Inverse Problems and Optimal Control for PDEs

23 – 27 May 2011

Warwick Mathematics Institute

Inverse Problems and Optimal Control for PDEs
23 – 27 May 2011 – Lecture room B3.03

Monday 23 May 2011

9.45-11.15	Coffee & Registration in B1.37 Maths	
11.15-12.45	**T Banks** (North Carolina)	*Aggregate Data and Inverse Problems*
12.45-2.00	Lunch in Maths Common Room	
2.00-3.30	**A Stuart** (Warwick)	*The Bayesian Approach to Inverse Problems Pt1*
3.30-4.15	Tea/Coffee in Maths Common Room	
4.15-5.45	**T Banks** (North Carolina)	*Propagation of Uncertainty in Dynamical Systems, Random Differential Equations and Stochastic Differential Equations*
5.45	Poster Session with Drinks & Snacks in Maths Common Room	

Tuesday 24 May 2011

9.00-10.30	**F Troltzsch** (Berlin)	*Aspects of numerical analysis in the optimal control of nonlinear PDEs Pt1*
10.30-11.15	Tea/Coffee in Maths Common Room	
11.15-12.45	**A Stuart** (Warwick)	*The Bayesian Approach to Inverse Problems Pt2*
12.45-2.00	Lunch in Maths Common Room	
2.00-3.30	**F Troltzsch** (Berlin)	*Aspects of numerical analysis in the optimal control of nonlinear PDEs Pt2*
3.30-4.15	Tea/Coffee in Maths Common Room	
4:15-5:00	**O Scherzer** (Vienna)	*Variational Methods for the Solution of Inverse Problems*
5.00-5.45	**M Hinze** (Hamburg)	*Identification of matrix parameters in elliptic PDEs*

Wednesday 25 May 2011

9:00-9.45	**C Elliott** (Warwick)	*Numerical analysis of an inverse problem for the eikonal equation*
9.45-10.30	**A Walther** (Paderborn)	*Solving large-scale inverse electromagnetic scattering problems: A parallel AD-based approach*
10.30-1.15	Tea/Coffee in Maths Common Room	
11.15-2.00	**A Szesspezy** (Stockholm)	*Hamilton-Jacobi theory for inverse PDE*
12.00-12.45	**V H Haong** (Nanyang)	*Bayesian inverse problems for Burgers and Hamilton-Jacobi equations with white-noise forcing*
12.45-2.00	Lunch in Maths Common Room	
2.00-2.45	**D Xiu** (Purdue)	*(TBC)*
2.45-3.30	**K Cliffe** (Nottingham)	*Flow in a Channel with a Sudden Expansion*
3.30-4.15	Tea/Coffee in Maths Common Room	
4.15-5.00	**A Wathen** (Oxford)	*Preconditioned iterative linear solvers for PDE-constrained Optimization problems*
5.00-5.45	**R Herzog** (Chemnitz)	*On Nonlinear Optimal Control Problems with an L^1 Norm*
7pm	Conference Dinner – Radcliffe House	

(cont.)

Thursday 26 May 2011

9.00-9.45	**N O Ghattas** (Texas)	*A stochastic Newton method for large-scale statistical inverse problems, with application to seismic inversion*
9.45-10.30	**O Dorn** (Manchester)	*Level set methods for structural inversion*
10.30-1.15	Tea/Coffee in Maths Common Room	
11.15-2.00	**P Barbone** (Boston)	*Two steps forward and one step back: Improved inverse problem solutions using improved forward solvers*
12.00-12.45	**E Casas** (Cantabria)	*A Paradox in the Approximation of Dirichlet Control Problems in Curved Domains*
12.45-2.00	Lunch in Maths Common Room	
2.00-2.45	**A Simoni** (Milan)	*Regularizing Priors for Linear Inverse Problems*
2.45-3.30	**S Siltanen** (Helsinki)	*Sparsity-promoting Bayesian inversion*
3.30-4.15	Tea/Coffee in Maths Common Room	
4.15-5.00	**N Nichols** (Reading)	*Conditioning, Preconditioning and Regularization of the Optimal State Estimation Problem*
5.00-5.45	**M Freitag** (Bath)	*Resolution of sharp fronts in the presence of model error in variational data assimilation*

Friday 27 May 2011

9.00-9.45	**M Hintermueller** (Humboldt)	*Automated regularization parameter selection in multi-scale total variation models for image restoration*
9.45-10.30	**C Schoenlieb** (Cambridge)	*Regularized optimal transport for smoothing and density estimation*
10.30-1.15	Tea/Coffee in Maths Common Room	
11.15-12.00	**E Zuazua** (BCAM)	*Control and Numeric : Continuous versus discrete approaches*
12.00-12.45	Lunch in Maths Common Room	

Lecture Titles

Surname	Christian Name	Affiliation	Title
Banks (Lecture 1)	H Thomas	(North Carolina)	Aggregate Data and Inverse Problems
Banks (Lecture 2)			Propagation of Uncertainty in Dynamical Systems, Random Differential Equations and Stochastic Differential Equations
Barbone	Paul	(Boston)	Two steps forward and one step back: Improved inverse problem solutions using improved forward solvers
Casas	Eduardo	(Cantabria)	A Paradox in the Approximation of Dirichlet Control Problems in Curved Domains
Cliffe	Kenneth Andrew	(Nottingham)	Flow in a Channel with a Sudden Expansion
Dorn	Oliver	(Manchester)	Level set methods for structural inversion
Elliott	Charlie	(Warwick)	Numerical analysis of an inverse problem for the eikonal equation
Freitag	Melina	(Bath)	Resolution of sharp fronts in the presence of model error in variational data assimilation
Ghattas	Omar	(Texas)	A stochastic Newton method for large-scale statistical inverse problems, with application to seismic inversion
Hoang	Viet	(Nanyang)	Bayesian inverse problems for Burgers and Hamilton-Jacobi equations with white-noise forcing
Herzog	Roland	(Chemnitz)	On Nonlinear Optimal Control Problems with an L^1 Norm
Hintermueller	Michael	(Berlin)	Automated regularization parameter selection in multi-scale total variation models for image restoration
Hinze	Michael	(Hamburg)	Identification of matrix parameters in elliptic PDEs
Nichols	Nancy	(Reading)	Conditioning, Preconditioning and Regularization of the Optimal State Estimation Problem
Scherzer	Otmar	(Vienna)	Variational Methods for the Solution of Inverse Problems
Schoenlieb	Carola-Bibiane	(Cambridge)	Regularized optimal transport for smoothing and density estimation
Siltanen	Samuli	(Helsinki)	Sparsity-promoting Bayesian inversion
Simoni	Anna	(Milan)	Regularizing Priors for Linear Inverse Problems
Stuart Lecture 1	Andrew	(Warwick)	The Bayesian Approach to Inverse Problems
Stuart Lecture 2			
Szepessy	Anders	(Stockholm)	Hamilton-Jacobi theory for inverse PDE
Tröltzsch Lecture 1	Fredi	(Berlin)	Aspects of numerical analysis in the optimal control of nonlinear PDEs
Tröltzsch Lecture 2			
Walther	Andrea	(Paderborn)	Solving large-scale inverse electromagnetic scattering problems: A parallel AD-based approach
Wathen	Andy	(Oxford)	Preconditioned iterative linear solvers for PDE-constrained Optimization problems
Xiu	Dongbin	(Purdue)	An Efficient Surrogate Approach to Bayesian Inference in Inverse Problems
Zuazua	Enrique	(BCAM)	Control and Numerics : Continuous versus discrete approaches

<u>Poster Titles</u>

Surname	**Christian Name**	**Affiliation**	**Title**
Agapiou	Sergios	(Warwick)	A Bayesian approach to the Laplacian-like Inverse Problem
Dashti	Masoumeh	(Warwick)	A Bayesian approach to an elliptic inverse problem
Knapik	Bartek	(Vu)	A Bayesian Approach to Inverse Problems
Lamm, Brett, McCormick, Scott		(Warwick)	Data Assimilation for the 2D Navier Stokes Equations
Law	Kody	(Warwick)	Data Assimilation for 2D Navier Stokes
Vollmer	Sebastian		Model Error in Bayesian Inverse Problems

ABSTRACTS

(Agapiou)

A Bayesian approach to the Laplacian-like Inverse Problem

(Poster)

We study the consistency of the Bayesian Approach to Linear Inverse Problems in the small noise limit in the separable Hilbert-Space setting.

We start with the Laplacian-like Inverse Problem with noise and prior covariances diagonalizable in the eigenbase of the Laplacian and generalize to more general situations. Numerical results are presented which support our theory.

(Banks)

Pt 1 Aggregate Data and Inverse Problems

Pt 2 Propagation of Uncertainty in Dynamical Systems, Random Differential Equations and Stochastic Differential Equations

We discuss the interplay between stochastic and deterministic formulations and propagation of uncertainty in distributed dynamical systems. Inverse problems for stochastic systems as well as systems with probability measures as parameters to be estimated are presented. The theoretical and computational methods discussed are motivated with examples from biomass growth and control problems and examples from labeled cell proliferation modeling.

(Barbone)

Two steps forward and one step back: Improved inverse problem solutions using improved forward solvers

Two steps forward and one step back: Improved inverse problem solutions using improved forward solvers

C.E. Rivas Aroni, P. E. Barbone, A. A. Oberai

May 5, 2011

A large class of inverse problems may be described as trying to infer medium properties in a domain based upon measurement of a field within or on the boundary of that domain. The ability to do this depends upon the measured field being sensitive to changes in the material properties. This sensitivity may be expressed in terms of an inequality which may be derived directly from the governing partial differential equations. More precisely, there exists a constant $0 < C < \infty$ such that:

$$\|\delta\mu\|_\mu \leq C\|\delta u\|_u \tag{1}$$

Here, $\delta\mu$ is an infinitesimal variation in material properties, δu is the corresponding change in the relevant field, C is a constant. The norms in equation (1) are application dependent.

Equation (1) indicates that any variation in material properties $\delta\mu$ results in a measurable variation in the field, δu. When equation (1) is satisfied for a particular problem, then we may hope to uniquely infer the properties from the given measurements. When (1) is violated, we cannot hope to infer the properties from the measurements without some external assumptions.

We consider a class of partial differential equations (PDEs) that govern inverse scalar and vector potential problems. For these (PDEs), we can show that equation (1) is satisfied. We then consider traditional FEM discretization of the PDEs, and examine the discrete counterpart to equation (1). We find in many cases that, though (1) is satisfied in the continuous case, it is not satisfied in the discrete case. We then derive improved FEM discretization methods that satisfy (1), even in the discrete case. Numerical examples shall also be presented.

(Casas)

A Paradox in the Approximation of Dirichlet Control Problems in Curved Domains

A Paradox in the Approximation of Dirichlet Control Problems in Curved Domains

Eduardo Casas* Andreas Günther† Mariano Mateos‡

ABSTRACT

In this talk we consider the following optimal control problem

$$(\mathrm{P})\begin{cases}\min J(u) = \int_\Omega L(x, y_u(x))\,dx + \frac{N}{2}\int_\Gamma u^2(x)\,d\sigma(x)\\ \text{subject to } (y_u, u) \in (L^\infty(\Omega)\cap H^1(\Omega))\times L^2(\Gamma),\\ \alpha \le u(x) \le \beta \quad \text{for a.e. } x\in\Gamma,\end{cases}$$

where Γ is a smooth manifold, y_u is the state associated to the control u, given by a solution of the Dirichlet problem

$$\begin{cases} -\Delta y + a(x,y) &= 0 \quad \text{in } \Omega,\\ y &= u \quad \text{on } \Gamma.\end{cases} \tag{1}$$

To solve the problem (P) numerically, it is usually necessary to approximate Ω by a (typically polygonal) new domain Ω_h. The difference between the solutions of both infinity dimensional control problems, one formulated in Ω and the second in Ω_h, was studied in [1], where an error of order $O(h)$ was proved. In [2], the numerical approximation of the problem defined in Ω was considered. The authors used a finite element method such that Ω_h was the polygon formed by the union of all triangles of the mesh of parameter h. They proved an error of order $O(h^{3/2})$ for the difference between continuous and discrete optimal controls. Here we show that the estimate obtained in [1] cannot be improved, which leads to the paradox that the numerical solution is a better approximation of the optimal control than the exact one obtained just by changing the domain Ω to Ω_h.

References

[1] E. Casas and J. Sokolowski, *Approximation of boundary control problems on curved domains*, SIAM J. Control Optim., 48 (2010), pp. 3746–3780.

[2] K. Deckelnick, A. Günther, and M. Hinze, *Finite element approximation of Dirichlet boundary control for elliptic PDEs on two- and three-dimensional curved domains*, SIAM J. Control Optim., 48 (2009), pp. 2798–2819.

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‡ Universidad de Oviedo, Spain (mmateos@uniovi.es)

(Cliffe)

Flow in a Channel with a Sudden Expansion

Numerical calculations of laminar flow in a two-dimensional channel with a sudden expansion exhibit a symmetry breaking bifurcation at Reynolds number 40.45 when the expansion ratio is 3:1. In the experiments reported by Fearn, Mullin and Cliffe (JFM, Vol 211, pp.595-608, 1990) there is a large perturbation to this bifurcation and the agreement with the numerical calculations is surprisingly poor. The talk will describe an attempt to explain this discrepancy using techniques from uncertainty quantification and Bayesian inverse methods.

(Dashti)

A Bayesian approach to an elliptic inverse problem

(Poster)

We consider the inverse problem of estimating a function from noisy, possibly nonlinear, observations. We adopt a Bayesian approach to the problem with Besov prior measures. In doing so a key technical tool is the development of a Fernique-like theorem for Besov measures. This theorem enables us to identify appropriate conditions on the forward solution operator which imply the well-definedness and well-posedness of the posterior measure, and to match the prior Besov measure to these conditions. We then consider the application of these results to the inverse problem of finding the diffusion coefficient of an elliptic partial differential equation, given noisy measurements of its solution.

(Dorn)

Level set methods for structural inversion

When solving inverse problems we typically have physical data given and are looking for an image (which might be 2D or 3D) which reproduces these data in some specified sense. Typically these images represent coefficients of a Partial Differential Equation, called the forward model, and a simulator is used in order to verify the data fidelity of any given image which is proposed as a possible solution. Classical inverse problems theory tells us that inverse problems are ill-posed, and a well-defined unique image which fits the data either does not exist, or the direct reconstruction of a useful image from data is unstable, or (typically) both. Then, in most deterministic approaches for calculating candidate images, regularization schemes are employed which provide images from a certain class, in most cases smooth images. Certainly, natural images are not necessarily smooth, but might contain discontinuities which provide the image with structure. This structure is of high importance in many applications. It might represent different lithologies in geophysical applications, or different materials in non-destructive testing, or different organs or tissue types in medical imaging applications. Structure can be imposed by brute force on the already reconstructed images by applying off-the-shelf or tailor-made image segmentation techniques. However, this approach has various drawbacks in many applications, and might simply not work at all. We propose in this talk a novel level-set based approach for finding such structured images (i.e. containing sharp interfaces between different characteristic regions) directly from the given data without making the detour via image post-processing techniques. Several examples from medical imaging, petroleum engineering and non-destructive testing applications are presented that show the performance of this novel technique in realistic situations.

(Elliott)

Numerical analysis of an inverse problem for the eikonal equation

We are concerned with the inverse problem for an eikonal equation of determining the speed function using observations of the arrival time on a fixed surface. This is formulated as an optimisation problem for a quadratic functional with the state equation being the eikonal equation coupled to the so—called Soner boundary condition. The state equation is discretised by a suitable finite difference scheme for which we obtain existence, uniqueness and an error bound. We set up an approximate optimisation problem and show that a subsequence of the discrete mimina converges to a solution of the continuous optimisation problem as the mesh size goes to zero. The derivative of the discrete functional is calculated with the help of an adjoint equation which can be solved efficiently by using fast marching techniques. Finally we describe some numerical results.

(Freitag)

Resolution of sharp fronts in the presence of model error in variational data assimilation

Data assimilation is an important tool for numerical weather prediction. In this talk we give an introduction to data assimilation and show that data assimilation using four-dimensional variation (4DVar) can be interpreted as a form of Tikhonov regularisation, a familiar method for solving ill-posed inverse problems. It is known from image restoration problems that L1-norm penalty regularisation recovers sharp edges in the image better than the L2-norm penalty regularisation. We apply this idea to 4DVar for problems where shocks are present and give some examples where the L1-norm penalty approach performs better than the standard L2-norm regularisation in 4DVar.

(Ghattas)

A stochastic Newton method for large-scale statistical inverse problems, with application to seismic inversion

We are interested in the solution of several inverse problems in solid earth geophysics, including the inference of mantle constitutive parameters from observed plate motions, earth seismic velocities from surface seismograms, and polar ice sheet basal friction from satellite
observations of ice sheet velocities. Each of these inverse problems is most naturally cast as a large-scale statistical inverse problem in the framework of Bayesian inference. The complicating factors are the high-dimensional parameter spaces (due to discretization of infinite-dimensional parameter fields) and very expensive forward problems.
Here we present a so-called stochastic Newton method in which MCMC is accelerated by constructing and sampling from a proposal density that builds a local Gaussian approximation based on local gradient and Hessian (of the log posterior) information. This can be interpreted as a Hessian-preconditioned Metropolized Langevin method. Hessian manipulations (inverse, square root) are made tractable by a low rank approximation that exploits the compact nature of the data misfit operator. This amounts to a reduced model of the parameter-to-observable map. We apply the method to 3D global seismic inverse problems, illustrating the efficacy and scalability of the low rank approximation. We discuss associated issues including elastic-acoustic wave propagation coupling, discontinuous Galerkin discretization, gradient and Hessian consistency, adaptivity on forest of octree meshes, and scalability on petascale and GPU-accelerated systems.
This work is joint with: Tan Bui-Thanh, Carsten Burstedde, James Martin, Georg Stadler, Lucas Wilcox

(Hoang)

Bayesian inverse problems for Burgers and Hamilton-Jacobi equations with white-noise forcing

We consider Bayesian inverse problems for inference in a topological probability space. We show that the condition established for the Banach space setting by S. Cotter, M. Dashti, J. Robinson and A. Stuart (Inverse Problems, Vol. 25, 2009) for the validity of the Bayes formula holds in the most general case. We also formulate a general formula for the wellposedness of the posterior measure, which is a generalization of the polynomial growth condition by Cotter et al. for Gaussian prior measures. We then apply the theory to Burgers and Hamilton-Jacobi equations with white noise in time forcing on a semi-infinite time interval. Inference is made on the white noise forcing, assuming the Wiener measure as the prior.

(Herzog)

On nonlinear optimal control problems with an L^1 norm

ON NONLINEAR OPTIMAL CONTROL PROBLEMS WITH AN L^1 NORM

joint work with Eduardo Casas and Gerd Wachsmuth

We consider optimal control problems governed by semilinear elliptic equations which feature an L^1 norm in the objective, i.e.,

$$(\mathbf{P}) \quad \begin{cases} \text{Minimize} & J(u) = \frac{1}{2}\|y - y_d\|^2_{L^2(\Omega)} + \frac{\nu}{2}\|u\|^2_{L^2(\Omega)} + \mu\|u\|_{L^1(\Omega)} \\ \text{such that} & Ay + d(\cdot, y) = u \quad \text{in } \Omega \\ & y = 0 \quad \text{on } \partial\Omega \\ \text{and} & u_a \leq u \leq u_b. \end{cases}$$

The L^1 norm is often a natural measure for the control cost. Moreover, it produces optimal controls which are sparsely supported, i.e., which are identically equal to zero on significant parts of the domain. This is a desirable property, for instance, in actuator placement problems.

Problem $(\mathbf{P})$ is non-differentiable due to the L^1 norm and non-convex due to the nonlinear control-to-state mapping. In this talk we address first and second order optimality conditions. The latter are also the basis for finite element discretization error estimates. Numerical examples complement the analysis.

(Hintermueller)

Automated regularization parameter selection in multi-scale total variation models for image restoration

A multi-scale total variation model for image restoration is introduced. The model utilizes a spatially dependent regularization parameter in order to enhance image regions containing details while still sufficiently smoothing homogeneous features. The fully automated adjustment strategy of the regularization parameter is based on local variance estimators. For robustness reasons, the decision on the acceptance or rejection of a local parameter value relies on a confidence interval technique based on the expected maximal local variance estimate. In order to speed-up the performance of the update scheme a generalized hierarchical decomposition of the restored image is used. The corresponding subproblems are solved by a superlinearly convergent algorithm based on Fenchel-duality and inexact semismooth Newton techniques. The talk ends by a report on numerical tests, a qualitative study of the proposed adjustment scheme and a comparison with popular total variation based restoration methods.

(Hinze)

Identification of matrix parameters in elliptic PDEs

We consider identification of the diffusion matrix in elliptic PDEs from measurements. We prove existence of solutions using the concept of H-convergence. We discretize the problem using variational discretization and prove Hd-convergence of the discrete solutions by adapting the concept of Hd-convergence introduced by Eymard and Gallouet for finite-volume discretizations to finite element approximations. Furhtermore, we prove strong convergence of the discrete coefficients in L2, and of the associated discrete states in the norm of the oberservation space.

(Knapik)

A Bayesian Approach to Inverse Problems

(Poster)

In my poster I present a Bayesian approach to estimating a parameter μ from an observation Y following the model

$$Y = K\mu + \frac{1}{\sqrt{n}}Z.$$

The unknown parameter μ is an element of a separable Hilbert space H_1, and is mapped into another Hilbert space H_2 by a known, compact, injective, linear operator $K : H_1 \to H_2$. The image $K\mu$ is perturbed by unobserved, scaled Gaussian white noise Z.

In order to make inference about μ one can put a Gaussian process prior on μ. The poster is focused on two aspects of inverse problems - estimation of the full parameter μ and linear functionals of μ.

Both in nonparametric and linear functional case, we show the rate of the contraction of the posterior distribution around the truth, and we inspect the frequentist coverage of credible sets. In particular, the behaviour of the posterior depends on the regularity of the element μ, the regularity of the prior, and the ill-posedness of the operator K, which is defined by its spectral properties. Correct combinations of these characteristics lead to the optimal rate of recovery.

The results are numerically illustrated by the problem of recovering a function form observation of a noisy version of its primitive.

(Lam, Brett, McCormick, Scott)

Data Assimilation for the 2D Navier Stokes Equations

(Poster)

We consider 3DVar data assimilation for the 2D Navier-Stokes equations. We prove analytic results to show that, for certain choices of the covariance operators, di fferent estimators with the same observations converge geometrically, and that estimators come within bounded distance from the true solution provided the observation noise is uniformly bounded. We back this up with numerical evidence, and predict that the theorems should hold for other choices of the covariance operators.

(Law)

Data Assimilation for 2D Navier Stokes

(Poster)

We investigate sequential approximations to the filtering posterior for the 2D Navier-Stokes equations. We compare the true Bayesian posterior distribution (obtained by MCMC) as well as the maximum a posteriori (MAP) estimator (obtained by 4DVAR) with sequential approximations (3DVAR and approximate Kalman filters).

(Nichols)

Conditioning, Preconditioning and Regularization of the Optimal State Estimation Problem

Abstract:

Data assimilation is a technique for determining an 'optimal' estimate of the current and future states of a dynamical system from a prior estimate, or model forecast, together with observations of the system. Applications arise in very large environmental problems where the number of state variables is $O(10^7 - 10^8)$ and the number of observations is $O(10^4 - 10^6)$. The errors in the prior estimate and in the observations are assumed to be random with known distributions and the solution to the optimization problem is taken to be the *maximum a posteriori* likelihood estimate. With the aid of Bayes Theorem, the problem reduces to a very large nonlinear least squares problem, subject to the dynamical system equations. The problem is treated in practice using an approximate Gauss-Newton iterative method, where a linearized least squares problem is solved at each step of the procedure by an 'inner' gradient iteration technique.

The convergence of the variational scheme and the sensitivity of the solution to perturbations are dependent on the conditioning of the least-squares variational equation. The problem is generally ill-conditioned and hence is difficult to solve quickly and accurately. In this study we examine how different components of the assimilation system influence the conditioning of the least squares problem. Theoretical bounds on the condition number are presented and used to predict how the condition number is affected by the observation distribution and accuracy and by the specified lengthscales in the background error covariance matrix. A commonly used preconditioning technique, in which the prior states are transformed to uncorrelated variables, is also analysed. We show that the problem becomes more ill-conditioned with increasingly dense and accurate observations. The preconditioned system can be viewed as a regularized least squares problem, where the problem becomes more ill-posed as the filtering of noise in the data is reduced.

(This is joint work with S.A. Haben and A. S. Lawless.)

(Scherzer)

Variational Methods for the Solution of Inverse Problems

In the paper we give an overview on variational regularization methods in Banach spaces. The theory is developed based on the classical theory of regularization me¬thods in Hilbert spaces. An essential ingredient of such are source conditions, which will be generalized to the Banach spaces setting.
After that the example of sparsity regularization, which has proven to be a powerful tool in imaging, will be analyzed in this framework. Such a result can be obtained (even in an infinite dimensional setting) from variational regularization theory in Banach spaces as well.
Finally we present some applications to Radar imaging, and, if time allows, to Pho-toacoustic Imaging.
This is joint work with M. Grasmair, M. Haltmeier, C. Pöschl and E. Resmerita.

(Schoenlieb)

Regularized optimal transport for smoothing and density estimation

In this presentation we investigate a novel nonparametric approach for estimating and smoothing densities based on a variational regularization method with the Wasserstein metric as a data fidelity. The approach allows a unified treatment of discrete and continuous probability measures and is hence attractive for various tasks. In particular the variational model for special regularization functionals yields a natural method for decomposing a probability measure into a density of particular structure and a remainder, e.g., into cartoons and oscillatory parts in the case of total variation regularization.
In order to compute solutions of the variational problems, a regularized optimal transport problem needs to be solved, for which we discuss several possible formulations and provide a detailed analysis. Moreover we compute special self-similar solutions for several standard regularization functionals in order to yield further insight into the structure of the favoured reconstructions, and we discuss several computational approaches and results. This is joint work with Marzena Franek and Martin Burger.

(Siltanen)

Sparsity-promoting Bayesian inversion

Consider an indirect measurement m = Au+e, where u is a function and e is random error. The related inverse problems is "given m, find an approximation of u in a noise-robust way". In Bayesian inversion one models m, u and e as random variables and constructs a finite-dimensional computational model of the measurement. Measurement data is complemented with a priori information using posterior distribution defined by the Bayes formula. Approximate reconstructions of u can then be achieved as point estimates from the posterior distribution. Discretization-invariance means in this context that if the computational model is refined, then the Bayesian estimates and probability distributions converge towards infinite-dimensional limiting objects. Computational examples are presented for model problems (1D deconvolution and 2D tomography) to demonstrate the edge-preserving properties of wavelet- based Besov space priors. Tese are the first non-Gaussian priors known to be discretization-invariant.

(Simoni)

Regularizing Priors for Linear Inverse Problems

Regularizing Priors for Linear Inverse Problems

Jean-Pierre Florens

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and

Anna Simoni

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Abstract: We consider statistical linear inverse problems in Hilbert spaces of the type $\hat{Y} = Kx + U$ where we want to estimate the function x from the noisy functional observation $\hat{Y}$. We use a Bayesian approach and a conjugate-Gaussian model. For a very general specification of the probability model the posterior distribution of x is known to be inconsistent in a frequentist sense. We construct a class of Gaussian prior distributions for x that are shrinking with the noise U. We show that, under mild conditions, the corresponding posterior distribution is consistent in a frequentist sense and converges at the optimal rate. Then, a class of posterior mean estimators for x is given. We propose an empirical Bayes procedure for selecting an estimator in this class that mimics the posterior mean with the smallest risk on the true x.

AMS 2000 subject classifications: Primary 62C10, 45Q05; secondary 60G15, 62G05.

Keywords and phrases: ill-posed inverse problems, g-prior, posterior consistency, adaptive estimator.

(Stuart)

The Bayesian Approach to Inverse Problems

I will overview the Bayesian approach to inverse problems, emphasizing connections with the classical theory of regularization,and methods from optimization. I will start by describing a mathematical framework in which bounds on the forward problem and its Lipschitz properties are combined with regularity properties of draws from the prior measure to obtain a well-posed inverse problem. Within this framework, and working with Gaussian random field priors, I will then demonstrate: (i) that maximizing the posterior probability links directly to solution of a classical Tikhonov-Phillips regularized least squares problem; (ii) that a Metropolis-Hastings algorithm for sampling the posterior probability distribution corresponds, in a certain parameter limit, to a noisy steepest descent/gradient flow algorithm for the Tikhonov-Phillips regularized least squares problem. I will illustrate the theory with two examples: (a) determining the initial condition and/or forcing of the 2D Navier-Stokes equations from observations of Lagrangian tracers; (b) determining the permeability from observations of the pressure in model for Darcy flow in a porous medium. The work in (i) is in collaboration with Masoumeh Dashti (Warwick); the work in (ii) is in collaboration with Natesh Pillai (Harvard) and Alexander Thiery (Warwick).

(Szepessy)

Hamilton-Jacobi theory for inverse PDE

In this talk I will show how the Hamilton-Jacobi theory can be used for constructing regularizations and error estimates for inverse problems for PDE, formulated by optimal control. I will give examples from calibration in mathematical finance, optimal design and error analysis in molecular dynamics.

The constructed Pontryagin method is a simple and general method for optimal design and reconstruction: the first, analytical, step is to regularize the Hamiltonian; next its Hamiltonian system (a nonlinear partial differential equation) is computed efficiently with the Newton method for a sparse Jacobian. I will also present an error estimate for the difference between exact and approximate objective functions, depending only on the difference of the Hamiltonian and its finite dimensional regularization along the solution path and its L^2 projection, i.e. not on the difference of the exact and approximate solutions to the Hamiltonian systems.

(Tröltzsch)

Aspects of numerical analysis in the optimal control of nonlinear PDEs

The lectures introduce to basic concepts of numerical analysis of optimal control problems with nonlinear partial differential equations. Main emphasis is placed on the difficulties related to nonlinearities in the PDEs. At the beginning, examples of optimal control of industrial processes are presented. In all of them, the state equations are nonlinear and constraints are imposed on the control and/or the state function. Motivated by these examples, the associated theory is discussed for simplified problems of semilinear elliptic type. In particular, first- and second-order optimality conditions, the problem of error estimation for the approximation of the problems by a finite element discretization, and some conceptual algorithms are addressed. These ideas will be extended to more challenging and important problems with state constraints or quasilinear state equation. In particular, it there is still enough time, problems of semi-infinite type are considered.

(Vollmer)

Model Error in Bayesian Inverse Problems

(Poster)

Data Assimilation is an important mathematical method for science and technology. The Input and the parameters of an underlying model are adapted according to the data. One aim of a Data Assimilation Scheme is to use a large number of data sets in order to make up for the observational error in each data set. In fact, the data is often generated by a physical system to which the model is an approximation. We study the model error in the context of the Bayesian Inverse Problem framework by investigation of a one-parameter family $\mathcal{G_{\epsilon}}$ of different forward models where $\mathcal{G}$ can be seen as an approximation for small ϵ. In this approach the solution is a probability distribution describing the probability of a particular input given the data.

(Walther)

Solving large-scale inverse electromagnetic scattering problems: A parallel AD-based approach

An example for an inverse scattering problem is the reconstruction of material properties of a 3D object based on the spatial and temporal analysis of intersecting radar waves. In the context of the CONSERT experiment an orbiter receives scattered emissions from a lander of a comet's surface. For a reconstruction domain of 2km, the permitivity of each of the 700^3 grid elements impacts on the signal at the orbiting receiver. For a given permittivity-field this signal can be approximated with the FDTD (finite differences in time domain) method for a given permitivity. To determine actual permitivity of the object, we formulate an inverse problem, i.e., adjust the permittivity values to minimize the difference between simulated and received signals. The resulting optimization problem is solved with a quasi-Newton-algorithm which requires the gradient of the objective function. The gradient is provided by Algorithmic Differentiation (AD). The numerical complexity of this problem and the memory requirements result in a parallel calculation of the objective function and the gradient.

We will present preliminary numerical results of a parallelization using MPI.

(Wathen)

Preconditioned iterative linear solvers for PDE-constrained Optimization problems

Many control problems for PDEs can be expressed as Optimization problems with the relevant PDEs acting as constraints. As is being discovered in other areas such as multi-physics, there seem to be distinct advantages to tackling such constrained Optimization problems `all-at-once' or with a `one-shot' method. That is, decoupling of the overall problem in some loosely coupled iterative fashion appears to be a rather poorer approach than to compute on the fully coupled problem.

The use of iterative methods for the relevant linear algebra is crucial here since the overall dimensions (including the Optimization and PDE) are usually very large, but matrix vector products as required in Krylov subspace methods such as MINRES are still readily computed. The work to ensure rapid convergence is in preconditioning and it is this topic that we will mostly focus on in this lecture. We will describe our general approach via block conditioning and demonstrate its use for the control of Poisson and Stokes problems and also for the fully time-dependent heat equations.

This is joint work with Tyrone Rees, Martin Stoll, Sue Thorne and John Pearson.

(Xiu)

An Efficient Surrogate Approach to Bayesian Inference in Inverse Problems

We present an efficient numerical strategy for the Bayesian solution of inverse problems. High order numerical methods, based on generalized polynomial chaos (gPC), are used to construct a surrogate model of the forward solution over the support of the prior distribution. This approximation then defines a surrogate posterior probability density that can be evaluated repeatedly at minimal computational cost. The ability to simulate a large number of samples from the posterior distribution results in very accurate estimates of the inverse solution and its associated uncertainty. Combined with high accuracy of the gPC-based forward solver, the new algorithm can provide great efficiency in practical applications. A rigorous error analysis of the algorithm is conducted, where we establish convergence of the approximate posterior to the true posterior and obtain an estimate of the convergence rate. It is proved that fast (exponential) convergence of the gPC forward solution yields similarly fast (exponential) convergence of the posterior. The numerical strategy and the predicted convergence rates are then demonstrated on nonlinear inverse problems of varying smoothness and dimension.

(Zuazua)

Control and Numerics : Continuous versus discrete approaches

Control Theory and Numerical Analysis are two disciplines that need to be combined when facing most relevant applications. This is particularly the case for problems involving Partial Differential Equation (PDE) modelling. There are two possible approaches. The continuous one, consisting on developing the control theory at the PDE level and, once controls are fully characterized, to implement the numerical approximation procedure. And the discrete one, consisting in doing the reverse, i.e. first discretizing the model and then controlling the resulting discrete system. In this lecture we shall compare these two approaches in two relevant examples: The control of vibrations and the control of flows in the presence of shocks. As we shall see, a number of unexpected phenomena occur and challenging problems arise both from a mathematical and applicational viewpoint.