(+Chernov, Koreparev)
Simanyi

Weak Invariance Principle (WIP)

Local Ergodicity without global hyperbolicity of singularities

SDB $(M, \{\phi^t\}, \mu)$

$x_0 \in \mathrm{Int}(M)$

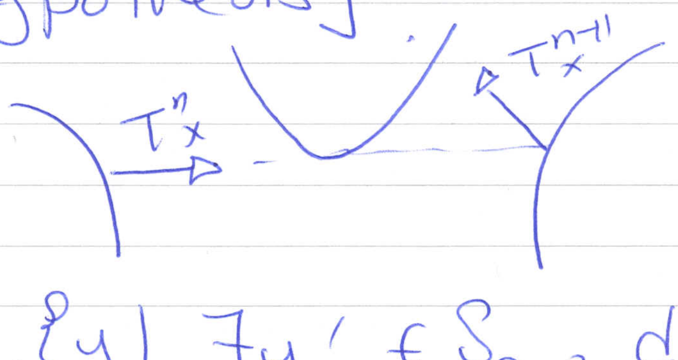
hyperbolic; $\Phi^R_{x_0}$ has ≤ 1 singularity

$U_0 \subseteq M$, $U_0 \subset$ one ergodic component

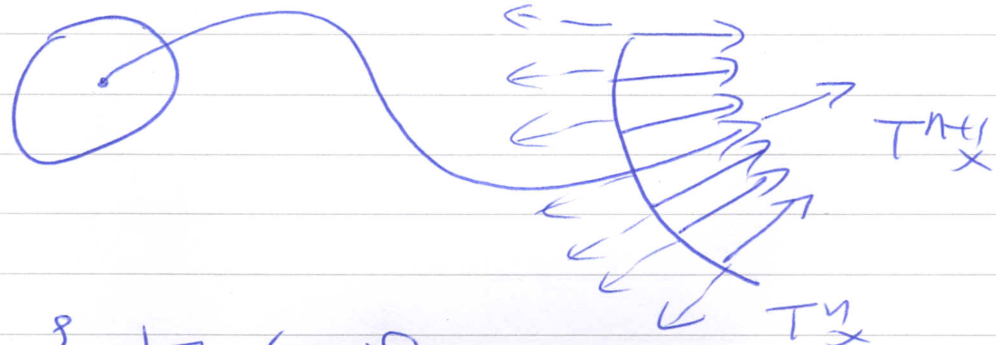
Ansatz: A.e. singular point is sufficient.

So ν : $\nu(S_0) < +\infty$

[Hypothesis]

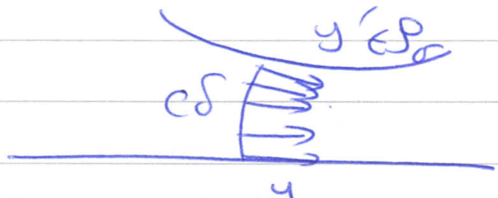


$\{y \mid \exists y' \in S_0,\ d(y,y') < \delta$ the backward map between y and $T^{-n+1} y$ is $\leq k$



$\mu \{ y \mid \exists y' \in S_0,$
$d(y,y') < \delta$, the backward ~~exp~~ exp
rate between y e $T^{-n+1}y \leq k \}$
$< c_0 \delta$

Make c_0 arbitrarily small
if $\delta < \delta_0$, $n \geq n_0$.

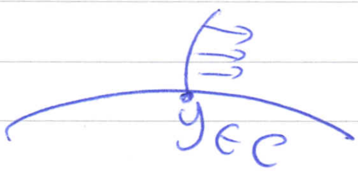


$\nu(S_{uff} \cap S_0)$
$= \nu(S_0)$

(Assume Ansatz)

$\forall \eta > 0$ $\quad \underset{C\,cpt}{C} \subseteq S_{uff} \cap S_c$

$\nu(S_0 \mid C) < \eta$



$\underbrace{\Phi^{(-\infty, c)} y''}_{\text{exp rate} \to +\infty}$ so hyperbolic.

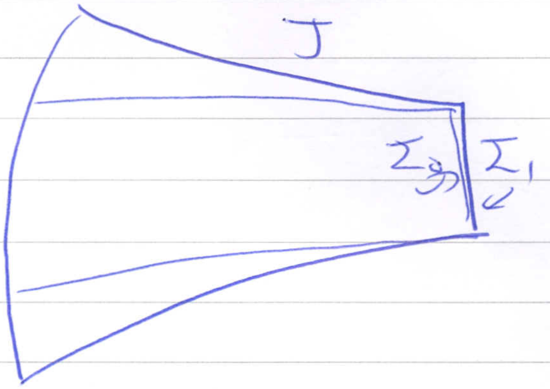
$V_0(y'') < M$

$\forall y'''' \in V_0(y'')$ for $n > n_0(y'')$ exp rate
between $T^{-n}y''''$ e $y'''' > k$.

$$\mathcal{C} \subset \underbrace{\bigcup_{i=1}^{k} V_c(y_i'')}_{\text{Nicely shaped}} \qquad \Big| \qquad \underbrace{y' \in (\mathcal{S}_{uff} \cap \mathcal{S}_0) \setminus \bigcup_{i=1}^{k} V_{\delta}}_{\text{small } \nu\text{-measure}}$$

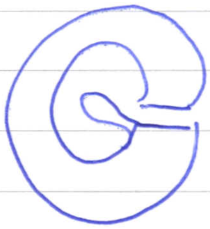
$$\bigcup_{0 \le t \le T} \Phi^t(\Sigma_1) = \mathcal{T}.$$

$$\mathcal{T}' = \bigcup_{0 \le t \le T} \Phi^t \Sigma_0 \subseteq \mathcal{J}.$$



Compare geodesic flows: (M, g) $K \le 0$ complete connected.

<u>Burns example</u> $(d = 3)$. hyperbolic space.



Needs convexity.