Iterative ensemble smoothers and applications to reservoir history matching

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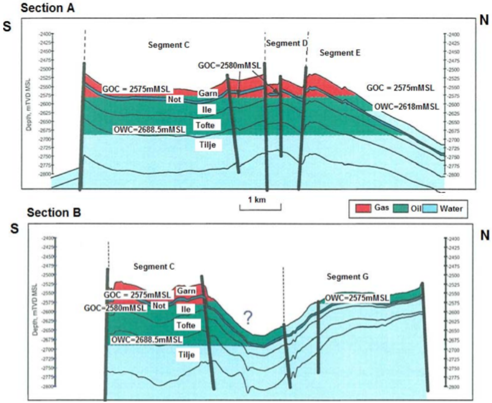
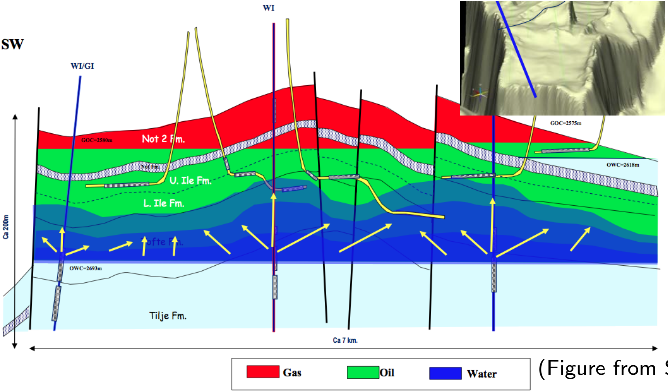
History matching: the Norne field



(Figures from Statoil reports)

- Multi-phase flow is primarily controlled by heterogeneous permeability and porosity
- Faults with unknown sealing properties

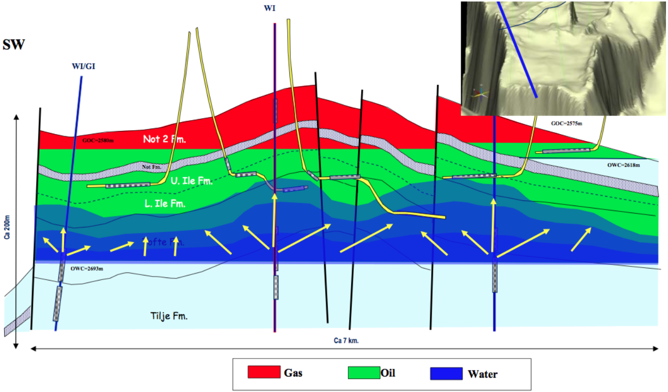
History matching: data and unknown variables



(Figure from Statoil report)

- Data are primarily limited to a small number of well locations
- Variables to be estimated
  1. Static variables: permeability and porosity of each simulation cell, fault transmissibility, etc.
  2. Dynamic variables: phase saturation and pressure, etc.

The Norne field drainage strategy

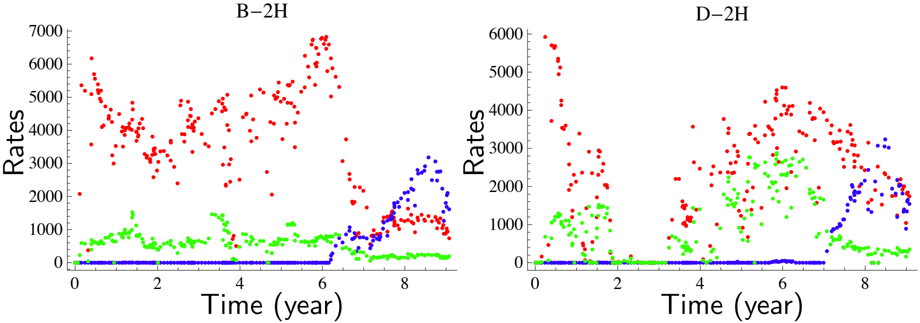


(Figure from Statoil report)

- Water alternating gas injection
- Use the iterative ES (ensemble smoother) to avoid updating saturation with EnKF (ensemble Kalman filter)

Noisy data: production rates

Production since 1997. Full field data (through 2006) released in August 2012



Oil, gas and water production rates. Unit is m^3/day for oil and water and 1000 m3/day for gas

Sampling from posterior pdf

Compute parameters that minimize a stochastic objective function:

$$J_i(\mathbf{x}) = \frac{1}{2} \overbrace{\left(\mathbf{g}(\mathbf{x}) - \mathbf{d}_i^o\right)^T \mathbf{C}_D^{-1} \left(\mathbf{g}(\mathbf{x}) - \mathbf{d}_i^o\right)}^{\text{Sum of squared data mismatch}} + \frac{1}{2} \underbrace{\left(\mathbf{x} - \mathbf{x}_i^{\mathrm{pr}}\right)^T \mathbf{C}_X^{-1} \left(\mathbf{x} - \mathbf{x}_i^{\mathrm{pr}}\right)}_{\text{Model parameter mismatch}}.$$

where

$\mathbf{x}$ is vector of model variables

$\mathbf{x}_i^{\mathrm{pr}}$ is the ith sample from the prior distribution

$\mathbf{d}^o = \mathbf{g}(\mathbf{x}^{\mathrm{tr}}) + \epsilon$ and $\epsilon \sim N[0, \mathbf{C}_D]$

$\mathbf{d}_i^o \sim N[\mathbf{d}^o, \mathbf{C}_D]$

Gauss-Newton method

Solving $\nabla J_i(\mathbf{x}) = 0$,

$$\delta \mathbf{x}_i^\ell = -(\mathbf{x}_i^\ell - \mathbf{x}_i^{\text{pr}}) - \mathbf{C}_X \mathbf{G}_\ell^T \left(\mathbf{C}_D + \mathbf{G}_\ell \mathbf{C}_X \mathbf{G}_\ell^T \right)^{-1} \left(\mathbf{g}(\mathbf{x}_i^\ell) - \mathbf{d}_i^o - \mathbf{G}_\ell (\mathbf{x}_i^\ell - \mathbf{x}_i^{\text{pr}}) \right).$$

where $\mathbf{G}_\ell = \nabla \mathbf{g}(\mathbf{x}^\ell)$.

At the first iteration ($\ell = 1$), when $\mathbf{x}_i^\ell = \mathbf{x}_i^{\text{pr}}$

$$\delta \mathbf{x}_i^1 = -\mathbf{C}_X \mathbf{G}_1^T \left(\mathbf{C}_D + \mathbf{G}_1 \mathbf{C}_X \mathbf{G}_1^T \right)^{-1} \left(\mathbf{g}(\mathbf{x}_i^{\text{pr}}) - \mathbf{d}_i^o \right).$$

Depending on the amount of data, Gauss-Newton and quasi-Newton methods have been used to obtain parameter estimates, from which state estimates were obtained.

Generating samples from the posterior pdf

- EnKF: Approximate $\mathbf{C}_X\mathbf{G}_1^T$ and $\mathbf{G}_1\mathbf{C}_X\mathbf{G}_1^T$ from ensemble

$$
\begin{aligned}
\delta\mathbf{x}_i^1 &= -\mathbf{C}_X\mathbf{G}_1^T\left(\mathbf{C}_D + \mathbf{G}_1\mathbf{C}_X\mathbf{G}_1^T\right)^{-1}\left(\mathbf{g}(\mathbf{x}_i^{\mathrm{pr}}) - \mathbf{d}_i^o\right) \\
&= -\Delta\mathbf{x}^{\mathrm{pr}}\Delta\mathbf{d}_1^T\left((N_e-1)\mathbf{C}_D + \Delta\mathbf{d}_1\Delta\mathbf{d}_1^T\right)^{-1}\left(\mathbf{g}(\mathbf{x}_i^{\mathrm{pr}}) - \mathbf{d}_i^o\right)
\end{aligned}
$$

Generating samples from the posterior pdf

- EnKF: Approximate $\mathbf{C}_X\mathbf{G}_1^T$ and $\mathbf{G}_1\mathbf{C}_X\mathbf{G}_1^T$ from ensemble

$$\begin{aligned}\delta\mathbf{x}_i^1 &= -\mathbf{C}_X\mathbf{G}_1^T\left(\mathbf{C}_D+\mathbf{G}_1\mathbf{C}_X\mathbf{G}_1^T\right)^{-1}\left(\mathbf{g}(\mathbf{x}_i^{\mathrm{pr}})-\mathbf{d}_i^o\right)\\ &= -\Delta\mathbf{x}^{\mathrm{pr}}\Delta\mathbf{d}_1^T\left((N_e-1)\mathbf{C}_D+\Delta\mathbf{d}_1\Delta\mathbf{d}_1^T\right)^{-1}\left(\mathbf{g}(\mathbf{x}_i^{\mathrm{pr}})-\mathbf{d}_i^o\right)\end{aligned}$$

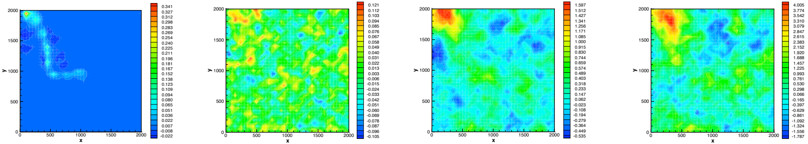
- GN-EnRML: Approximate $\mathbf{G}_\ell\mathbf{C}_X\mathbf{G}_\ell^T$ and $\mathbf{C}_X\mathbf{G}_\ell^T$ and $\mathbf{G}_\ell$ from ensemble

$$\delta\mathbf{x}_i^\ell = -\beta_\ell(\mathbf{x}_i^\ell-\mathbf{x}_i^{\mathrm{pr}})-\beta_\ell\mathbf{C}_X\mathbf{G}_\ell^T\left(\mathbf{C}_D+\mathbf{G}_\ell\mathbf{C}_X\mathbf{G}_\ell^T\right)^{-1}\left(\mathbf{g}(\mathbf{x}_i^\ell)-\mathbf{d}_i^o-\mathbf{G}_\ell(\mathbf{x}_i^\ell-\mathbf{x}_i^{\mathrm{pr}})\right).$$

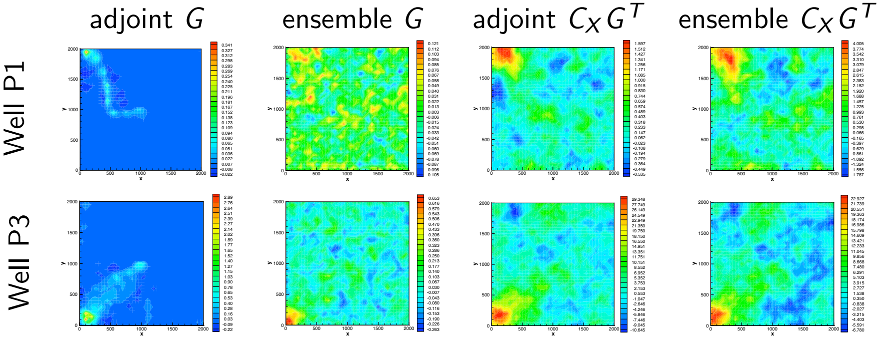
Five-spot waterflood example

adjoint G | ensemble G | adjoint $C_X G^T$ | ensemble $C_X G^T$

Well P1



Five-spot waterflood example



Regularized ensemble-based iterative updating

1) Levenberg-Marquardt regularization

$$\delta \mathbf{x}_i^\ell = -\left[(1+\lambda_\ell)\mathbf{P}_\ell^{-1} + \mathbf{G}_\ell^T \mathbf{C}_D^{-1} \mathbf{G}_\ell\right]^{-1} \times \left[\mathbf{C}_X^{-1}(\mathbf{x}_i^\ell - \mathbf{x}_i^{\mathrm{pr}}) + \mathbf{G}_\ell^T \mathbf{C}_D^{-1}(g(\mathbf{x}_i^\ell) - \mathbf{d}_i^{\mathrm{o}})\right].$$

2) Matrix inversion lemma

$$\delta \mathbf{x}_i^\ell = -\left[(1+\lambda_\ell)\mathbf{P}_\ell^{-1} + \mathbf{G}_\ell^T \mathbf{C}_D^{-1} \mathbf{G}_\ell\right]^{-1} \mathbf{C}_X^{-1}(\mathbf{x}_i^\ell - \mathbf{x}_i^{\mathrm{pr}}) - \mathbf{P}_\ell \mathbf{G}_\ell^T \left[(1+\lambda_\ell)\mathbf{C}_D + \mathbf{G}_\ell \mathbf{P}_\ell \mathbf{G}_\ell^T\right]^{-1} (g(\mathbf{x}_i^\ell) - \mathbf{d}_i^{\mathrm{o}}).$$

3) Set $\mathbf{P}_\ell = \Delta \mathbf{x}_\ell \, \Delta \mathbf{x}_\ell{}^T / (N_e - 1)$, then LM-EnRML is

$$\delta \mathbf{x}_i^\ell = -\left[(1+\lambda_\ell)\mathbf{P}_\ell^{-1} + \mathbf{G}_\ell^T \mathbf{C}_D^{-1} \mathbf{G}_\ell\right]^{-1} \mathbf{C}_X^{-1}(\mathbf{x}_i^\ell - \mathbf{x}_i^{pr}) - \Delta \mathbf{x}_\ell \Delta \mathbf{d}_\ell^T \left[(1+\lambda_\ell)(N_e - 1)\mathbf{C}_D + \Delta \mathbf{d}_\ell \Delta \mathbf{d}_\ell^T\right]^{-1} (\mathbf{g}(\mathbf{x}_i^\ell) - \mathbf{d}_i^o)$$

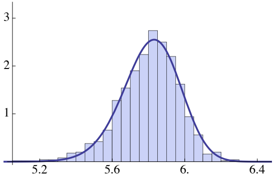
Levenberg-Marquardt form of iterative ensemble smoother (LM-EnRML)

$$\delta \mathbf{x}_i^\ell = - \left[(1+\lambda_\ell) \mathbf{P}_\ell^{-1} + \mathbf{G}_\ell^T \mathbf{C}_D^{-1} \mathbf{G}_\ell \right]^{-1} \mathbf{C}_X^{-1} (\mathbf{x}_i^\ell - \mathbf{x}_i^{pr}) \\ - \Delta \mathbf{x}_\ell \Delta \mathbf{d}_\ell^T \left[(1+\lambda_\ell)(N_e - 1) \mathbf{C}_D + \Delta \mathbf{d}_\ell \Delta \mathbf{d}_\ell^T \right]^{-1} (\mathbf{g}(\mathbf{x}_i^\ell) - \mathbf{d}_i^o)$$

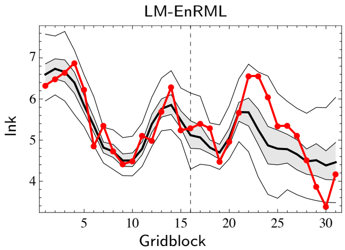
- First iteration is exactly the same as would be obtained with the ensemble Kalman filter (except that $\mathbf{C}_D \to (1+\lambda)\mathbf{C}_D$).
- The initial value for λ is typically quite large in reservoir flow problems ($\lambda_1 \sim 10^4$).
- If the objective function decreases, then decrease λ.
- Note that the gradient of the objective function was not modified — only the approximation to the Hessian.

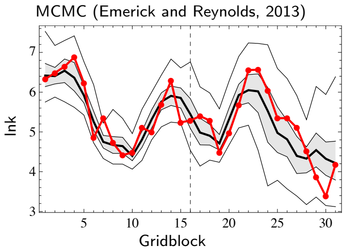
Various test problems using LM-EnRML

- One-variable nonlinear (validated against true posterior pdf)



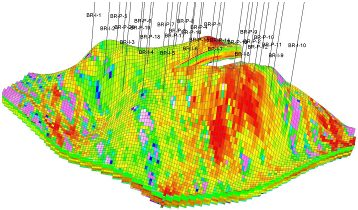
- One-dimensional flow (validated against MCMC)



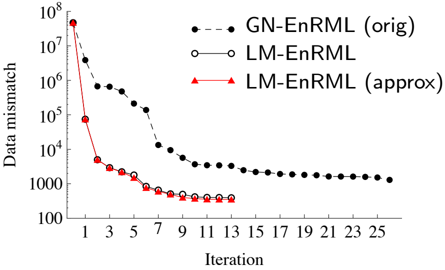


Rate of convergence

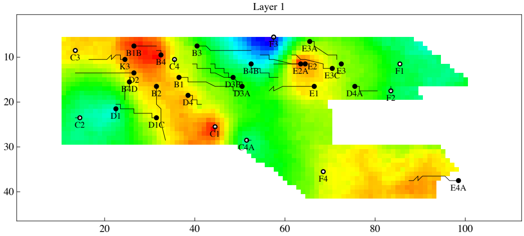
- Brugge benchmark case



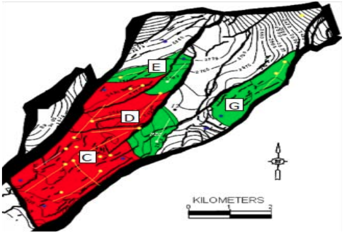
Data mismatch: $\left(g\left(\mathbf{x}\right)-\mathbf{d}^{\mathrm{o}}\right)^{T}\mathbf{C}_{D}^{-1}\left(g(\mathbf{x})-\mathbf{d}^{\mathrm{o}}\right)$



The Norne field simulation model



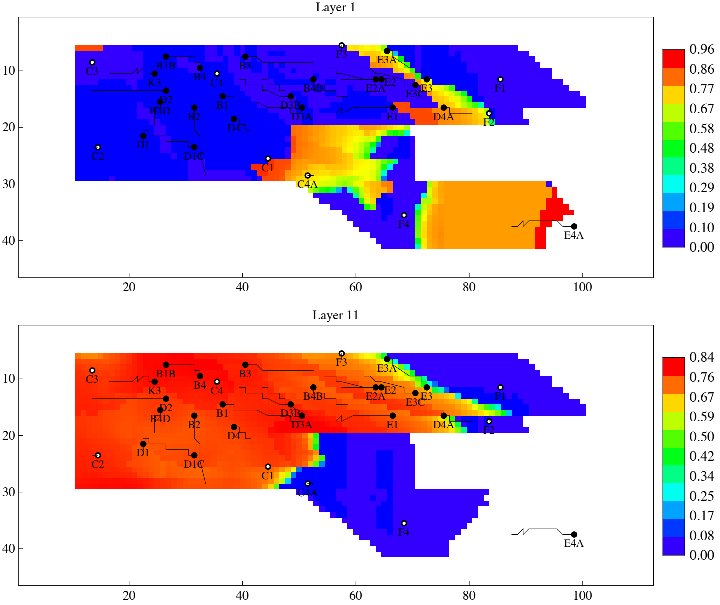
one layer of simulation model



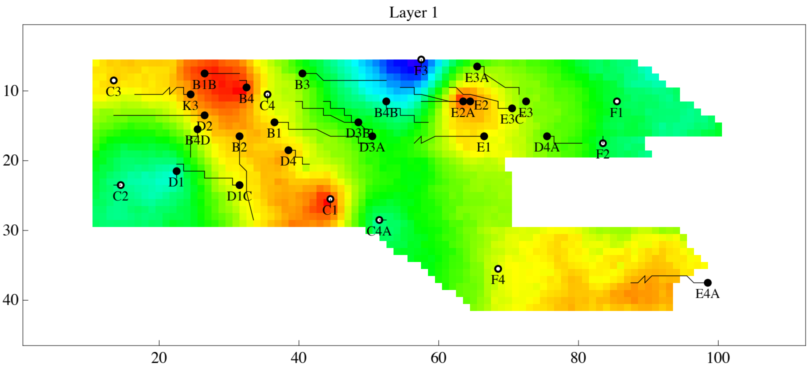
structure map (statoil report)

- The dimension is $46 \times 112 \times 22$, 44927 active cells
- Historical rate from Nov 1997 to Dec 2006
- RFT pressure at 14 time instances from 14 wells
- Four main fault blocks and in total 60 faults
- Five equilibration regions with different depth of initial fluid contacts

Initial oil saturation



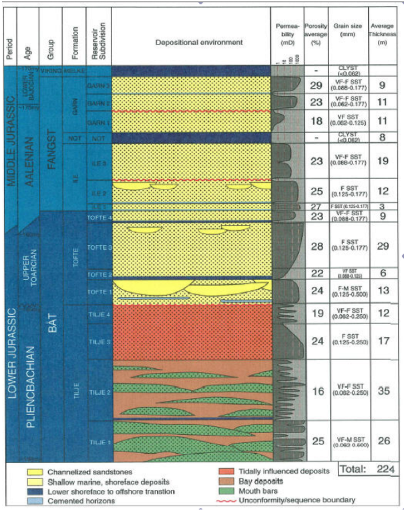
The Norne field



As of December 2006, 17 active wells: 11 oil producers, 3 water injectors, 3 gas producers.

Total of 31 wells with production data (9 injectors, 22 producers) from Nov 1997 to Dec 2006.

Vertical communication



Inactive

MULTZ $= 0.05$

MULTZ $= 0.01$

MULTZ $= 0.05$

MULTZ $= 0.001$

MULTZ $= 0.0$

MULTZ: vertical transmissibility multiplier

(Figure from Statoil report)

Manual history matching

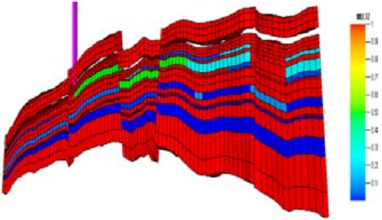


Figure 36 - Local modifications to the field-wide MULTZ barriers in the 2004 model

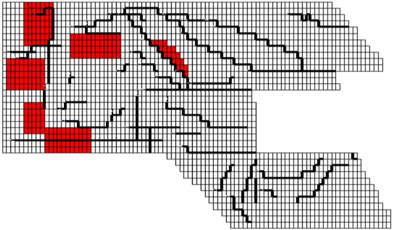


Figure 54 - Local MULTZ modifications in the carbonate barrier to allow observed water rise.

(Figures from thesis of Eirik Morell, NTNU, 2010)

Selected field-wide vertical transmissibility multiplier values, then modified them locally to try to match RFT and water rise.

Manual history matching

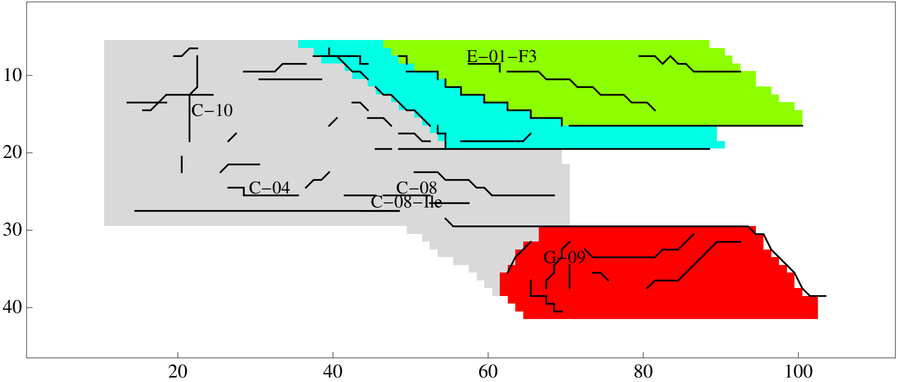


Figure: main fault blocks and location of faults

- Modified transmissibility between main fault blocks
- Modified transmissibility across faults

Variables for history matching (iterative ES)

Porosity	Grid-based property
Permeability	Grid-based property, correlated with porosity
Net-to-Gross	Grid-based property
MULTZ	Grid-based property, at six layers
MULTFLT	Fault transmissibility multipliers (53 parameters)
Rel perm	End-point water and gas relative permeability of four zones (8 parameters)
MULTREGT	Transmissibility multiplier between a few fault blocks (3 parameters)
WOC	Initial water-oil contacts (5 parameters)

- Number of model parameters is about 150,000
- Used 100 realizations in the iterative ensemble smoother

Production data and noise assumed in data

Producers are under reservoir volume constraint

- Water injection rate (20 m^3/day except for C-1H)
- Gas injection rate (20 m^3/day)

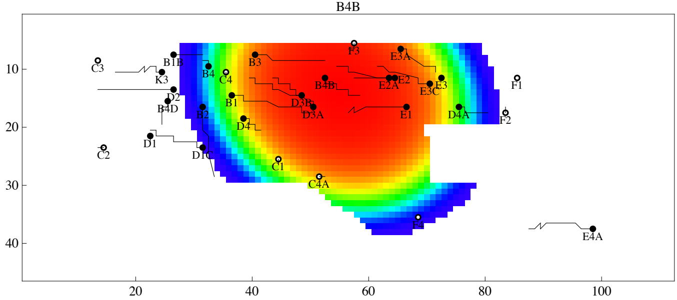
Other data:

- Oil production rate (100 m^3/day)
- Water production rate (200 m^3/day)
- Gas production rate (20000 m^3/day)
- RFT pressure (2 bar)

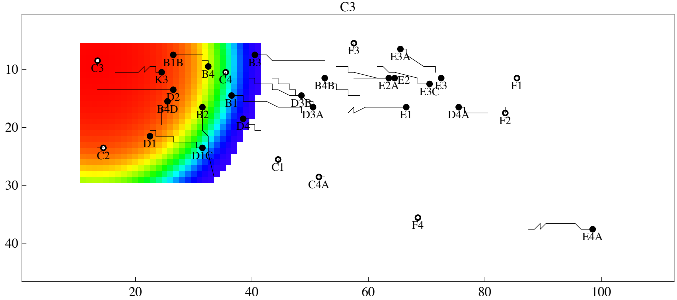
The number of effective data is about 2000

Distance-based localization

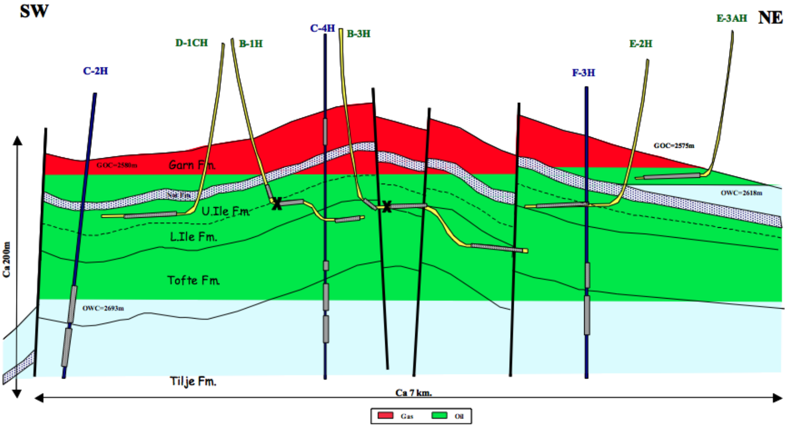
B-4BH



C-3H



Vertical localization by location of completions



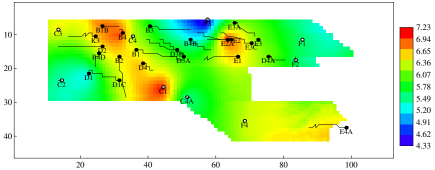
(Figure from Statoil report)

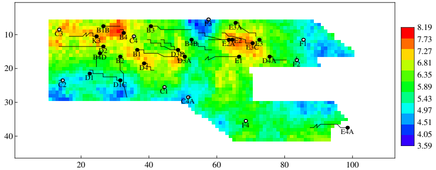
Localization

Area of localization changes with layer and with time at which data were obtained

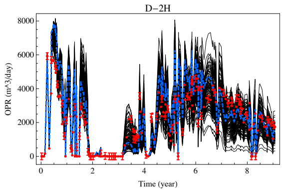
- Updates to permx, porosity, NTG localized to layers within zones of completion
- Shale barrier (gridblock vertical transmissibility) is updated when well is completed in zones immediately above or below the barrier
- Fault transmissibility is updated by data within the same fault block (no distance discrimination)
- Relative permeability endpoint of one zone is only updated by data in the same zone
- Water-oil contact of a fault block is only updated by data in the same fault block

Generation of initial ensemble

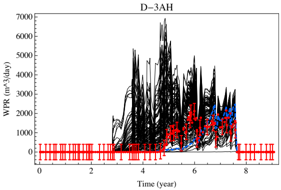




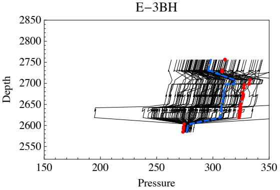
Prediction from the initial ensemble



OPR



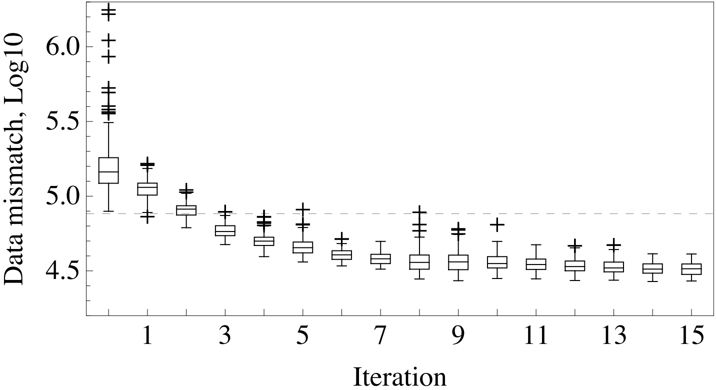
WPR



RFT

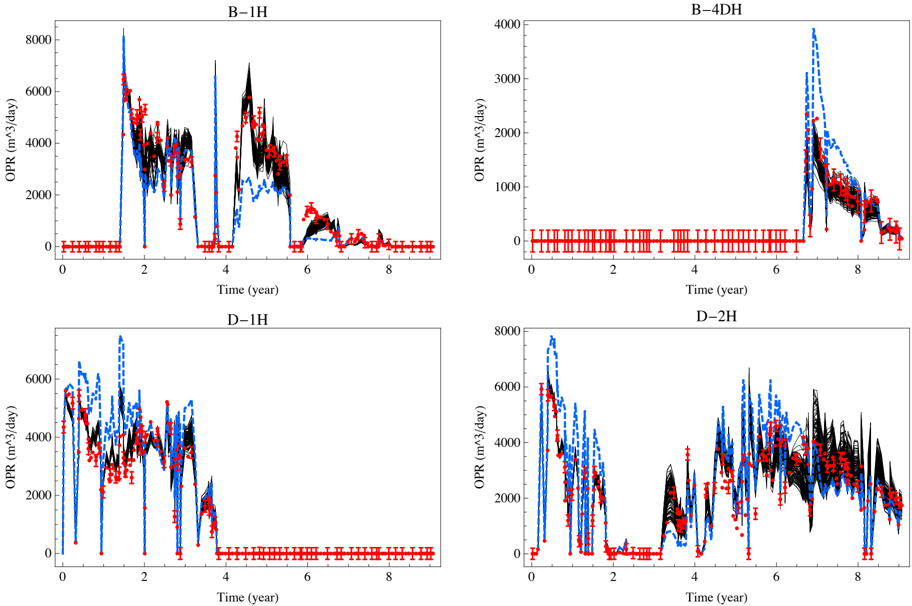
Data mismatch

$$O_d = (d_{sim} - d_{obs})^T C_D^{-1} (d_{sim} - d_{obs})$$

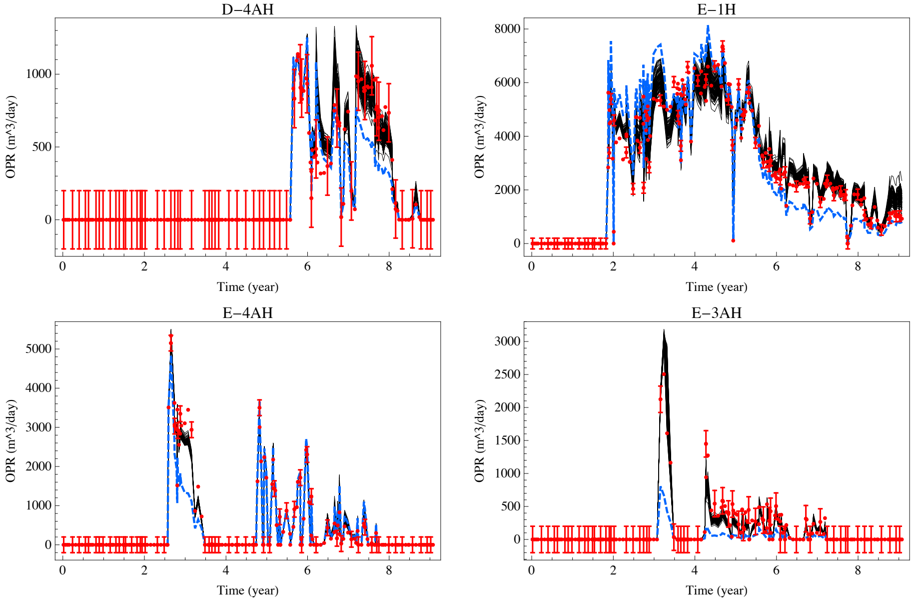


Dashed horizontal line shows O_d of the manual history matched model. The number of effective data is about 2000.

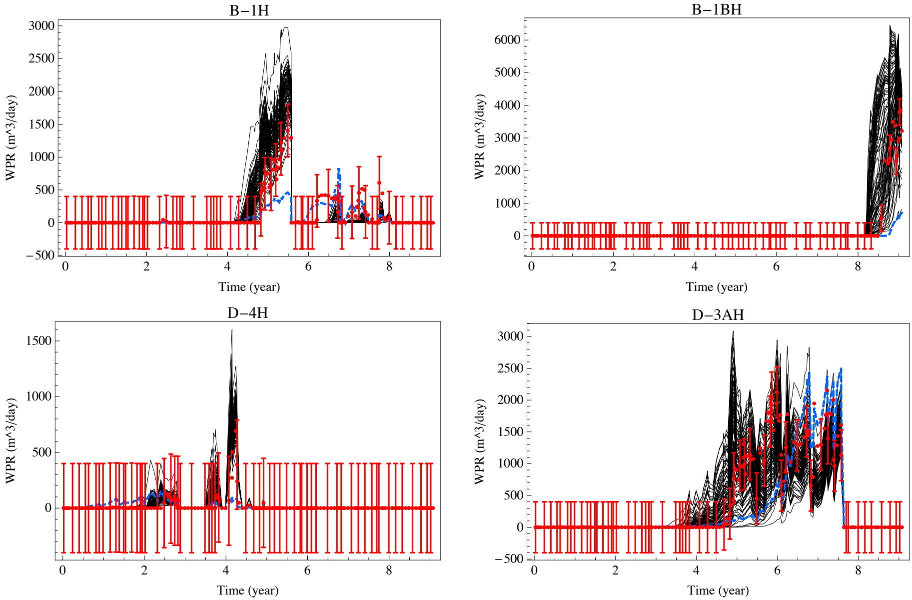
Data match: oil production rate



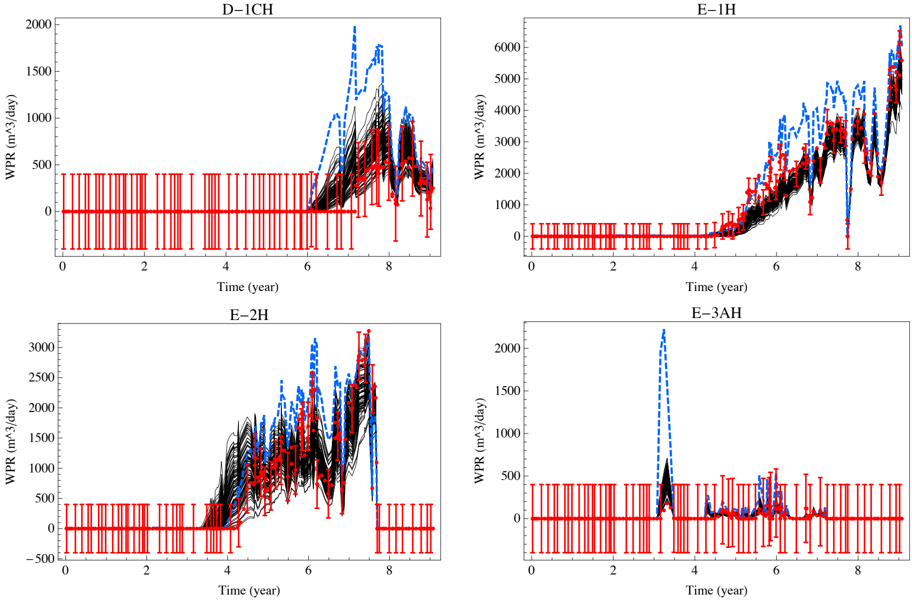
Data match: oil production rate



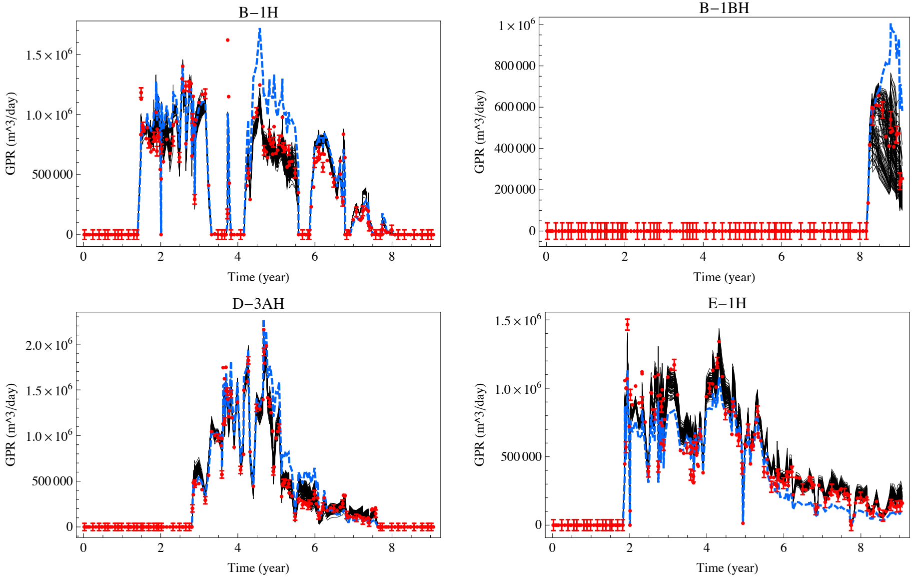
Data match: water production rate



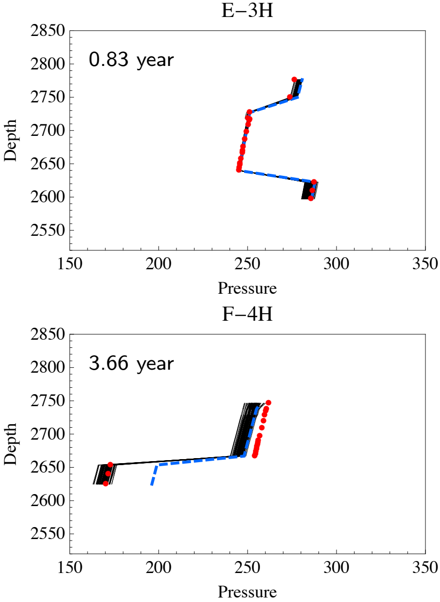
Data match: water production rate



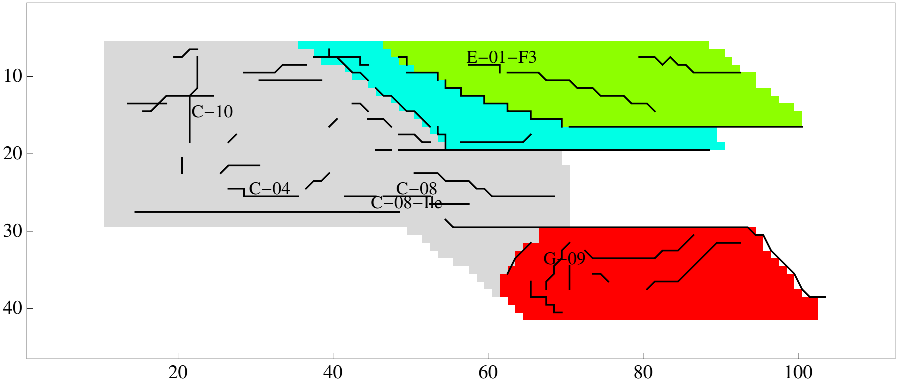
Data match: gas production rate



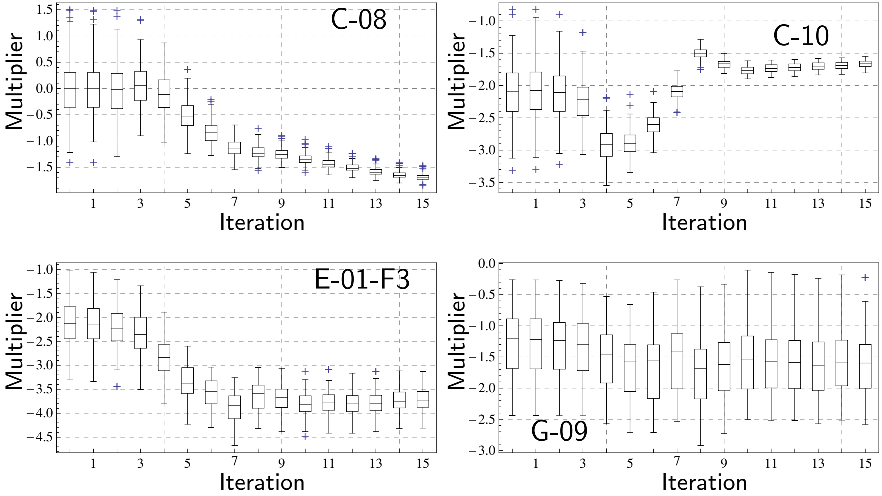
Data match: RFT pressure



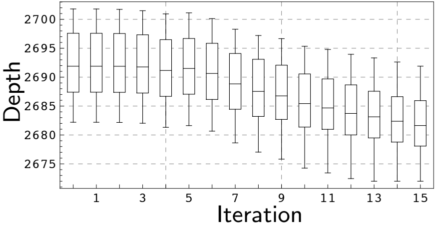
Faults in Norne model



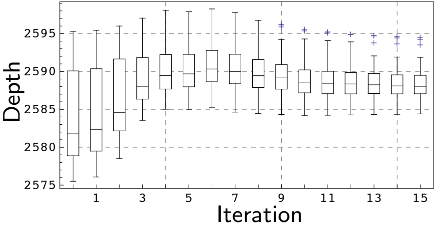
Estimation: fault transmissibility multiplier



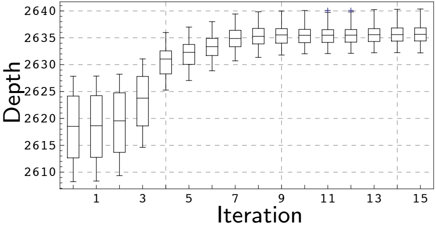
Estimation: oil-water contact



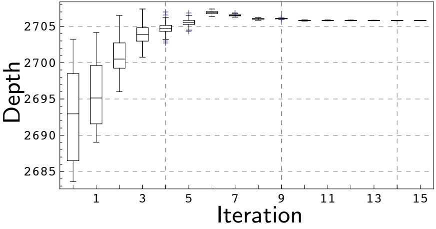
C&D – Garn



G – Garn

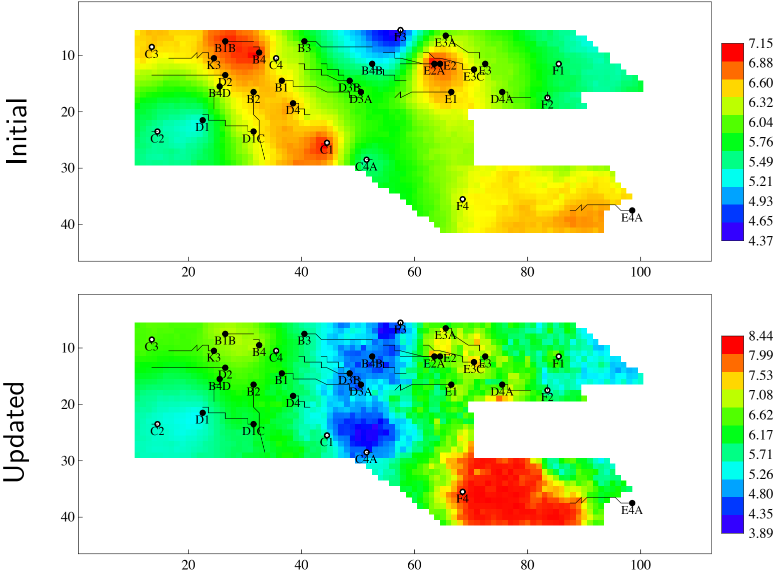


E – Garn

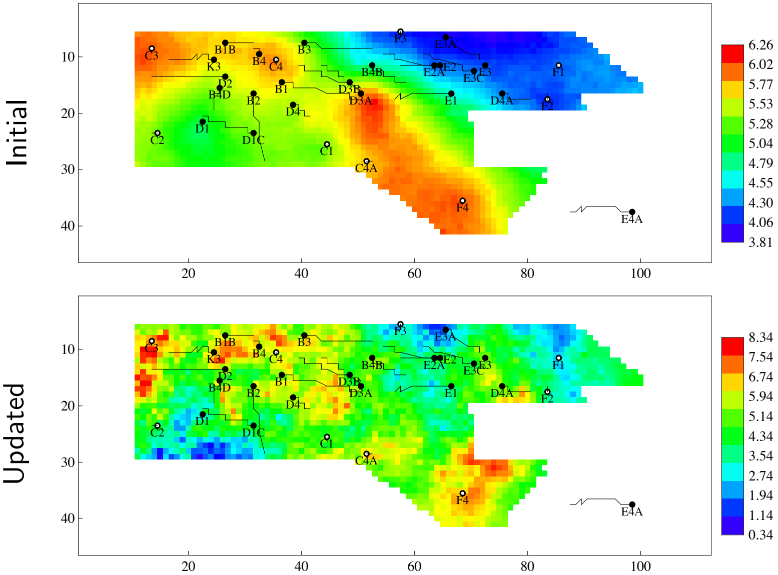


C&D&E – Ile to Tilje

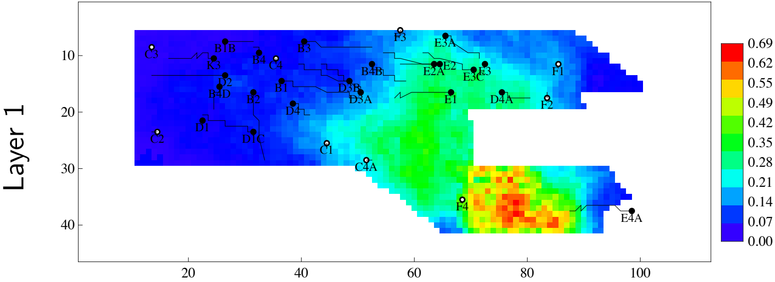
Estimation: mean gridblock permeability of Layer 1

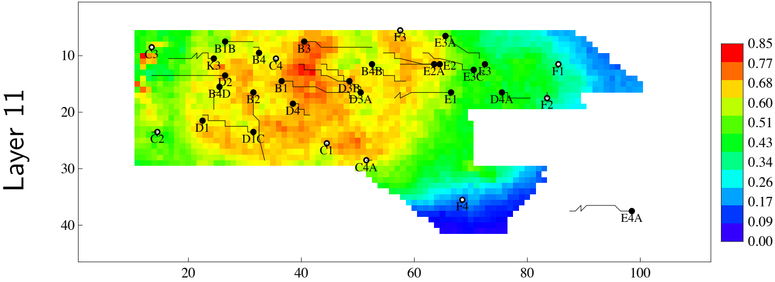


Estimation: mean gridblock permeability of Layer 11

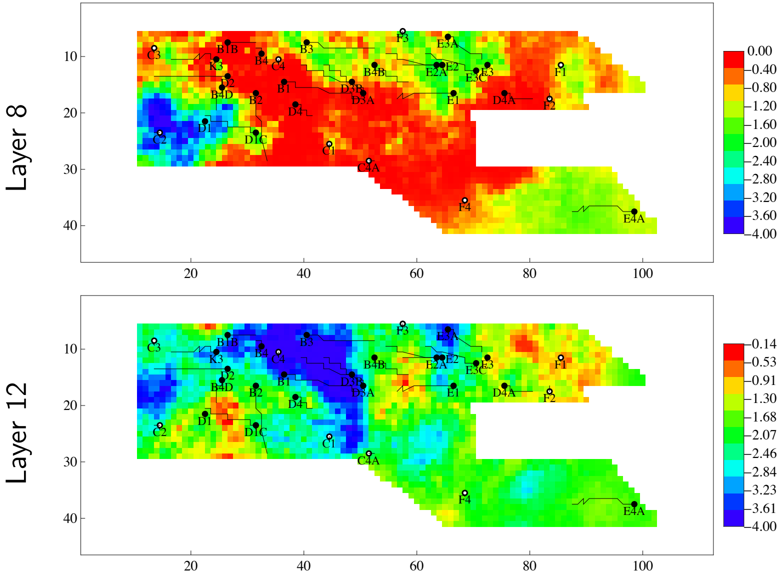


Reduction in uncertainty: gridblock permeability





Estimation: mean vertical transmissibility multiplier



Summary

- Fairly complex case (uncertainty in compartmentalization, point properties, fluid contacts, rock-fluid properties) with great nonlinearity handled almost routinely
- Was not necessary to spend much time "improving the prior" (as might be required for ES)
- Method was quite robust with respect to parameters of minimization (initial choice of λ, reduction rule, etc.)
- In general, should not worry about "the danger of over parameterization" if localization is done properly
- Probably still missing some sources of uncertainty: structure, skin distribution, fault transmissibility distribution, etc. (but then places greater requirement on localization)
- Localization needs simplification and improvement (some collapse of variability of perm in C block)

Emerick, A. A. and A. C. Reynolds, Investigation of the sampling performance of ensemble-based methods with a simple reservoir model, *Computat. Geosci.*, 2013.