The family of generalized bounded type numbers and its bifurcation locus

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Credits

Work in (slow) progress in collaboration with Giulio Tiozzo.

Dynamical interpretation

The Gauss map $G : [0,1] \to [0,1]$

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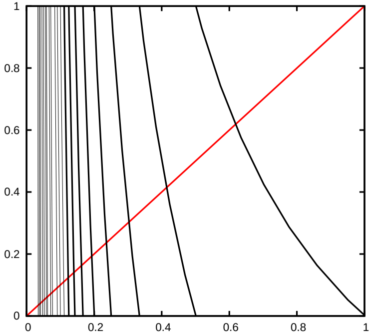

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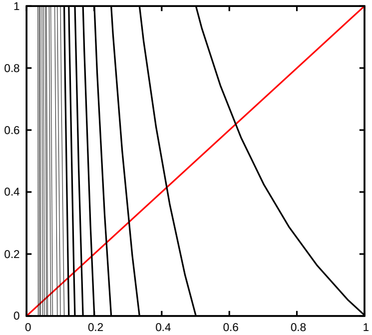

Using C.F. expansion the action of G is given by a shift

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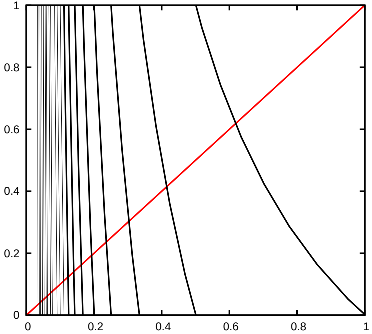

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Constant type numbers are those value whose orbit is bounded away from 0.

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The set function $t \mapsto \mathcal{B}(t)$ is locally constant outside $\mathcal{E}$.

Hausdorff dimension

Theorem

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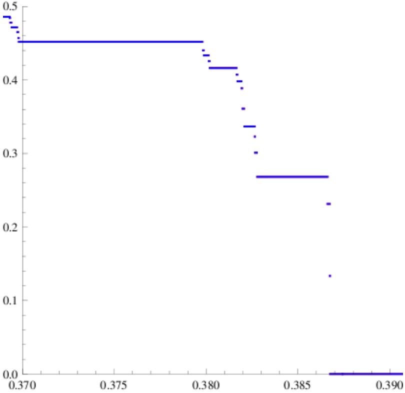

Other directions

The set $\mathcal{E}$ appears (in more or less disguised form) elswhere:

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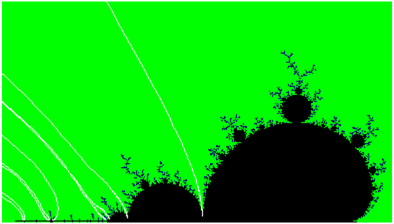
- *dynamics of quadratic polynomials*...

Gaps of $\mathcal{E}$ and symmetric limbs of the mandelbrot set



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The end (and some references)

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C.Bonanno, C.C., S.Isola, G.Tiozzo: *Dynamics of continued fractions and kneading sequences of unimodal maps*, arXiv:1012.2131 [math.DS]

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