

Functional Ito calculus and harmonic functionals on path spaces

Rama CONT

Laboratoire de Probabilités et Modèles Aléatoires, CNRS– Université de Paris VI

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Abstract:

The *functional Ito calculus* [1, 2, 3, 6] is a non-anticipative functional calculus which extends the Ito calculus to path-dependent functionals of stochastic processes, using a notion of pathwise directional derivative [6, 1] for functionals on the space $D([0, T], \mathbb{R}^d)$ of right-continuous paths. Given a square integrable Ito martingale X , we construct a derivative operator ∇_X which is shown to be the adjoint of the Ito integral with respect to X . When X is the Wiener process, the operator ∇_X is shown to be a non-anticipative "lifting" of the Malliavin derivative [3].

Using the tools of functional Ito calculus, we show that a large class of Brownian martingales, viewed as time-dependent functionals on Wiener space, may be characterized as *space-time harmonic functionals*, solutions of a functional PDE on $[0, T] \times C^0([0, T], \mathbb{R}^d)$. This construction is then generalized to functionals of a Brownian semimartingale X : we derive a *functional Kolmogorov equation* which is shown to characterize the martingale property for a large class of path-dependent functionals of X [4]. This new class of functional PDEs is shown to share various properties with parabolic PDEs in finite dimensions : comparison principle, maximum principle, functional Feynman-Kac formula [4].

Joint work with David FOURNIE (Columbia University).

References

- [1] R Cont and D Fournié (2010) A functional extension of the Ito formula, *Comptes Rendus de l'Académie des Sciences*, Volume 348, Issues 1-2, January 2010, Pages 57-61.
- [2] R Cont and D Fournié (2009) Functional Ito calculus and stochastic integral representation of martingales, to appear in *Annals of Probability*, <http://arxiv.org/abs/1002.2446>.
- [3] R Cont and D Fournié (2010) Change of variable formulas for non-anticipative functionals on path space, *Journal of Functional Analysis*, Volume 259, No 4, Pages 1043-1072.
- [4] R Cont and D Fournié (2010) Functional Kolmogorov equations, Working Paper.
- [5] R Cont (2011) Functional Ito calculus and functional Kolmogorov equations, *Barcelona Summer School on Stochastic Analysis 2012*.
- [6] B Dupire (2009) Functional Ito calculus, Working Paper.