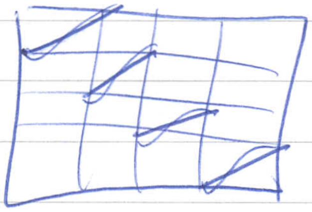
Yoccoz

Dynamics of Generalized Interval Exchange Maps

Thm. Let $r \in \mathbb{Z}$, $r \geq 2$ and T_0 = siem of "restricted Roth type".
Then amongst C^{r+3} simple deformations of T_0 those which are C^r conj to T_0 form a C^1 submanifold of codim $(g-1)(2r+1)+s$.



$T_0 \to T$.

$\to r \geq 3$ (Herman method).

$\to r = 2$ $\quad cod = 2r+2$

$\to r = 1$? $\quad g = 2r+2$.

(Bol. Br. Math Soc. '85)

Thm (Herman) Let $r \geq 3$ be an integer, $\omega \in \mathbb{T}$ satisfying a diophantine condition

$$|\omega - p/q| \geq C/q^{2+\tau}, \quad \mathrm{Diff}_+^{r+3}(\mathbb{T}),$$

for some $C > 0$, $\tau < 1$. Then for any f suff close to R_ω (in C^{r+3} topology) can be uniquely written as

$$f = R_t \circ h \circ R_\omega \circ h^{-1} \quad \text{for } h \in \mathrm{Diff}_+^r(\mathbb{T}), \quad \int (h(x) - x)\,dx = 0$$

h close to id, t close to 0.

$f = R_t \circ h \circ R_\omega \circ h^{-1}$ (*)

$f \mapsto (t, h) \in C^1$.

$Sf = D^2 \log Df - \frac{1}{2}(D \log Df)^2$.

$S(f \circ g) = Sf \circ g \cdot (Dg)^2 + Sg$

(*) $\Longrightarrow Sf \circ h \cdot (Dh)^2 + Sh = Sh \circ R_\omega$

<u>Prop^n</u>: $\forall \phi \in C^{r+2}(\mathbb{T})$, $\int \phi(x)\,dx = 0$.

Then $\begin{cases} \phi = \psi \circ R_\omega - \psi, \ \psi \in C^r(\mathbb{T}) \\ \phi \longrightarrow \psi \text{ bdd} \end{cases}$

<u>Lemma</u> $h \in \mathrm{Diff}^{r+3}_+ \longrightarrow Sh - [Sh] \in C^r_0$

is a local C^1 diffeo at $h = id$.

(Derivative at $h = id$ is

<u>Pf</u>): $\delta \mapsto D^3 \delta h$ is invertible (and Inverse f^n Thm)

$f, h \in \mathrm{Diff}^r_{+0} \longrightarrow (Sf \circ h)(Dh)^2 = \phi(f, h) \in C^{r-1}$

$\cap$

$\mathrm{Diff}^{r+3}_+(\mathbb{T})$ ϕ of class C^1 $\psi \in C^{r-3}_0(\mathbb{T})$

$Sf \circ h \cdot (Dh)^2 + c(f, h) = \psi \circ R_\omega - \psi$; $\psi = S\tilde{h} + c$

$= S\tilde{h} \circ R_\omega - S\tilde{h}$; $\tilde{h} \in \mathrm{Diff}^r_{+0}(\mathbb{T})$

At $f = R_\omega$: $\frac{\partial}{\partial h}\Psi \equiv 0$.

$\Psi|_{\text{fixed } f}$ is a contraction.

$\delta f \circ h \, (Dh)^2 + c(f,h) = \delta h \circ R_\omega - \delta h$

$\Rightarrow \delta(f \circ h) + c(f,h) = \delta(h \circ R_\omega)$

$\Rightarrow f \circ h = R_t \circ h \circ R_\omega$.

$\mathcal{E}^r = \mathcal{C}^r_{comp}(\sqcup I_\alpha^t) = \{\phi \in \mathcal{C}^r(\sqcup I_\alpha^t)$
$(r \geq 2)$ ϕ has cpt support in each $I_\alpha^t\}$.

Thm (MMY). For T_0 of restricted Roth type. One has $\mathcal{E}^r = \mathcal{I}^r \oplus \mathcal{F}^r$ with

- $\dim \mathcal{F}^r = (g-1)(2r+1)+1$
- $\mathcal{I}^r = \{\phi \in \mathcal{E}^r, \phi = \psi \circ T_\lambda - \psi$, $\psi \in \mathcal{C}^{r-2}(I)\}$.

$$\delta T \circ h \, . \underset{\underset{D(T,h)}{\parallel}}{(Dh)^2} = \delta h \circ T_0 - \delta h \quad (\Psi).$$

$\phi(T,h) = \Psi \circ T_0 - \Psi + \chi \; (\in \mathcal{F}^R)$

To kill integrals for cobdry.

$\psi = S\tilde{h} + c$

$S T_0 h (Dh)^2 = S\tilde{h} \circ T_0 - S\tilde{h} + \psi_{T,h}$

$(T,h) \longrightarrow \tilde{h}$ has a unique fixed pt.

$S T_0 h . (Dh)^2 = Sh \circ T_0 - Sh + \kappa_{T,h}$, $h = h(T)$

From this we can recover the theorem.

- $r \geq 3$
- $r = 2$ — use primitive of Schwarzian derivative: $N_1 f = D\log Df - \frac{1}{2}\int (D\log Df)^2 - [(D\log Df)]^2$
- $r = 1$?

$d = 2g+s-1$

Semi-conjugacy class of T_0 should have codimension $2g-2+s$.

C^r conj - dim of T_0 has $2(r+1)(g-1)+s$

$r \geq 2$

C^1 conj class of T_0 should have $(3g-3+s)$

C^0 ? ⇑ ? Conj class of T_0 should have $(3g-3+s)$

T_0

Global geometry of the conjugacy class

Q: Characterize those T_0 for which the C^r conj. class amongst C^r simple deformations has locally finite codimension.

Does this set depend on r?