A Paradox in the Approximation of Dirichlet Control Problems in Curved Domains

Eduardo Casas* Andreas Günther† Mariano Mateos‡

ABSTRACT

In this talk we consider the following optimal control problem

$$(\mathrm{P}) \begin{cases} \min J(u) = \displaystyle\int_\Omega L(x, y_u(x))\,dx + \frac{N}{2}\int_\Gamma u^2(x)\,d\sigma(x) \\ \text{subject to } \ (y_u, u) \in (L^\infty(\Omega) \cap H^1(\Omega)) \times L^2(\Gamma), \\ \alpha \le u(x) \le \beta \quad \text{for a.e. } \ x \in \Gamma, \end{cases}$$

where Γ is a smooth manifold, y_u is the state associated to the control u, given by a solution of the Dirichlet problem

$$\begin{cases} -\Delta y + a(x, y) &= 0 \quad \text{in } \Omega, \\ y &= u \quad \text{on } \Gamma. \end{cases} \tag{1}$$

To solve the problem (P) numerically, it is usually necessary to approximate Ω by a (typically polygonal) new domain Ω_h. The difference between the solutions of both infinity dimensional control problems, one formulated in Ω and the second in Ω_h, was studied in [1], where an error of order $O(h)$ was proved. In [2], the numerical approximation of the problem defined in Ω was considered. The authors used a finite element method such that Ω_h was the polygon formed by the union of all triangles of the mesh of parameter h. They proved an error of order $O(h^{3/2})$ for the difference between continuous and discrete optimal controls. Here we show that the estimate obtained in [1] cannot be improved, which leads to the paradox that the numerical solution is a better approximation of the optimal control than the exact one obtained just by changing the domain Ω to Ω_h.

References

[1] E. Casas and J. Sokolowski, *Approximation of boundary control problems on curved domains*, SIAM J. Control Optim., 48 (2010), pp. 3746–3780.

[2] K. Deckelnick, A. Günther, and M. Hinze, *Finite element approximation of Dirichlet boundary control for elliptic PDEs on two- and three-dimensional curved domains*, SIAM J. Control Optim., 48 (2009), pp. 2798–2819.

*Universidad de Cantabria, Spain (eduardo.casas@unican.es)
†Konrad-Zuse-Zentrum für Informationstechnik Berlin (ZIB), Germany (guenther@zib.de)
‡Universidad de Oviedo, Spain (mmateos@uniovi.es)