sum_{i,j} \varepsilon_{i,j}^2$$

Optimal Design: Minimal L1-Norm ➔ Sparsest Network



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Part I:

Inferring Network Topology From Complex Dynamics

Idea: local dynamics and coupling functions known,
interaction topology unknown, optimal solution to over-determined linear eqns.

New Journal of Physics 13:013004 (2011) presented on the board

Part II

Reconstructing Network Topology from Response Dynamics

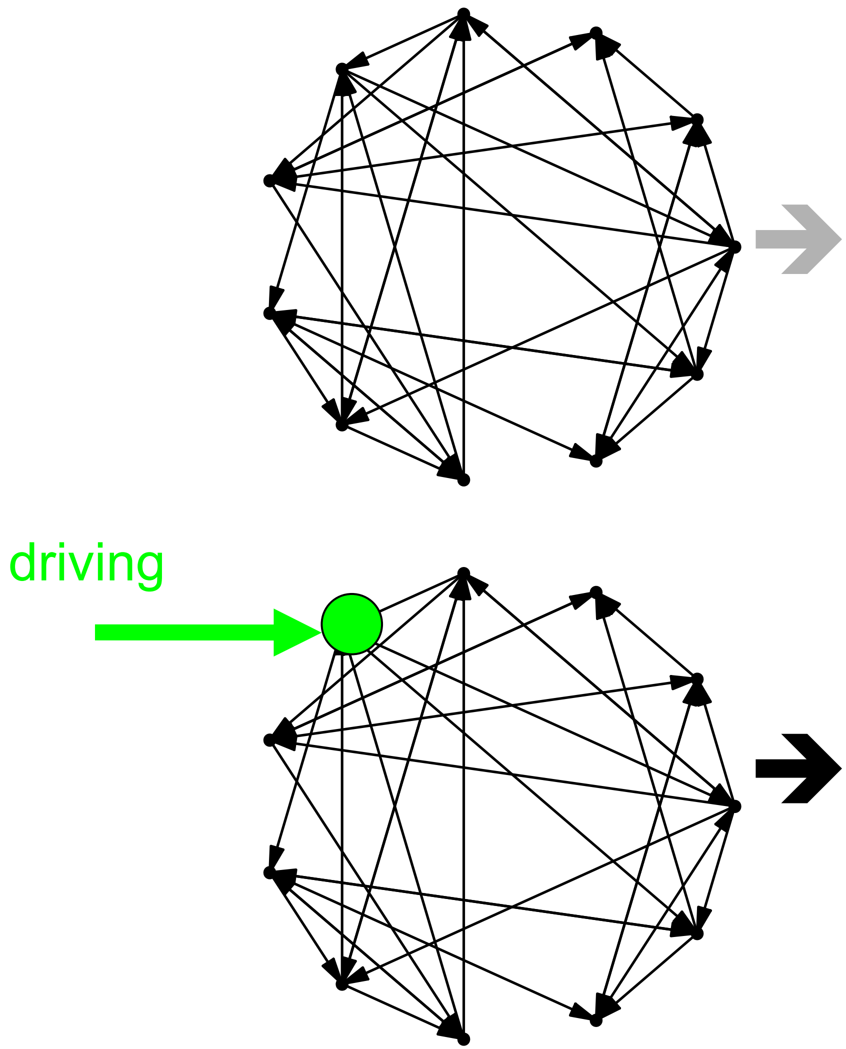
Idea: Given any stable fixed point or other stable trajectory:
Suff. small driving signals move fixed point. Where it moves depends on driving signal and location/s *as well as topology*. Europhys. Lett. 76:367 (2006)

➔ Topology can be inferred from evaluating several driving experiments.

Linear response for small driving sufficient ➔ no detailed knowledge of local dynamics and coupling functions required

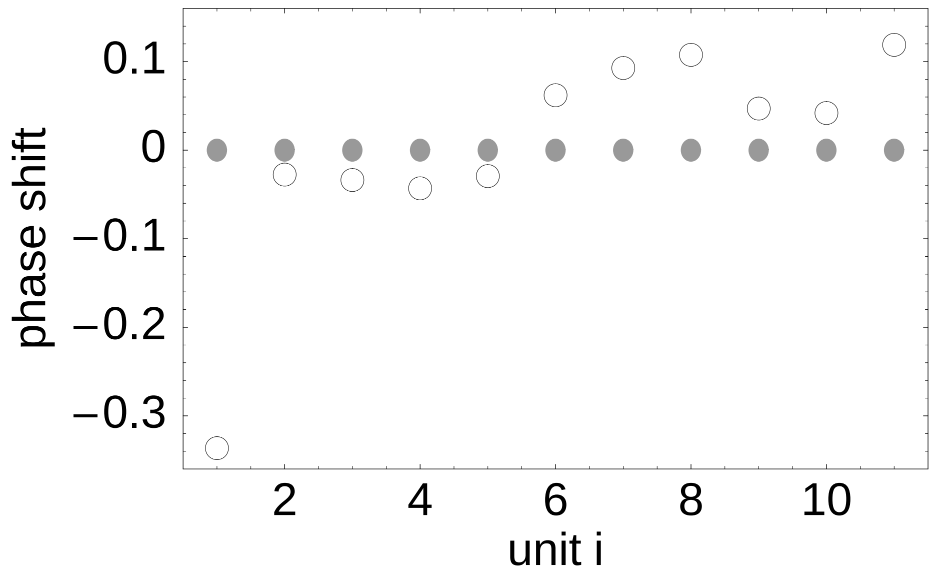
Phys. Rev. Lett. 98:224101 (2007) presented in the following

Inferring Network Connectivity by accessing the dynamics only



Phase Response to Driving

Homogeneous frequencies



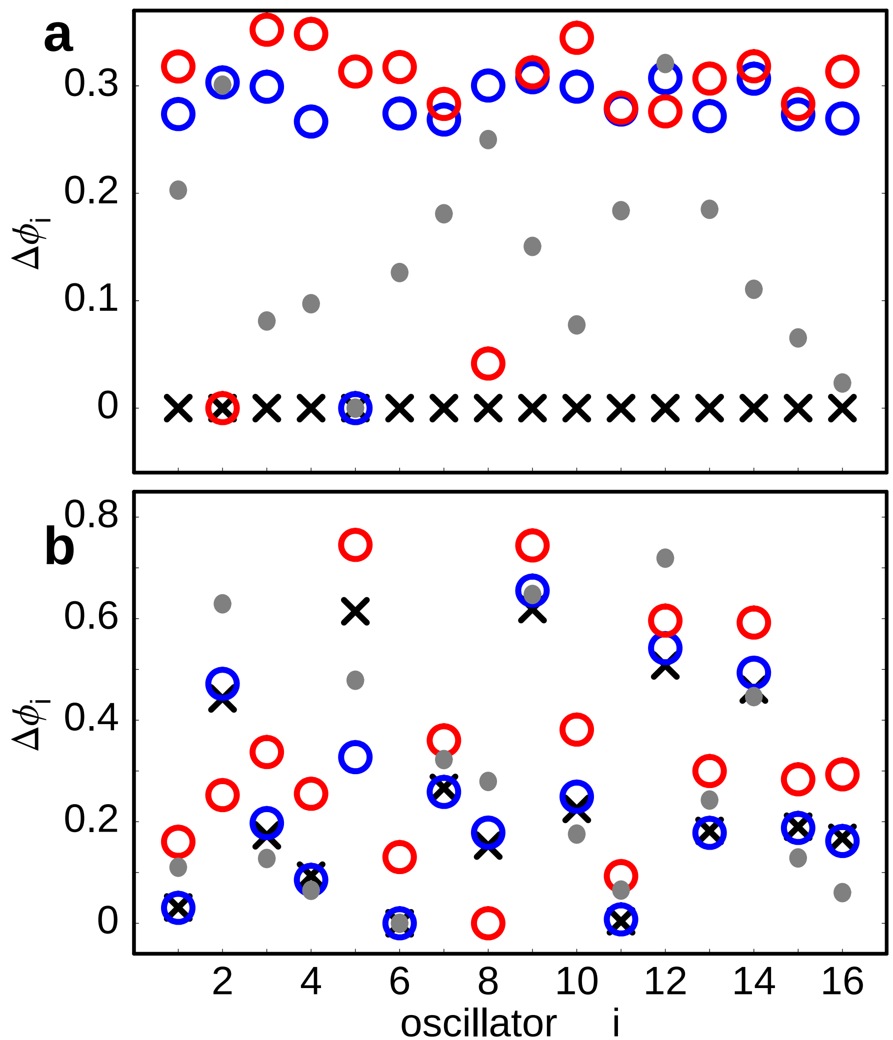
M.T., Europhys. Lett. 76:367 (2006)

Response to Driving (Phase Patterns)

$$d_t\phi_i = \omega_i + \sum_{j=1}^{N} J_{ij}\sin(\phi_j - \phi_i) + \ I_{i,m}$$

$m \in \{1,\ldots,M\}$ labels experimental condtion

Homogeneous frequencies →



x undriven

O i=5 driven $I_{i,m} = 0.2$

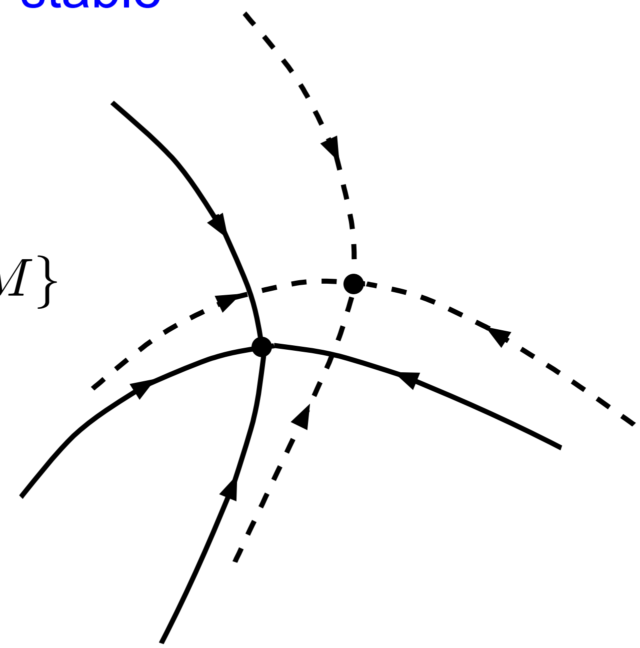
O i=2 and i=8 driven

● all units randomly driven

Heterogeneous frequencies →

Response Analysis I

Idea: dynamically stable states are structurally stable
+ many networks exhibit additive coupling



$$\dot{\phi}_{i,m} = \omega_i + \sum_{j=1}^{N} J_{ij} \sin(\phi_{j,m} - \phi_{i,m}) + I_{i,m}$$

driving condition $m \in \{1, \ldots, M\}$

Stationary phase-locked solution:

$$\Omega_m = \omega_i + \sum_{j=1}^{N} J_{ij} \sin(\phi_{j,m} - \phi_{i,m}) + I_{i,m}$$

Difference to unperturbed:

$$\Omega_m - \Omega_0 - I_{i,m} = \sum_{j=1}^{N} J_{ij} (\sin(\phi_{j,m} - \phi_{i,m}) - \sin(\phi_{j,0} - \phi_{i,0}))$$

for all $i \in \{1, \ldots, N\}$

***N* equations restrict the coupling matrix** J_{ij}

Response Analysis II

$$\Omega_m - \Omega_0 - I_{i,m} = \sum_{j=1}^{N} J_{ij}(\sin(\phi_{j,m} - \phi_{i,m}) - \sin(\phi_{j,0} - \phi_{i,0}))$$

Linearize:

$$D_{i,m} = \sum_{j=1}^{N} \hat{J}_{ij}\theta_{j,m}$$

$$D_{i,m} = \Omega_m - \Omega_0 - I_{i,m}$$

$$\theta_{j,m} = \phi_{j,m} - \phi_{j,0}$$

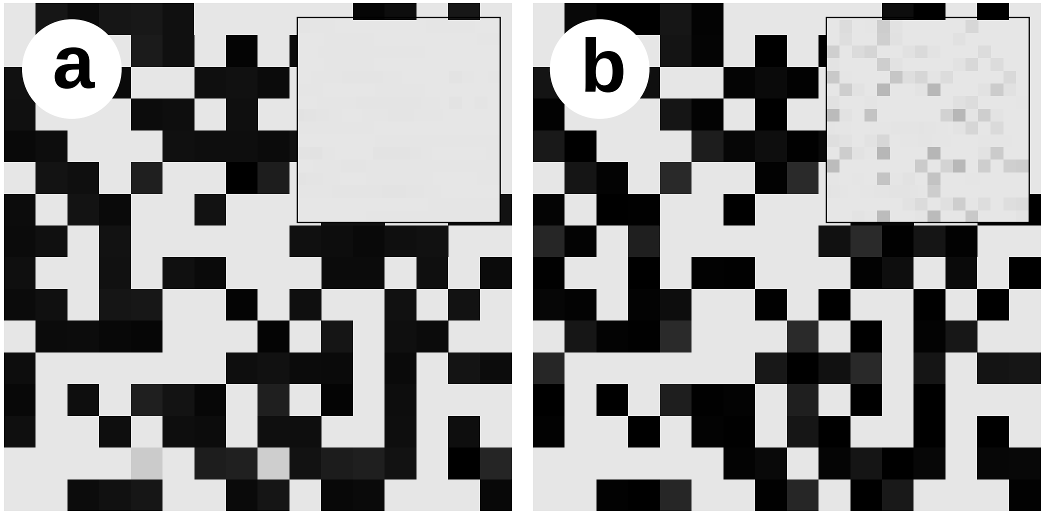
$$\hat{J}_{ij} = \begin{cases} \cos(\phi_{j,0} - \phi_{i,0})J_{ij} & \text{for} \quad i \neq j \\ -\sum_{k,k\neq j} \hat{J}_{ik} & \text{for} \quad i = j \end{cases}$$

Measurements under *M*=*N* driving conditions result in network topology $\hat{J} = D\theta^{-1}$

Response to Driving (Network Reconstruction)

M=N=16, k=8 random connections per unit

Homogeneous, heterogeneous frequencies



Reconstruction with M<N experiments ? - Analysis

Assume: **many links absent** (sparse network)
➔ sparsest network consistent with restrictions

$$D_{i,m} = \sum_{j=1}^{N} \hat{J}_{ij} \theta_{j,m}$$

Rewrite fairly using singular value decomposition

$$\theta^{\mathsf{T}} = USV^{\mathsf{T}} \quad \Rightarrow \quad \hat{J} = DU\tilde{S}V^{\mathsf{T}} + PV$$

Parameters with $P_{ij} = 0 \text{ for } j \leq M$

$$\min_P \sum_{j=1, j \neq i}^{N} \left| \hat{J}_{ij} \right|$$

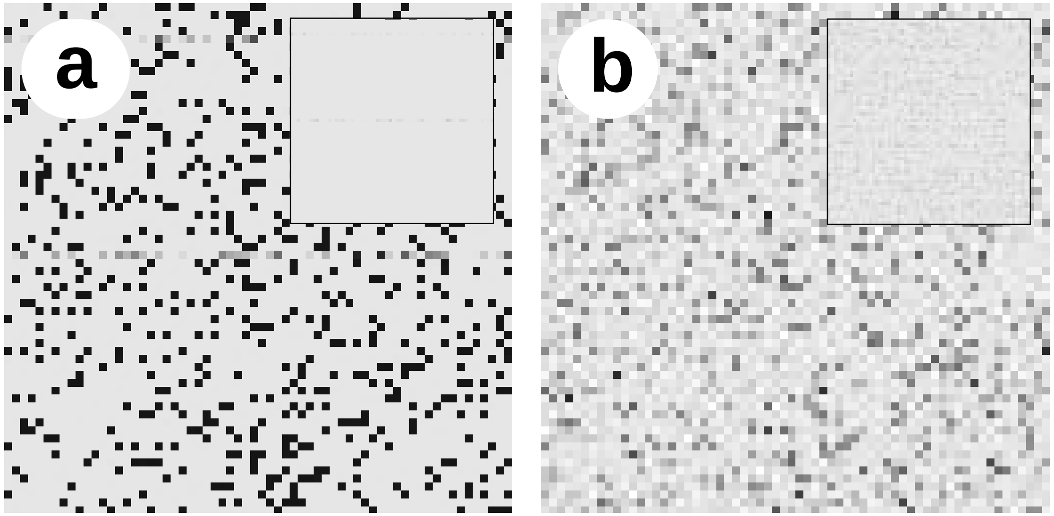
➔ Optimal network solution with **low number** of links

M.T., Phys. Rev. Lett. 98:224101 (2007)

Reconstruction with M<N experiments ? - Numerics

(N=64, k=10 random connections per unit)

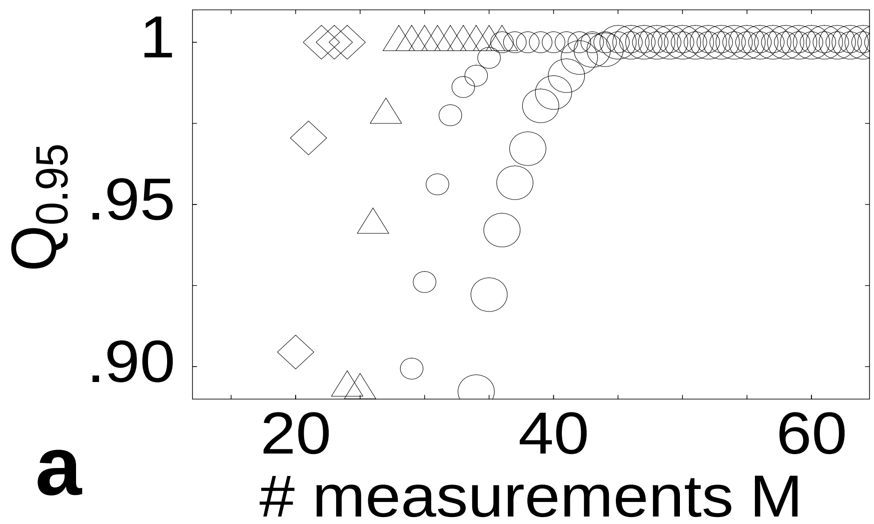
M=38 M=24



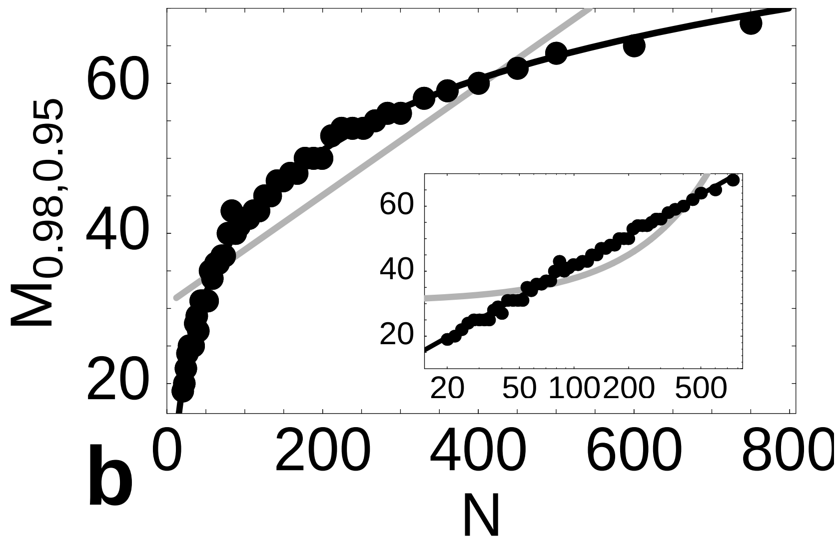
➔ Optimal network solution with low number of links

Quality of Reconstruction (k=10)

fraction of coupling strengths
considered correct (95% level)



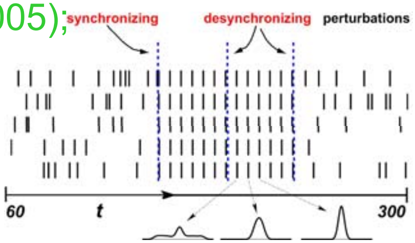
experiments needed



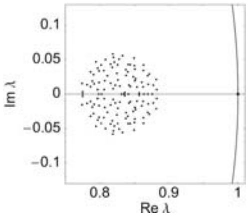
➔ Sublinear scaling of necessary # experiments

Challenges in Network Dynamics:
New Mathematics joins Neuroscience, Engineering & Physics

- **Unstable Attractors:** New mathematics from neural models
  Phys. Rev. Lett. 89:154105 (2002a); *Chaos* 13:377 (2003); *Nonlinearity* 18:20 (2005);
  Nature 436:36 (2005); *Phys. Rev. E* 78:065201(R) (2008).



- **Synchronization** in Networks: Multi-operator problems
  Phys. Rev. Lett. 89:258701 (2002c); *Phys. Rev. Lett.* 92:074103 (2004a)
  Phys. Rev. Lett. 93 (2004c); *Nonlinearity* 21:1579 (2008);



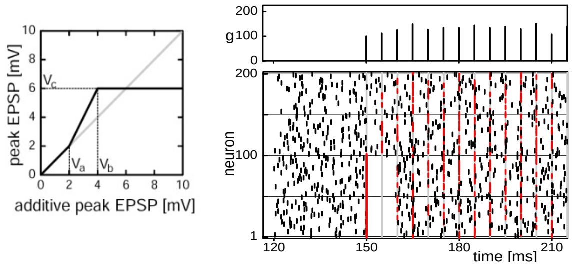
- **Speed Limits:** Explained by Random Matrix Theory
  Phys. Rev. Lett. 92:074101 (2004b); *Chaos* 16:015108 (2006);

- **Designing** networks exhibiting predefined patterns: 1st Inverse problem
  ***Phys. Rev. Lett.* 97:188101 (2006); *Physica D* 224:182 (2006);**

- **Reconstructing** complex network connectivity: 2nd Inverse problem
  ***Europhys. Lett.* 76:367 (2006); *Phys. Rev. Lett.* 98:224101 (2007);**
  ***New J. Phys.* 13:013004 (2011);** *Frontiers Comp. Neurosci.*, under review (2011)

- **Data Analysis Methods** to detect spatio-temporal relations: spikes/LFPs
  Neurocomputing 70:2096 (2007); *Neurosci. Res.* 61:S280 (2008).

Challenges in Network Dynamics:
New Mathematics joins Neuroscience, Engineering & Physics



- Theory of **spatio-temporal spike patterns**
  Frontiers in Neurosci. 3:2 (2009);
  Discr. Cont. Dyn, Syst. 28:1555 (2010);
  Handbook on Biological Networks (Chapter on ‘Spike Patterns’) (2010).

- **Novel routes to desynchronization:** Sequential bifurcations,…
  Phys. Rev. Lett. 102:068101 (2009); *Nonlinearity*, under review (2011);
  Europhys. Lett., 90:48002 (2010); *SIAM J. Appl. Math.* 70:2119 (2010)

- **Cortical ‘ground state’:** Chaos does NOT generate irregularity!
  Phys. Rev. Lett. 100:048102 (2008); *Frontiers in Comput. Neurosci*. 3:13 (2009);

- Nonlinear dynamics for **computation** and **autonomous systems**
  Nature Phys., 6:224 (2010); *J. Phys. A: Math. Theor.* 42:345103 (2009)

- **Complex disordered systems & counting problems** on graphs
  Phys. Rev. Lett. 88:245501 (2002); *Cornell Rep.* 1813:1352 (2007); *New J Phys.* 11:023001 (2009);
  Nature Phys., in press (2011); *J. Phys. A: Math. Theor.*, 43:175002 (2010)

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YOU all for your attention !

Questions & Comments Welcome!