Optimal Order Placement

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(joint work with Antje Fruth)

The execution of large transactions on a financial market will typically affect market prices in an adverse manner, thus leading to possibly significant execution costs. Minimizing these costs requires to trade-off projections of future market depth vs. market resilience and vs. the urgency to trade. We present an extension of the model proposed by Obizhaeva and Wang which allows for these key market parameters to change over time and we show how to produce a closed-form solution to the resulting optimal control problem.

Optimal stopping and incomplete information

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We consider optimal stopping problems in cases where the drift of the underlying process is not directly observable. By filtering theory, these problems have natural formulations as several-dimensional optimal stopping problems. Using a change of measure technique, we reduce the dimension in certain examples, including a natural model for an asset sale.

Executive Stock Options: Portfolio Effects

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Executives compensated with stock options generally receive grants periodically and so on any given date, may have a portfolio of options of differing strikes and maturities on their companys stock. Non-transferability and trading restrictions in the company stock result in the executive facing unhedgeable risk. We employ utility indifference pricing to analyse the optimal exercise thresholds for each option, option values and cost of the options to shareholders. Portfolio interaction effects mean that each of these differ, depending on the composition of the remainder of the portfolio. We demonstrate that the exercise threshold for a particular option can be discontinuous at the time that the option's position in the exercise order changes. In particular, the cost to shareholders of an option portfolio is lowered relative to its cost computed on a per-option basis. The model can explain a number of empirical observations - which options are attractive to exercise first, how exercise changes following a new grant, and early exercise.

Joint with Jia Sun and Elizabeth Whalley (Warwick Business School).

Shuttling a diffusion — the dynamic case

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We consider the problem of shuttling an instantaneously reflecting diffusion to 1 and back to zero. The control is to choose the drift at each level on first arriving at that level. For the case where the process starts at zero we will give the optimal drift and show that it can be chosen in advance. We will give the optimal dynamic drift in the case where the process starts above zero but the drift is fixed at all higher levels. Here, the dynamic drift gives a strictly lower payoff than the static optimum.

Backward martingale representation and the existence of a complete equilibrium

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Let $\mathbb{Q}$ and $\mathbb{P}$ be equivalent probability measures and let ψ be a J-dimensional vector of random variables such that $\frac{d\mathbb{Q}}{d\mathbb{P}}$ and ψ are defined in terms of a weak solution X to a d-dimensional stochastic differential equation. Motivated by the problem of *endogenous completeness* in financial economics we present conditions which guarantee that any local martingale under $\mathbb{Q}$ is a stochastic integral with respect to the J-dimensional martingale $S_t \triangleq \mathbb{E}^{\mathbb{Q}}[\psi|\mathcal{F}_t]$. While the drift $b = b(t,x)$ and the volatility $\sigma = \sigma(t,x)$ coefficients for X need to have only minimal regularity properties with respect to x, they are assumed to be analytic functions with respect to t. We provide a counter-example showing that this t-analyticity assumption for σ cannot be removed. The presentation is based on joint papers with Silviu Predoiu.

Capped American lookback

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(joint work with Curdin Ott)

We consider the case of a capped American Lookback option in the setting of a risk asset driven by a spectrally negative Levy process. The resulting optimal stopping problem is solved by looking at excursions and a threshold which moves as a function of local time at the maximum. We also make some interesting connections with Peskir's maximality principle.

On the exercise boundary of American options in exponential Lévy models

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This talk is based on joint work with M. Mikou on one hand and A. Bouselmi on the other hand. We will discuss the behavior of the exercise boundary near maturity. It turns out that, in exponential Lévy models, the picture is quite different from the Black Scholes case.

Unspanned endowment and face-lifting

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(joint work with Gordan Zitkovic and Mete Soner)

We study the relation between optimizing over finite additive and countable additive probability measures. Standard arguments show that the optimizer is always attained in the finite additive class. We will then discuss and derive the influence a possible singular component has on the value function close to maturity. More specifically, in general incomplete Brownian settings, we explicitly identify the face-lift (boundary layer) in the value function.

Semimartingale Backward Equations and an Optimal Equivalent Change of Measure

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We study optimization problems, where the control choice implies an equivalent transformation of some initial probability measure. We shall use R. Chitashvili's construction (Lect. Notes in Math. N. 1021, pp. 73-92, 1983) of controlled measures based on the notion of stochastic line integral. We show that the value process of the problem is a solution of corresponding semimartingale backward equation in case where the decision set is not necessarily a compact. As an example we deal with the martingale analogue of the so-called linear-quadratic regulator problem. The problem of sufficiency of partial information of the optimization problem is also studied.

MSC2010: 93E20, 60G44.

Keywords: Stochastic Optimal Control, Semimartingale Backward Equation.

Homogenization and asymptotics for small transaction costs

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The problem of investment and consumption in a market with transaction costs was formulated by Magill & Constantinides in 1976. Since then, starting with the classical paper of Davis & Norman an impressive understanding has been achieved. This problem of proportional transaction costs is a special case of a singular stochastic control problem in which the state process can have controlled discontinuities. The related partial differential equation for this class of optimal control problems is a quasi-variational inequality which contains a gradient constraint. Technically, the multi-dimensional setting presents intriguing free boundary problems.

It is well known that in practice the proportional transaction costs are small and in the limiting case of zero costs, one recovers the classical problem of Merton. Then, a natural approach to simplify the problem is to obtain an asymptotic expansion in terms of the small transaction costs. This was initiated in the pioneering paper of Constantinides. The first proof in this direction was obtained in the appendix of Shreve & Soner.

In this talk, we consider this classical problem of small proportional transaction costs and develop a unified approach to the problem of asymptotic analysis. We also relate the first order asymptotic expansion to an ergodic singular control problem.

This convergence result also provides a connection with homogenization. Indeed, the dynamic programming equation of the ergodic problem is the *corrector equation* in the homogenization terminology. This identification allows us to construct a rigorous proof similar to the ones in homogenization. Moreover, the ergodic problem is a singular one and its continuation region also describes the asymptotic shape of the no-trade region in the transaction cost problem.

The main proof technique is the viscosity approach of Evans to homogenization. This powerful method combined with the relaxed limits of Barles & Perthame provides the necessary tools. As well known, this approach has the advantage of using only a simple L^∞ bound.

MSC2010: 91B28, 35K55, 60H30.

Keywords: transaction costs, homogenization, viscosity solutions, asymptotic expansions.

Coupling and tracking of regime-switching martingales

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In this talk we describe two explicit couplings of a pair of standard Brownian motions, which yield the stochastic minimum of (a) the coupling time and (b) the tracking error of two regime-switching martingales. It turns out that the optimal strategies for the two problems are explicit in the initial data, identical in size and opposite in sign. This is joint work with Saul Jacka.

Long-run Investment under Drawdown Constraints: optimal portfolios and numeraire property

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(joint work with Vladimir Cherny; Constantinos Kardaras, Eckhard Platen)

We consider long-run investment problems under drawdown constraints: the current wealth can not fall below a given function of its past maximum. We work in a general semimartingale setting. First, we show that this problem is equivalent to an unconstrained problem but with a modified utility function: both the value and the optimal portfolio are given explicitly in terms of their unconstrained counterparts (joint work with Vladimir Cherny). Second, we analyse in more detail the growth optimal portoflio under linear drawdowns. We show it enjoys the numeraire property along specific sequences of stopping times and asymptotically but not for all times. A turnpike theorem shows it is a limit of numeraire strategies on finite horizons (joint work with Constantinos Kardaras and Eckhard Platen).

MSC2010: 91B16.

Keywords: drawdown constrains, numeraire, long-run investment, utility maximisation.

Optimal mean-variance stopping strategies

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In this talk, I will present trading strategies (stopping times) that are optimal for a mean-variance criterion. The objective is to maximize – over all trading strategies (stopping times) – the mean-variance criterion. When the stock prices are modelled as a geometric Brownian motion, optimal strategies will be described and illustrated.

Backward SDEs with partially nonpositive jumps and Hamilton-Jacobi-Bellman IPDEs

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We consider a class of BSDEs where the jumps component of the solution is subject to a partial nonpositive constraint. After proving existence and uniqueness of a minimal solution under mild assumptions, we give a dual representation of this solution as an essential supremum over a family of equivalent change of probability measures. We then show how minimal solutions to our BSDE class provide actually a new probabilistic representation for integro-partial differential equations (IPDEs) of Hamilton-Jacobi-Bellman (HJB) type, when dealing with a suitable Markovian framework. Connections with the recent theory of 2BSDEs are discussed.

Optimal selling of a defaultable stock in a spectrally negative Lévy model

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In this talk we consider the problem to identity the optimal time to sell a stock in the sense of minimizing the ratio of the maximum over the investment horizon and the value at the time of transaction. We assume that the price of the stock is given by an exponential Lévy process, which is a model that captures the features of fat tails and asymmetry, which are commonly observed features in financial returns data. In this study we restrict ourselves to spectrally negative Lévy processes, which have no positive jumps. We assume furthermore that the stock is subject to a possibility of (equity) default. This problem is phrased as an optimal stopping problem which we solve explicitly. This talk is based on joint work with A. Mijatovic.

Extremal Martingales

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Availability of market prices of call options of all strikes determines the risk-neutral distribution of the underlying asset at the terminal time. Finding the maximum and minimum price of various derivatives whose prices depend on the maximal value and the terminal value (such as barrier options) has been studied in the last 15 years or so by Hobson, Cox, Obloj, Brown, and others, and some quite complete results are known. This talk takes as its starting point some older work characterizing the possible joint laws of the maximum and terminal value of a martingale; this converts the problem of finding the extremal martingale into a linear programming problem, an observation which allows effective numerical solution. I hope to be able to talk about more recent work with Moritz Duembgen characterizing the possible joint distributions of the maximum, minimum and terminal value of a continuous martingale.

Optimal stopping of strong Markov processes

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(joint work with Sören Christensen and Bao Q. Ta)

We characterize the value function and the optimal stopping time for a large class of optimal stopping problems where the underlying process to be stopped is a fairly general Markov process. The main result - a new verification theorem - is inspired by recent findings for Lévy processes obtained essentially via the Wiener-Hopf factorization. The main ingredient in our approach is the representation of the β-excessive functions as expected suprema. A variety of examples is given. A preprint is available at arXiv:1203.4726v2

MSC2010: 60G40, 60J30.

Keywords: Hunt process, Lévy process, diffusion, Riesz representation, supremum representation for excessive functions.

A Measure Approach to Impulse Control Problems

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(joint work with Kurt L. Helmes and Chao Zhu)

Only finitely many actions will occur in any bounded interval of time for control problems in which each action incurs a positive cost. Typically these problems are formulated so that the decision-maker selects sequences of stopping times $\tau := \{\tau_k : k \in \mathbb{N}\}$ and jump amounts $Y := \{Y_k : k \in \mathbb{N}\}$ such that at time τ_k the state process X instantaneously jumps from $X(\tau_k-)$ to $X(\tau_k-)+Y_k$. The paired sequeneces (τ, Y) form an impulse control policy. A criterion is then used to compare the efficacy of the policies.

This talk establishes a general formulation of impulse control problems for state processes characterized as solutions to martingale problems for their generators. It shows how to reformulate a discounted control problem in terms of expected occupation measures, resulting in an imbedding in an infinite-dimensional linear programme. Specific examples are considered in which the analysis of auxiliary linear programmes having fewer constraints determines the optimal value and identifies an optimal impulse control policy.

MSC2010: 93E20, 60G40, 60G44.

Keywords: impulse control, expected discounted occupation measures, infinite-dimensional linear programming.

Viscosity Solutions of Fully Nonlinear Path-Dependent PDEs

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We propose a notion of viscosity solutions for path dependent fully nonlinear parabolic PDEs. One typical example is the path dependent HJB equations, which can also be viewed as viscosity solutions of second order Backward SDEs and G-martingales. The definition is based on a nonlinear optimal stopping problem, and is consistent with the notion of classical solution in the sense of Dupire's functional Itô calculus. We prove the existence, uniqueness, stability, and comparison principle for the viscosity solutions. Our approach is to use a variation of the Peron's approach to prove the comparison principle.

Optimal investment decision for a cash-constrained firm

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The large and growing literature on real options is based on three influential factors: cash-flows uncertainty, non-recovery of investment costs and flexibility in the timing of decision. Nevertheless, the real option literature relies also on the assumption of perfect capital markets and thus put aside the funding needs. In this presentation based on several previous and on-going research, we depart from the assumption of perfect capital markets and analyse the investment decision for a cash-constrained firm.

MSC2010: 60G40, 91G80, 91G50.

Keywords: singular control, optimal stopping, corporate finance.

Optimal stopping of one-dimensional diffusions with generalised drift

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(joint work with Pui Chan Lon)

We consider the problem of optimally stopping a one-dimensional diffusion with generalised drift over an infinite horizon. We derive a variational inequality that the problem's value function should satisfy and we prove a verification theorem. We then solve the special case that arises when the state process is a skew geometric Brownian motion and the reward function is the one of a financial call option. We show that the optimal stopping strategy can take several qualitatively different forms, depending on parameter values.

MSC2010: 60G40, 60J60.

Keywords: optimal stopping, diffusions with generalised drift, skew Brownian motion.