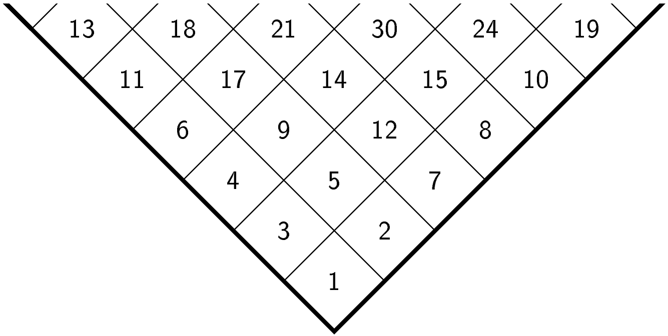
Second class particles and random Young tableaux

joint work with Dan Romik

Piotr Śniady

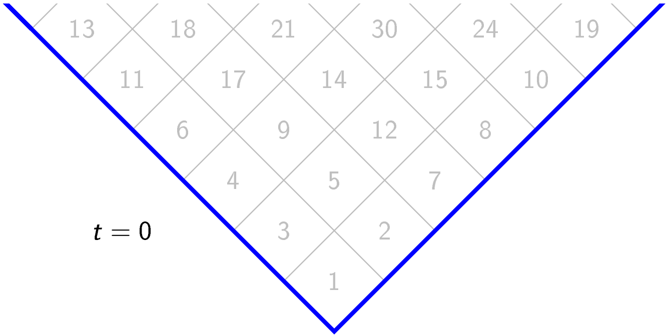
Polish Academy of Sciences
and
University of Wrocław

infinite Young tableau



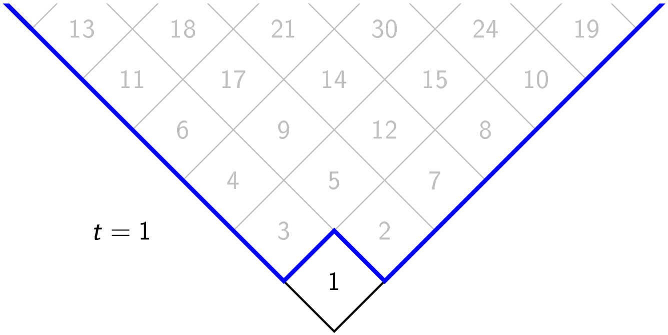
Ω — set of infinite Young tableaux

Young tableau as a growth process

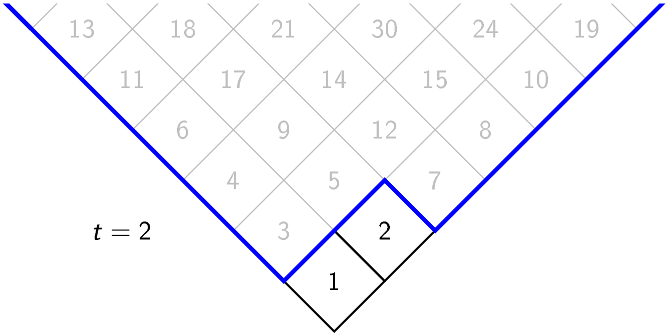


$t = 0$

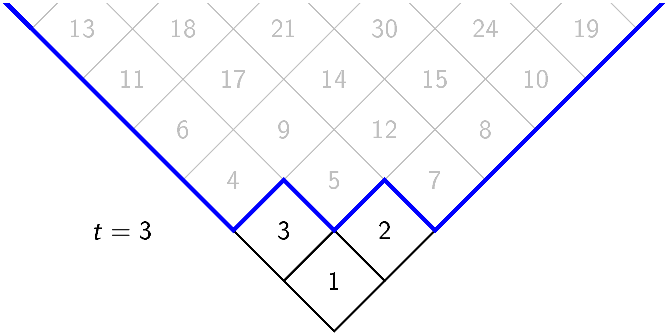
Young tableau as a growth process



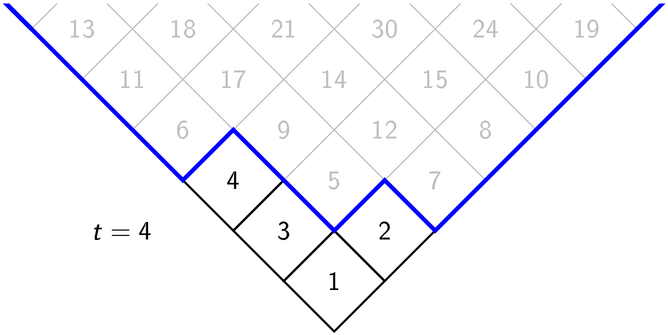
Young tableau as a growth process



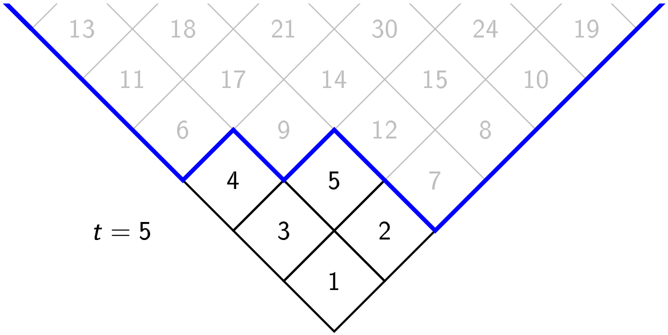
Young tableau as a growth process



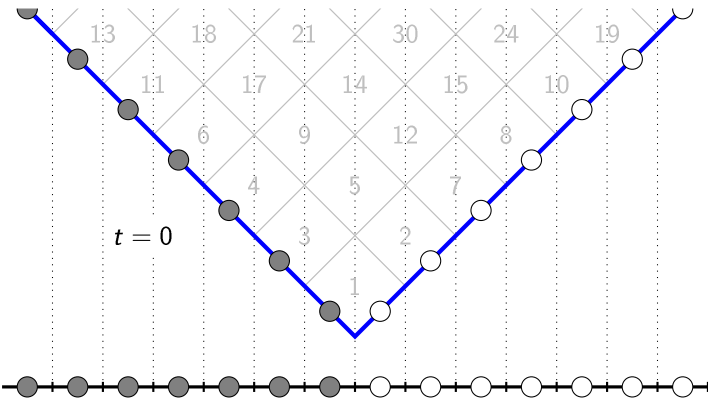
Young tableau as a growth process



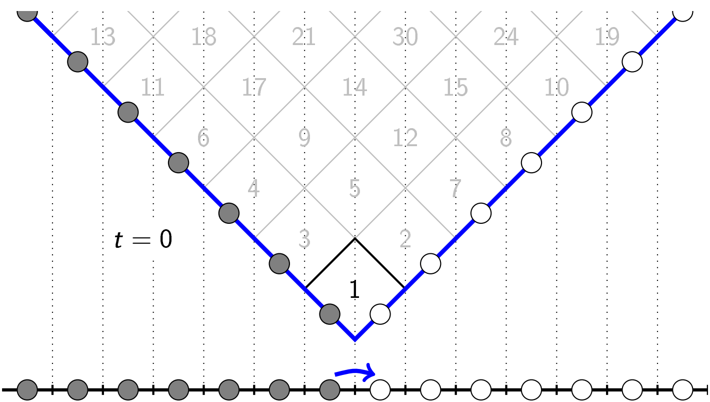
Young tableau as a growth process



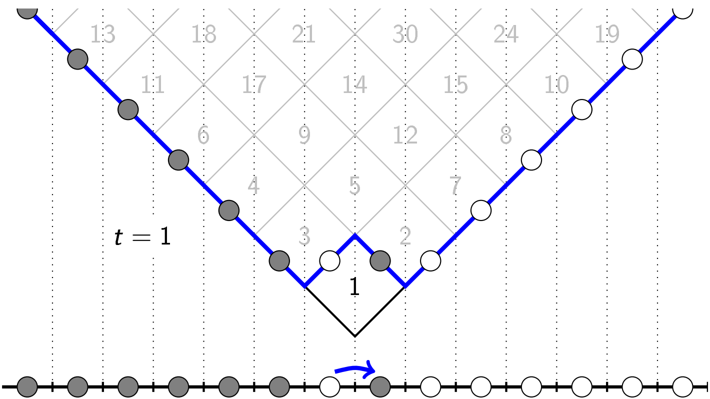
Young tableau and dynamics of particles



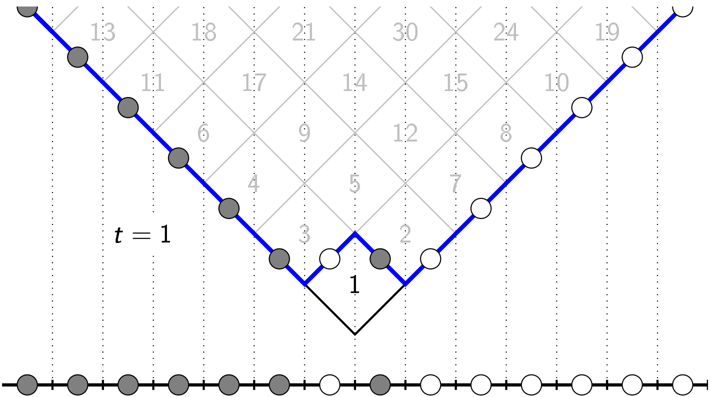
Young tableau and dynamics of particles



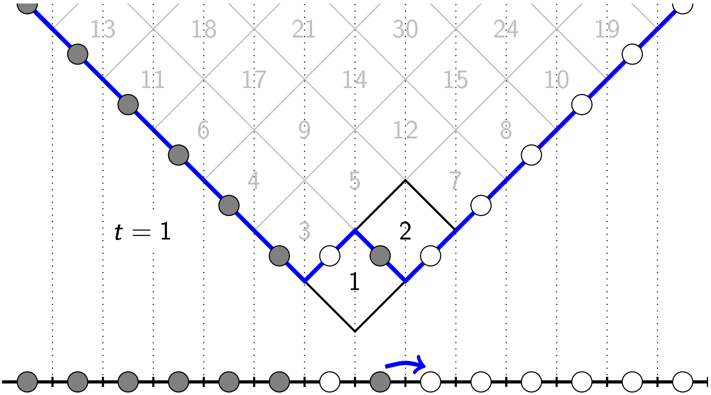
Young tableau and dynamics of particles



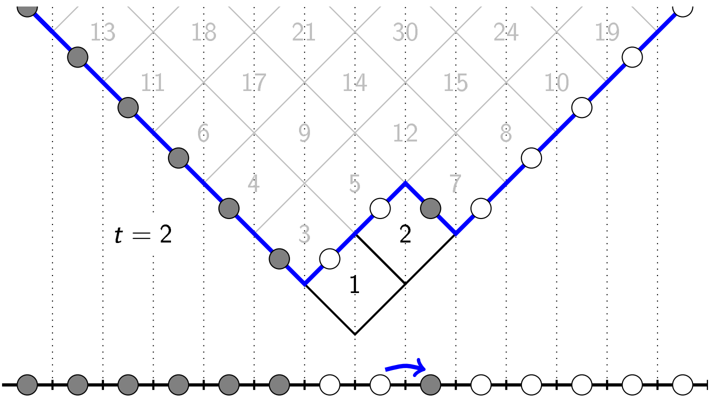
Young tableau and dynamics of particles



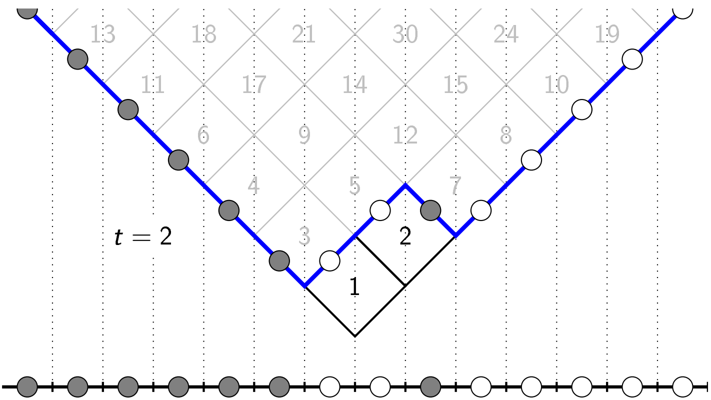
Young tableau and dynamics of particles



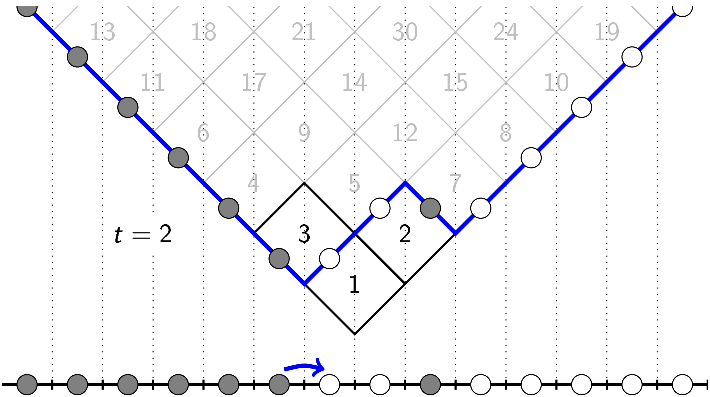
Young tableau and dynamics of particles



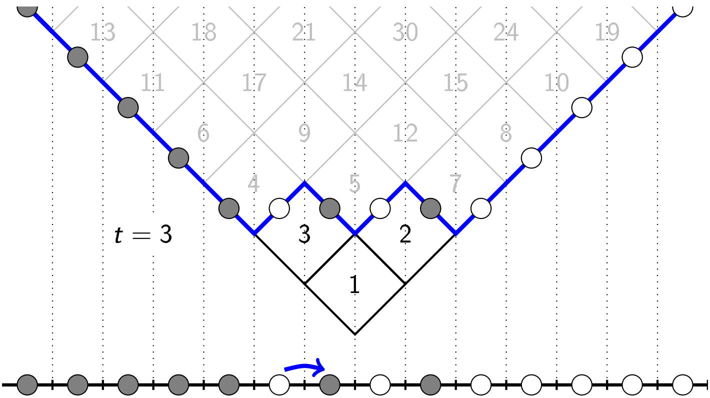
Young tableau and dynamics of particles



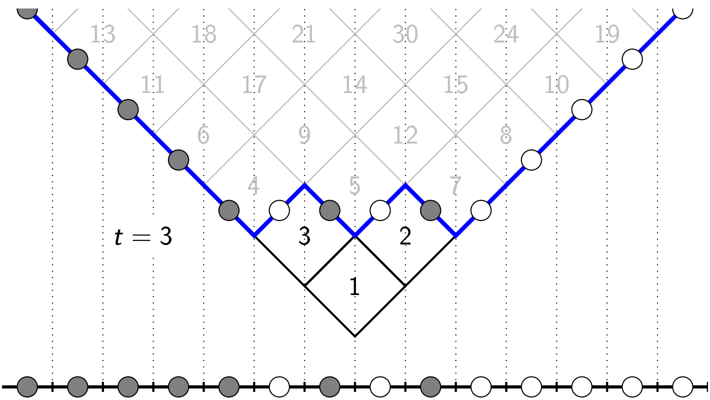
Young tableau and dynamics of particles



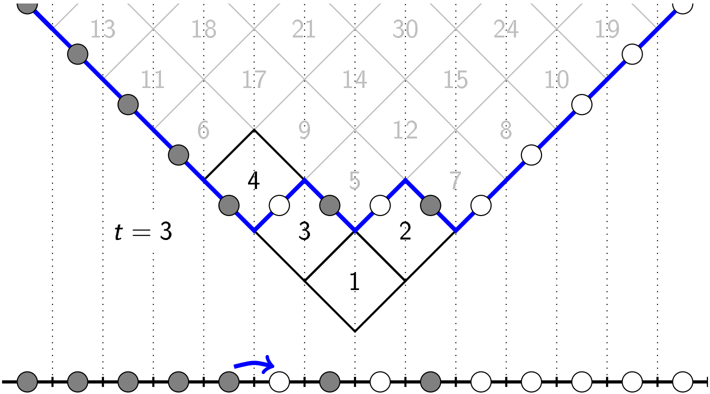
Young tableau and dynamics of particles



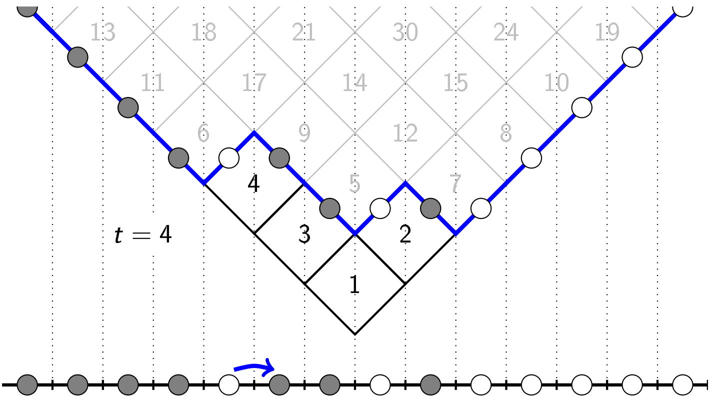
Young tableau and dynamics of particles



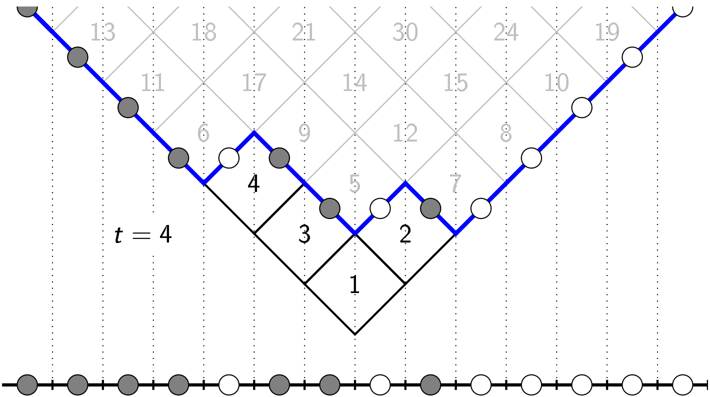
Young tableau and dynamics of particles



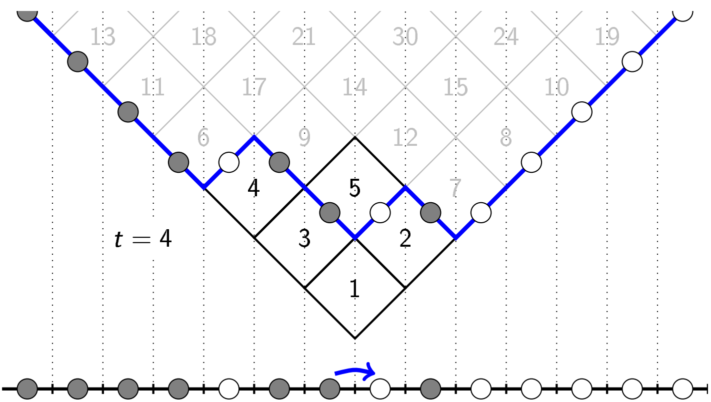
Young tableau and dynamics of particles



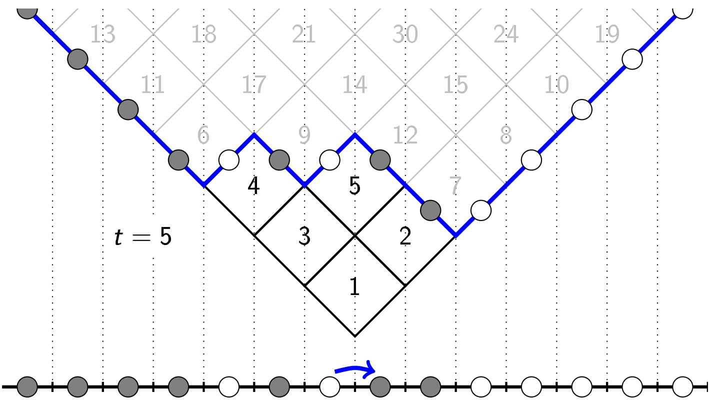
Young tableau and dynamics of particles



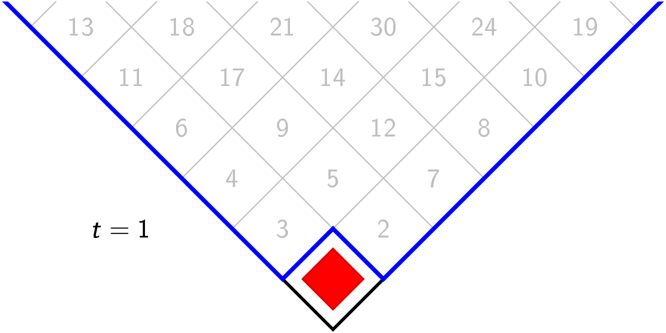
Young tableau and dynamics of particles



Young tableau and dynamics of particles



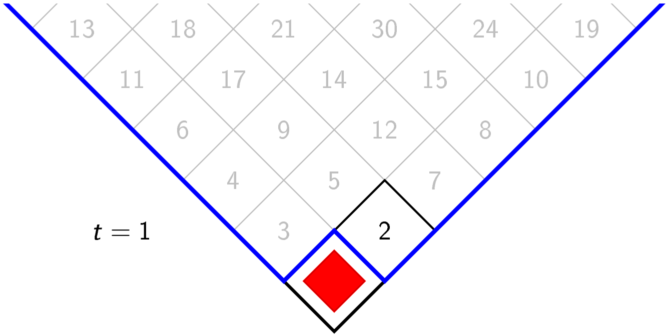
jeu de taquin and second class particles



$t = 1$

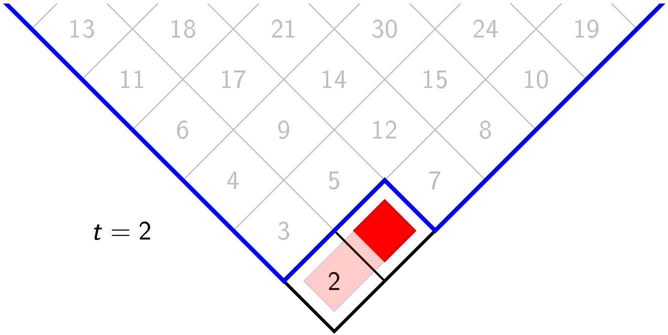
jeu de taquin trajectory — rouge path of the gap

jeu de taquin and second class particles



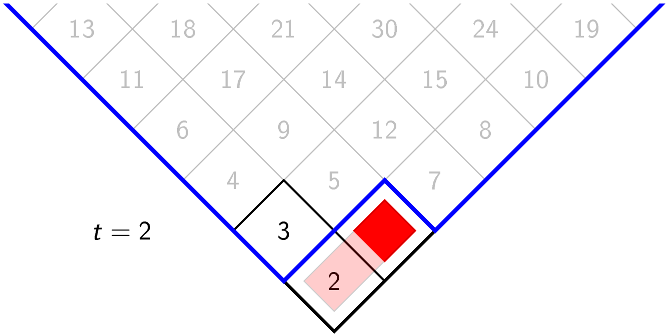
jeu de taquin trajectory — rouge path of the gap

jeu de taquin and second class particles



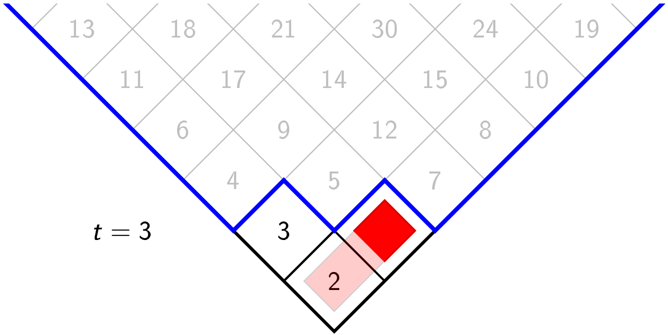
jeu de taquin trajectory — rouge path of the gap

jeu de taquin and second class particles



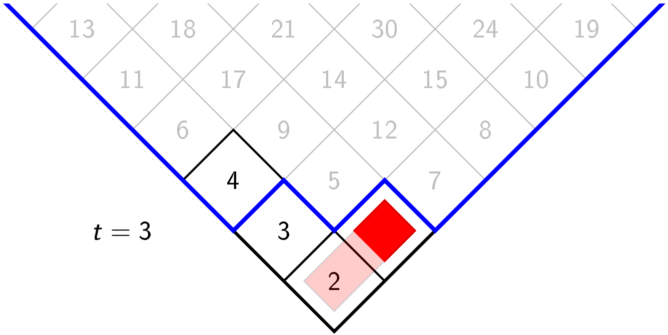
jeu de taquin trajectory — rouge path of the gap

jeu de taquin and second class particles



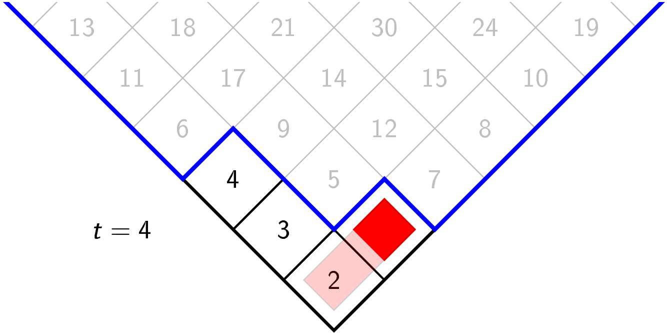
jeu de taquin trajectory — rouge path of the gap

jeu de taquin and second class particles



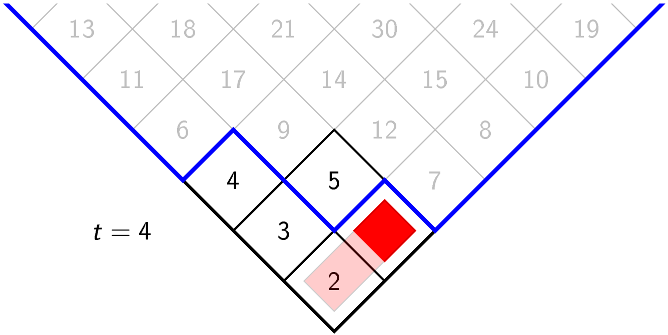
jeu de taquin trajectory — rouge path of the gap

jeu de taquin and second class particles



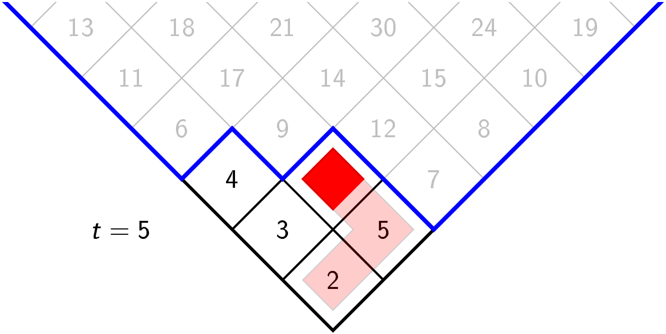
jeu de taquin trajectory — rouge path of the gap

jeu de taquin and second class particles



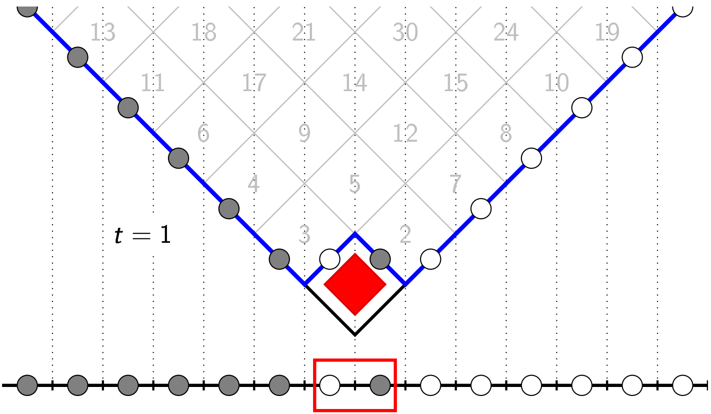
jeu de taquin trajectory — rouge path of the gap

jeu de taquin and second class particles

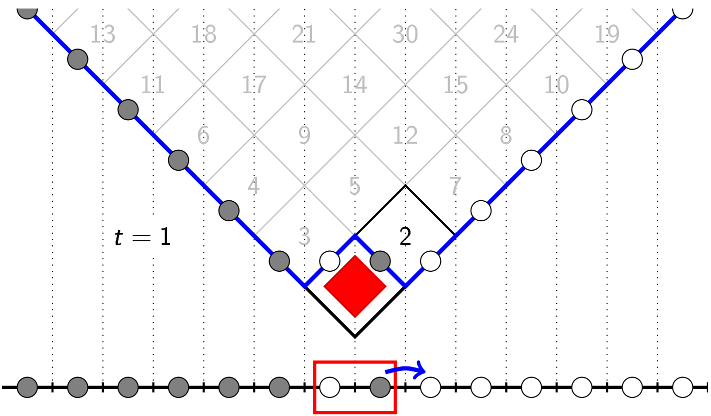


jeu de taquin trajectory — rouge path of the gap

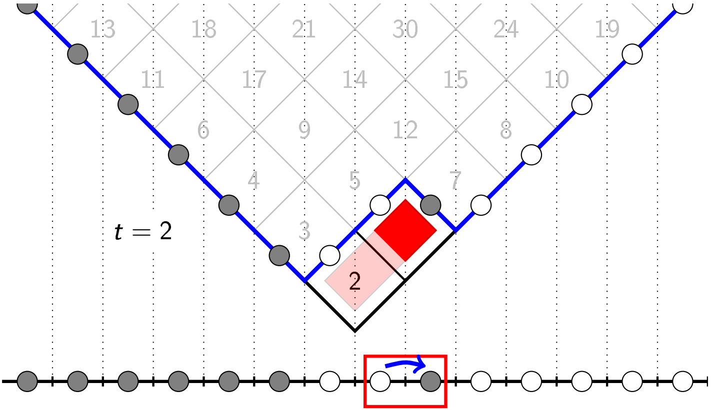
jeu de taquin and second class particles



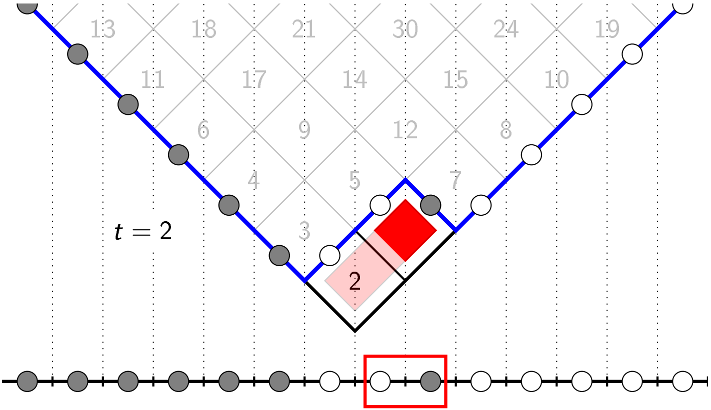
jeu de taquin and second class particles



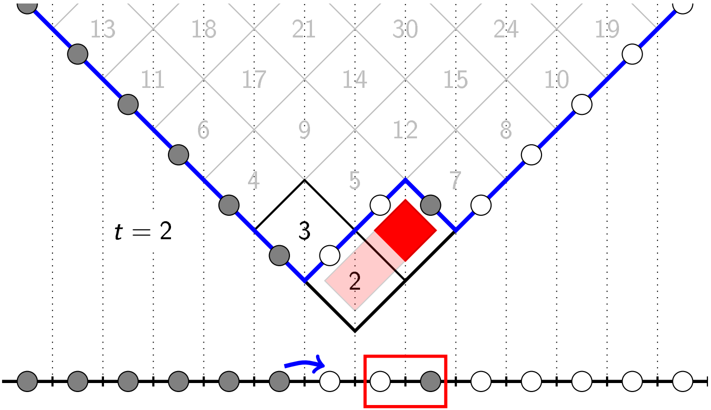
jeu de taquin and second class particles



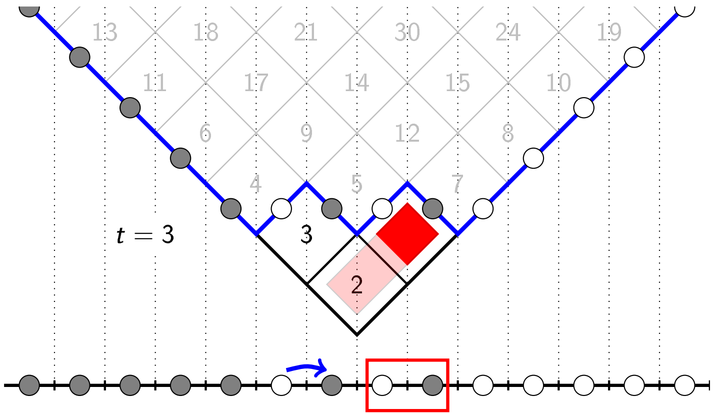
jeu de taquin and second class particles



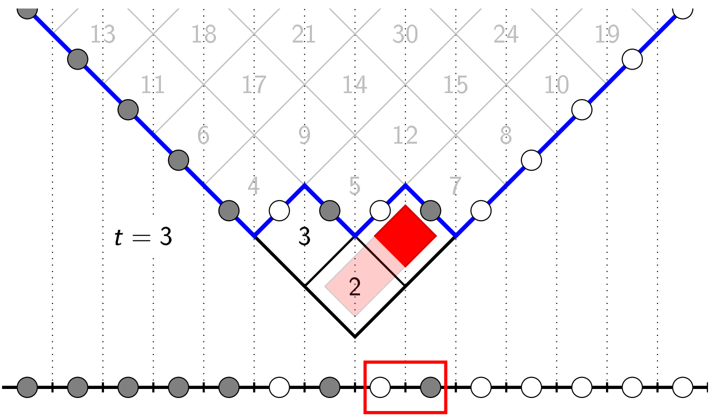
jeu de taquin and second class particles



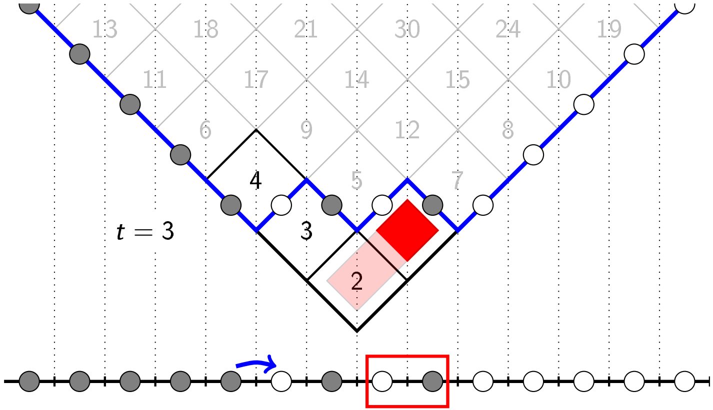
jeu de taquin and second class particles



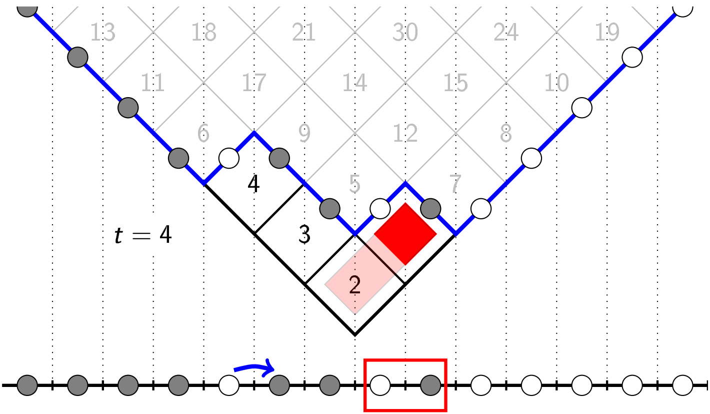
jeu de taquin and second class particles



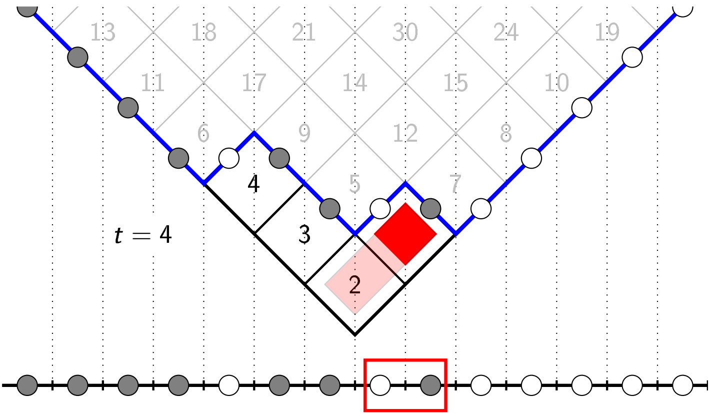
jeu de taquin and second class particles



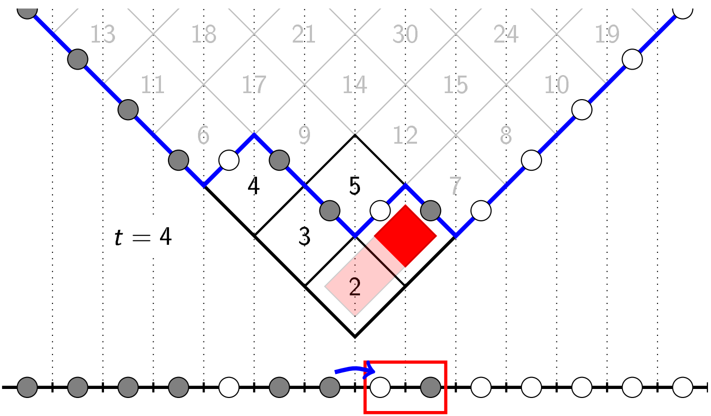
jeu de taquin and second class particles



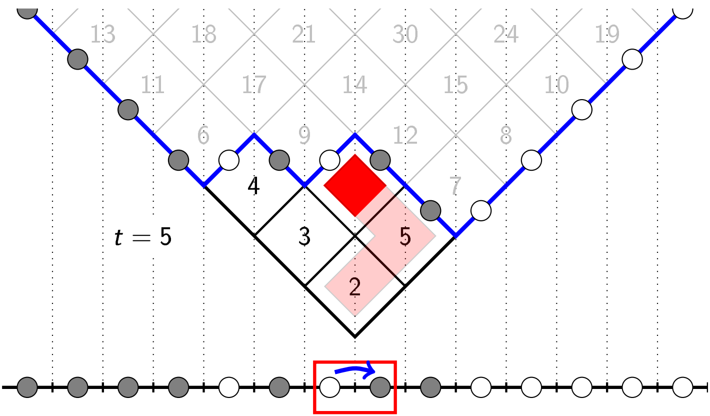
jeu de taquin and second class particles



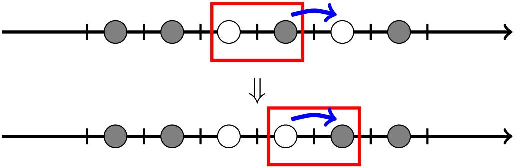
jeu de taquin and second class particles

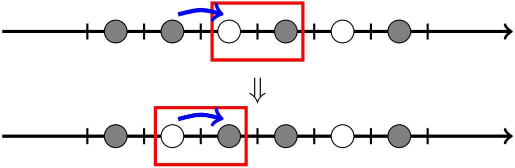


jeu de taquin and second class particles



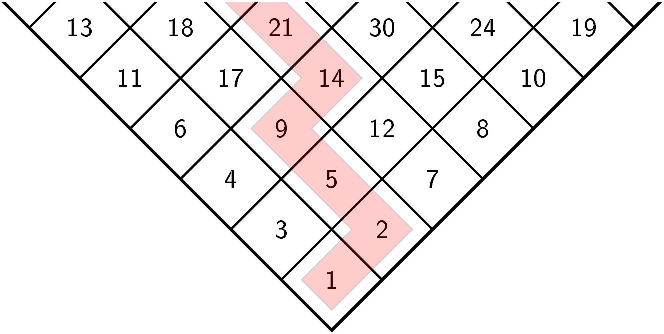
second class particle





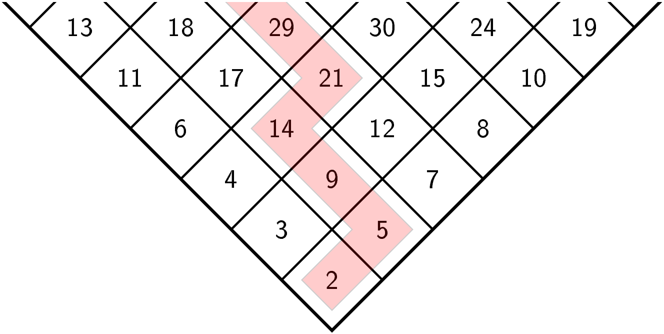
second class particle = jeu de taquin

jeu de taquin — overview



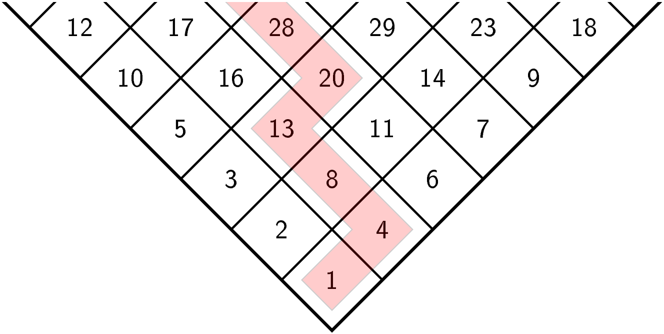
original tableau T

jeu de taquin — overview



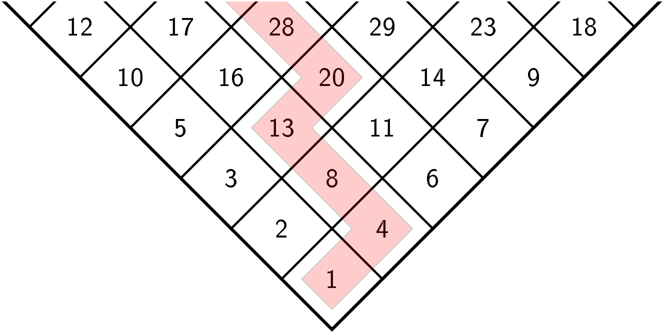
outcome of slidings

jeu de taquin — overview



new tableau $J(T)$

jeu de taquin — overview

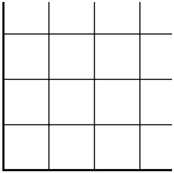


jeu de taquin applied to T gives two pieces of information:

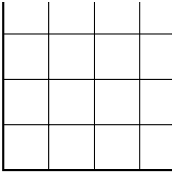
- trajectory $\mathbf{p}(T)$ of jeu de taquin,
- the new tableau $J(T)$,

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\mapsto} \text{recording tableau} \in \Omega$$



insertion tableau

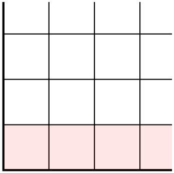


recording tableau

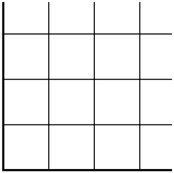
F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\mapsto} \text{recording tableau} \in \Omega$$



insertion tableau



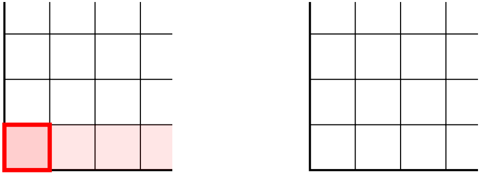
recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*

infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\overset{\text{RSK}}{\mapsto}$ recording tableau $\in \Omega$



insertion tableau   recording tableau

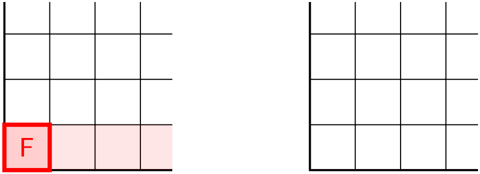
F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*

② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\mapsto} \text{recording tableau} \in \Omega$$



insertion tableau   recording tableau

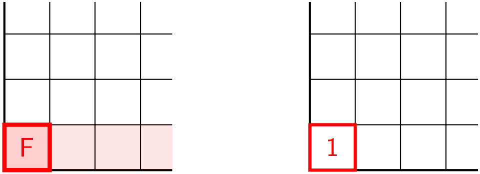
F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*

② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\mapsto} \text{recording tableau} \in \Omega$$



insertion tableau　　　　recording tableau

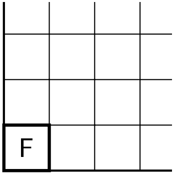
F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

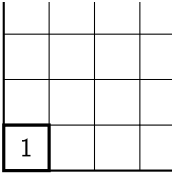
④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\overset{\text{RSK}}{\mapsto}$ recording tableau $\in \Omega$



insertion tableau



recording tableau

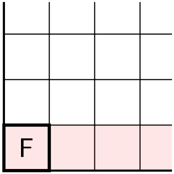
F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

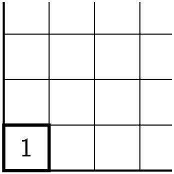
④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\mapsto} \text{recording tableau} \in \Omega$$



insertion tableau



recording tableau

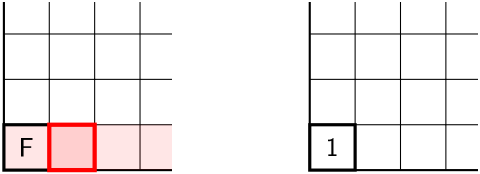
F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\mapsto} \text{recording tableau} \in \Omega$$



insertion tableau recording tableau

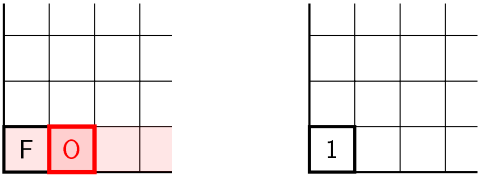
F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\overset{\text{RSK}}{\mapsto}$ recording tableau $\in \Omega$



insertion tableau recording tableau

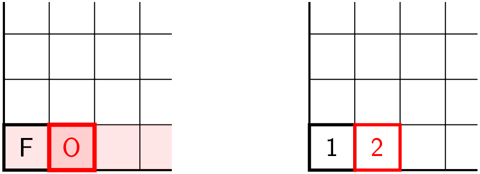
F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\mapsto} \text{recording tableau} \in \Omega$$



insertion tableau   recording tableau

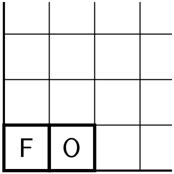
F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

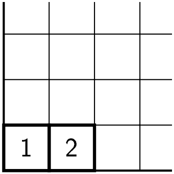
④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\mapsto} \text{recording tableau} \in \Omega$$



insertion tableau



recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

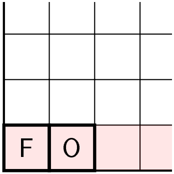
① *start from the first row,*

② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

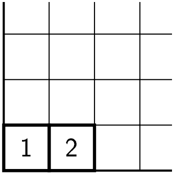
④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\overset{\text{RSK}}{\mapsto}$ recording tableau $\in \Omega$



insertion tableau



recording tableau

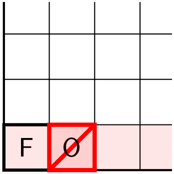
F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

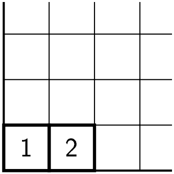
① *start from the first row,*
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\mapsto} \text{recording tableau} \in \Omega$$





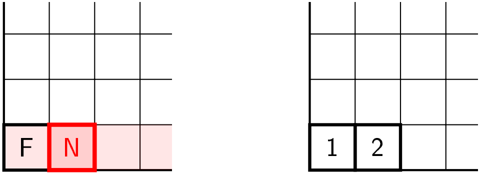
F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\mapsto} \text{recording tableau} \in \Omega$$



insertion tableau recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

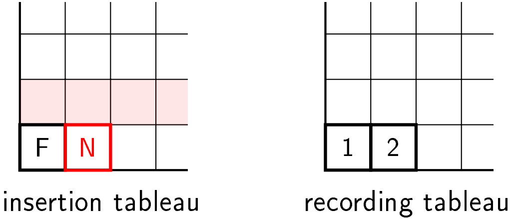
① *start from the first row,*

② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\overset{\text{RSK}}{\mapsto}$ recording tableau $\in \Omega$



F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*

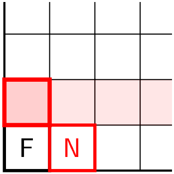
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

③ *insert the bumped element into the next row,*

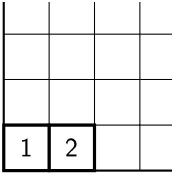
④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\overset{\text{RSK}}{\mapsto}$ recording tableau $\in \Omega$



insertion tableau



recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*

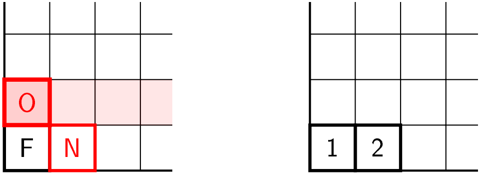
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

③ *insert the bumped element into the next row,*

④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\mapsto} \text{recording tableau} \in \Omega$$



insertion tableau recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*

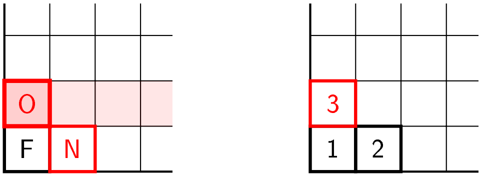
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

③ *insert the bumped element into the next row,*

④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\mapsto} \text{recording tableau} \in \Omega$$



insertion tableau recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*

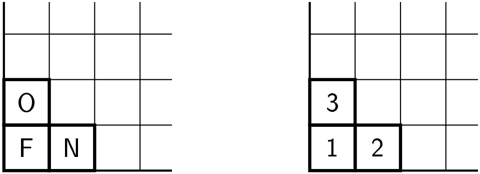
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

③ *insert the bumped element into the next row,*

④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\mapsto} \text{recording tableau} \in \Omega$$



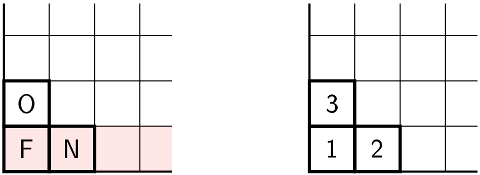
insertion tableau | recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*
③ *insert the bumped element into the next row,*
④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\overset{\text{RSK}}{\mapsto}$ recording tableau $\in \Omega$



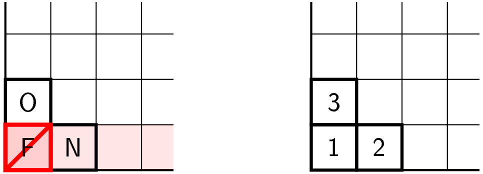
insertion tableau　　　recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*
③ *insert the bumped element into the next row,*
④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\overset{\text{RSK}}{\mapsto}$ recording tableau $\in \Omega$



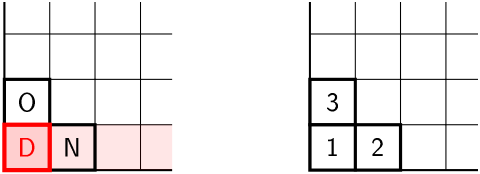
insertion tableau recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*
③ *insert the bumped element into the next row,*
④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\overset{\text{RSK}}{\mapsto}$ recording tableau $\in \Omega$



insertion tableau recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*

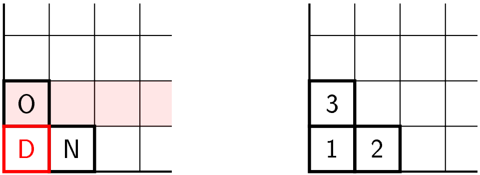
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

③ *insert the bumped element into the next row,*

④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\mapsto} \text{recording tableau} \in \Omega$$



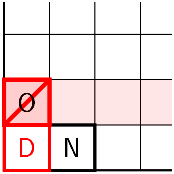
insertion tableau   recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

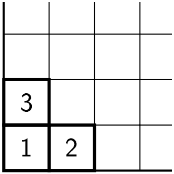
① *start from the first row,*
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*
③ *insert the bumped element into the next row,*
④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\overset{\text{RSK}}{\mapsto}$ recording tableau $\in \Omega$



insertion tableau



recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*

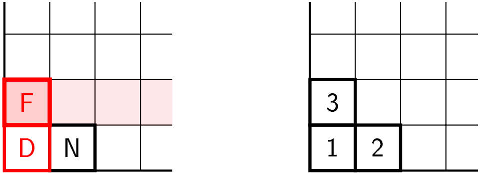
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

③ *insert the bumped element into the next row,*

④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\mapsto} \text{recording tableau} \in \Omega$$



insertion tableau recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*

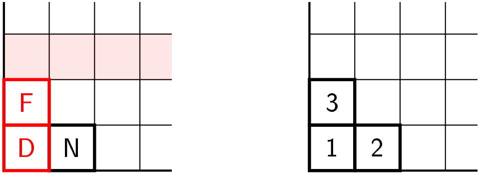
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

③ *insert the bumped element into the next row,*

④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\overset{\text{RSK}}{\mapsto}$ recording tableau $\in \Omega$



insertion tableau recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*

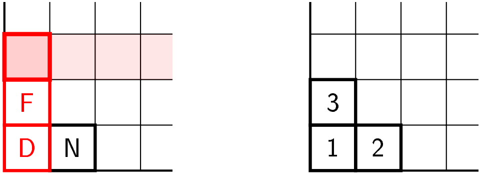
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

③ *insert the bumped element into the next row,*

④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\overset{\text{RSK}}{\mapsto}$ recording tableau $\in \Omega$



insertion tableau   recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*

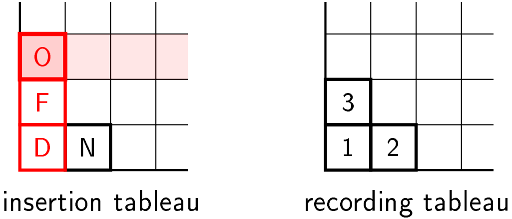
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

③ *insert the bumped element into the next row,*

④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\mapsto} \text{recording tableau} \in \Omega$$



F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*

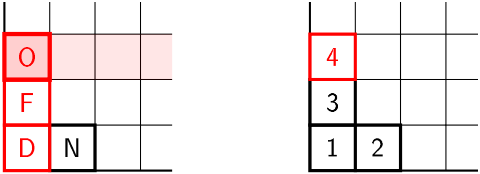
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

③ *insert the bumped element into the next row,*

④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\mapsto} \text{recording tableau} \in \Omega$$



insertion tableau   recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*

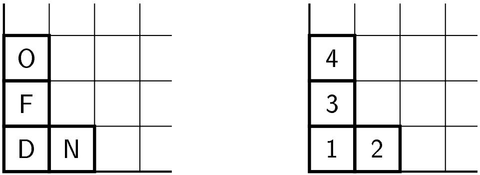
② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

③ *insert the bumped element into the next row,*

④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\mapsto} \text{recording tableau} \in \Omega$$



insertion tableau recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*

② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

③ *insert the bumped element into the next row,*

④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\mapsto} \text{recording tableau} \in \Omega$$

F	L	P	W
D	G	M	R
B	E	J	Q
A	C	H	I

insertion tableau

7	11	22	24
4	10	16	17
3	9	14	15
1	2	5	6

recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

① *start from the first row,*

② *insert the letter as far to the right as possible, so that the row is increasing and no gaps are created,*

③ *insert the bumped element into the next row,*

④ *information about the new box into the recording tableau,*

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\mapsto} \text{recording tableau} \in \Omega$$

7	11	22	24
4	10	16	17
3	9	14	15
1	2	5	6

~~insertion tableau~~ recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\mapsto} \text{recording tableau} \in \Omega$$

7	11	22	24
4	10	16	17
3	9	14	15
1	2	5	6

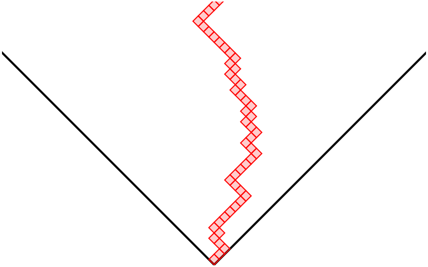
~~insertion tableau~~ recording tableau

F O N D P X B Z U L G E A T W R S M Y V C J H Q I K

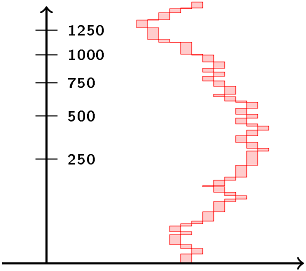
if $X_0, X_1, \dots$ are i.i.d. $U(0,1)$ random variables then

$$\text{RSK}(X_0, X_1, \dots) \overset{\text{distribution}}{=} \text{Plancherel measure}$$

trajectories

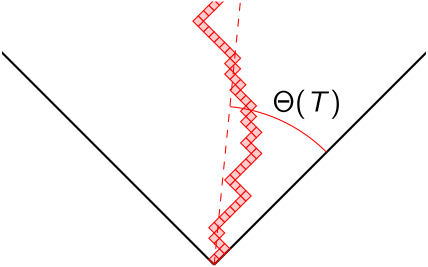


jeu de taquin



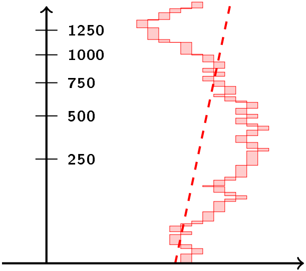
second class particle

trajectories



jeu de taquin

$\Theta(T)$ — asymptotic angle of jeu de taquin



second class particle

asymptotic speed of second class particle

$$
\begin{tikzcd}
T_0 \arrow[r, mapsto, "J"] \arrow[d, mapsto, "\Theta"'] & T_1 \\
\theta_0 &
\end{tikzcd}
$$

$$
\begin{tikzcd}
T_0 \arrow[r, mapsto, "J"] \arrow[d, mapsto, "\Theta"'] & T_1 \arrow[r, mapsto, "J"] \arrow[d, mapsto, "\Theta"'] & \cdots \\
\theta_0 & \theta_1 &
\end{tikzcd}
$$

$$
\begin{array}{ccccc}
T_0 & \xmapsto{J} & T_1 & \xmapsto{J} & \cdots \\
\Theta \downarrow & & \Theta \downarrow & & \\
\theta_0 & & \theta_1 & &
\end{array}
$$

jeu de taquin dynamical system $(\Omega, \text{Plancherel}, J)$

$$
\begin{tikzcd}
(x_0, x_1, \dots) \arrow[d, mapsto, "\mathrm{RSK}"'] & & \\
T_0 \arrow[r, mapsto, "J"] \arrow[d, mapsto, "\Theta"'] & T_1 \arrow[r, mapsto, "J"] \arrow[d, mapsto, "\Theta"'] & \cdots \\
\theta_0 & \theta_1 &
\end{tikzcd}
$$

jeu de taquin dynamical system $\left(\Omega, \text{Plancherel}, J\right)$

$$\begin{tikzcd}
(x_0, x_1, \dots) \arrow[r, mapsto, "?"] \arrow[d, mapsto, "\mathsf{RSK}"'] & ? \arrow[r, mapsto, "?"] \arrow[d, mapsto, "\mathsf{RSK}"'] & \cdots \\
{\color{red}T_0} \arrow[r, mapsto, red, "J"] \arrow[d, mapsto, "\Theta"'] & {\color{red}T_1} \arrow[r, mapsto, red, "J"] \arrow[d, mapsto, "\Theta"'] & {\color{red}\cdots} \\
\theta_0 & \theta_1 &
\end{tikzcd}$$

jeu de taquin dynamical system $(\Omega, \text{Plancherel}, J)$

$$
\begin{tikzcd}
(x_0, x_1, \dots) \arrow[r, mapsto, "s"] \arrow[d, mapsto, "\mathsf{RSK}"'] & (x_1, x_2, \dots) \arrow[r, mapsto, "s"] \arrow[d, mapsto, "\mathsf{RSK}"'] & \cdots \\
{\color{red} T_0} \arrow[r, mapsto, red, "J"] \arrow[d, mapsto, "\Theta"'] & {\color{red} T_1} \arrow[r, mapsto, red, "J"] \arrow[d, mapsto, "\Theta"'] & {\color{red} \cdots} \\
\theta_0 & \theta_1 &
\end{tikzcd}
$$

jeu de taquin dynamical system $(\Omega, \text{Plancherel}, J)$

i.i.d. shift dynamical system $\left([0,1]^{\mathbb{N}}, \prod \text{Lebesgue}, s\right)$

$$\begin{tikzcd}
(x_0, x_1, \dots) \arrow[r, mapsto, "s"] \arrow[d, mapsto, "\mathrm{RSK}"'] & (x_1, x_2, \dots) \arrow[r, mapsto, "s"] \arrow[d, mapsto, "\mathrm{RSK}"'] & \cdots \\
\color{red}{T_0} \arrow[r, mapsto, red, "J"] \arrow[d, mapsto, "\Theta"'] & \color{red}{T_1} \arrow[r, mapsto, red, "J"] \arrow[d, mapsto, "\Theta"'] & \color{red}{\cdots} \\
\theta_0 & \theta_1 &
\end{tikzcd}$$

jeu de taquin dynamical system $\left(\Omega, \text{Plancherel}, J\right)$

i.i.d. shift dynamical system $\left([0,1]^{\mathbb{N}}, \prod \text{Lebesgue}, s\right)$

$$\begin{tikzcd}
(x_0, x_1, \dots) \arrow[r, mapsto, "s"] \arrow[d, mapsto, shift right, "\text{RSK}"'] & (x_1, x_2, \dots) \arrow[r, mapsto, "s"] \arrow[d, mapsto, "\text{RSK}"'] & \cdots \\
T_0 \arrow[r, mapsto, "J"] \arrow[u, mapsto, shift right, "\text{inverse?}"'] \arrow[d, mapsto, "\Theta"'] & T_1 \arrow[r, mapsto, "J"] \arrow[d, mapsto, "\Theta"'] & \cdots \\
\theta_0 & \theta_1 &
\end{tikzcd}$$

jeu de taquin dynamical system $\left(\Omega, \text{Plancherel}, J\right)$

i.i.d. shift dynamical system $\left([0,1]^{\mathbb{N}}, \prod \text{Lebesgue}, s\right)$

$$
\begin{tikzcd}
(x_0, x_1, \dots) \arrow[r, mapsto, "s"] \arrow[d, mapsto, "\text{RSK}"', shift right] & (x_1, x_2, \dots) \arrow[r, mapsto, "s"] \arrow[d, mapsto, "\text{RSK}"'] & \cdots \\
T_0 \arrow[r, mapsto, "J"] \arrow[u, mapsto, "\text{inverse!}"', shift right] \arrow[d, mapsto, "\Theta"'] & T_1 \arrow[r, mapsto, "J"] \arrow[d, mapsto, "\Theta"'] & \cdots \\
\theta_0 & \theta_1 &
\end{tikzcd}
$$

jeu de taquin dynamical system $(\Omega, \text{Plancherel}, J)$

i.i.d. shift dynamical system $\left([0,1]^{\mathbb{N}}, \prod \text{Lebesgue}, s\right)$

$$
\begin{tikzcd}
(x_0, x_1, \ldots) \arrow[r, mapsto, "s"] \arrow[d, mapsto, "\mathrm{RSK}"', shift right] & (x_1, x_2, \ldots) \arrow[r, mapsto, "s"] \arrow[d, mapsto, "\mathrm{RSK}"'] & \cdots \\
T_0 \arrow[r, mapsto, "J"] \arrow[u, mapsto, "\text{inverse!}"', shift right] \arrow[d, mapsto, "\Theta"'] & T_1 \arrow[r, mapsto, "J"] \arrow[d, mapsto, "\Theta"'] & \cdots \\
\theta_0 = f(x_0) & \theta_1 = f(x_1) &
\end{tikzcd}
$$

jeu de taquin dynamical system $\left(\Omega, \text{Plancherel}, J\right)$

i.i.d. shift dynamical system $\left([0,1]^{\mathbb{N}}, \prod \text{Lebesgue}, s\right)$

$$\begin{tikzcd}
(x_0, x_1, \dots) \arrow[r, mapsto, "s"] \arrow[d, mapsto, shift right, "\text{RSK}"'] & (x_1, x_2, \dots) \arrow[r, mapsto, "s"] \arrow[d, mapsto, "\text{RSK}"'] & \cdots \\
T_0 \arrow[r, mapsto, "J"] \arrow[u, mapsto, shift right, "\text{inverse!}"'] \arrow[d, mapsto, "\Theta"'] & T_1 \arrow[r, mapsto, "J"] \arrow[d, mapsto, "\Theta"'] & \cdots \\
\theta_0 = f(x_0) & \theta_1 = f(x_1) &
\end{tikzcd}$$

jeu de taquin dynamical system $(\Omega, \text{Plancherel}, J)$

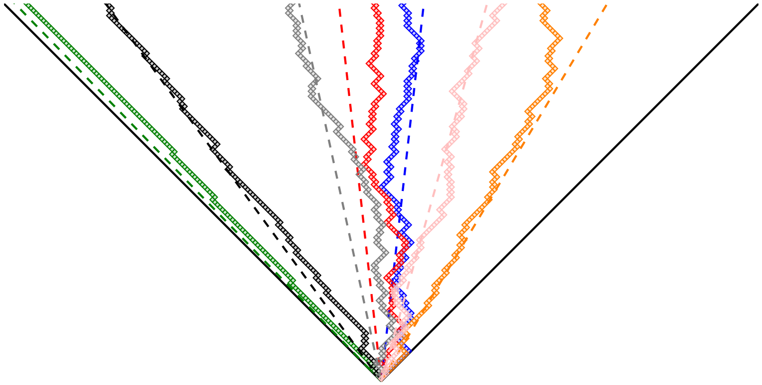
the jeu de taquin dynamical system is isomorphic to i.i.d. shift

the inverse map is given by $x_i = f^{-1}(\theta_i)$

main results

- slope angles $\theta_0, \theta_1, \ldots$
  (and hence asymptotic speeds of second class particles!)
  exist almost surely,
- they are independent random variables with explicit distribution,
- RSK is an isomorphism between the dynamical system of shift and jeu de taquin,
- jeu de taquin is an ergodic transformation,

open problems



Dan Romik, Piotr Śniady
Jeu de taquin dynamics on infinite Young tableaux and second class particles
arXiv:1111.0575