Kinetically constrained models, from classical to quantum

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£ EPSRC The Leverhulme Trust

1.- Why & what of **KCMs** ?

2.- Generalising to **quantum dissipative KCMs**

3.- How **KCM** dynamics **emerges in Rydberg** gases

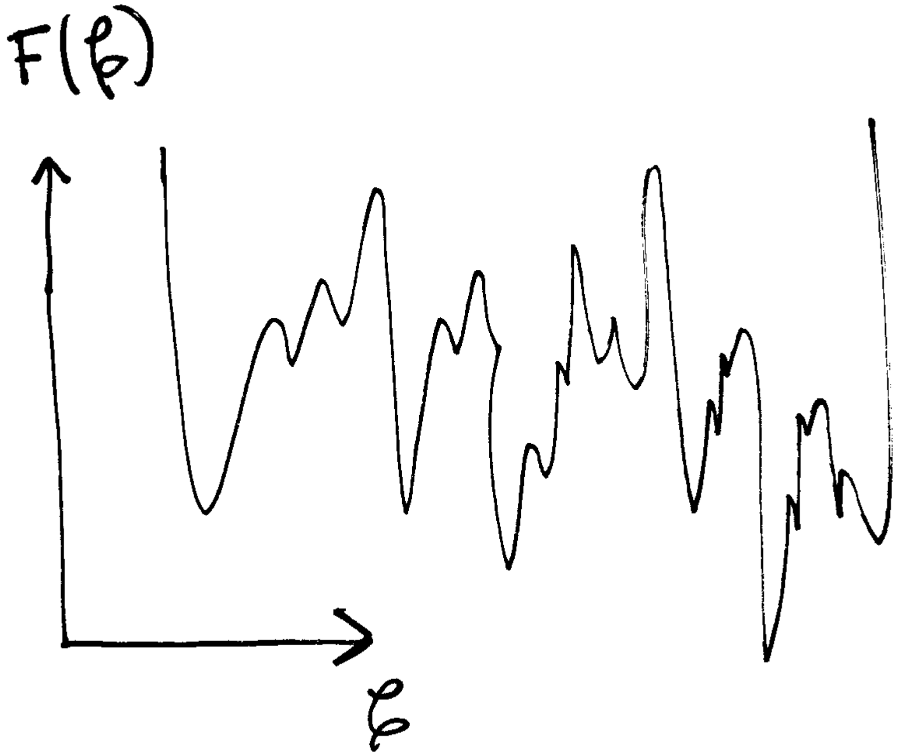
4.- **KCMs** and **many-body localisation** in quantum systems

5.- Outlook

Basics of KCMs

Competing perspectives on glass transition & how to model them

Thermodynamic



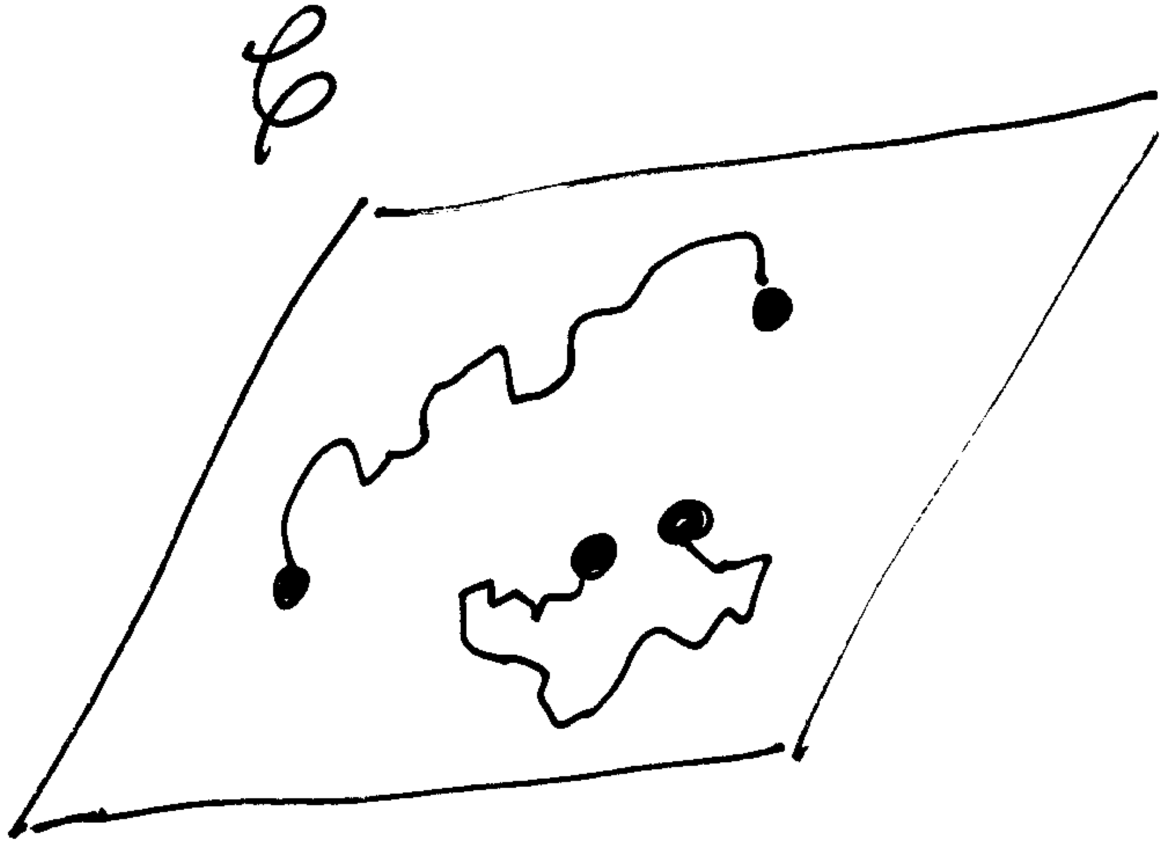
Statics ⇒ Dynamics

eg. **RFOT**

{Parisi+Wolynes+many others}

ideal models e.g. **p-spin spin glass**

Dynamic



Statics **does not** ⇒ Dynamics

metric → Dynamic facilitation

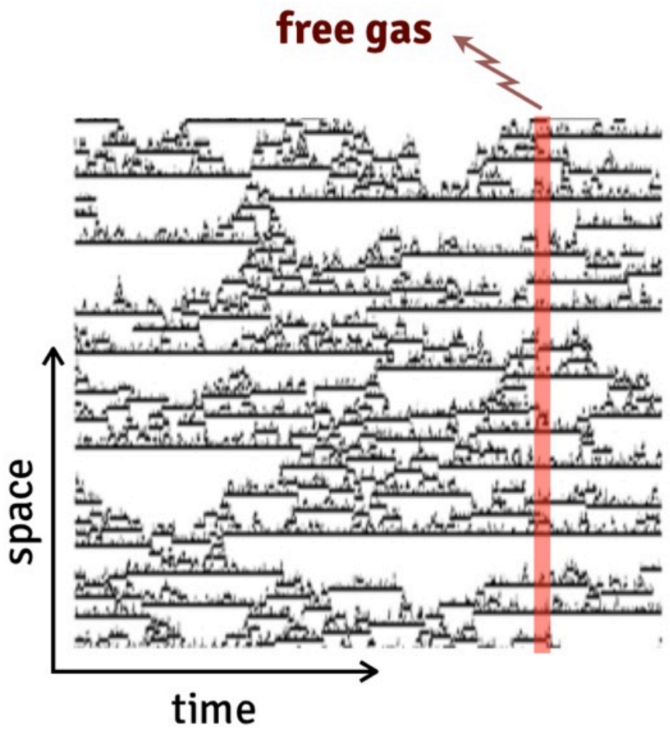
ideal models **KCMs**

{Anderson+Andersen+Jackle+many others}

Basics of KCMs

$$\partial_t|P\rangle = \mathbb{W}|P\rangle \quad \longrightarrow \quad \mathbb{W} = \sum_i (n_{i-1}+\delta)\left[\epsilon\sigma_i^+ + \sigma_i^- - \epsilon(1-n_i) - n_i\right] + (i \longleftrightarrow i-1)$$

Basics of KCMs



trivial statics

heterogeneous & **hierarchical** dynamics

East model

$$\partial_t|P\rangle = \mathbb{W}|P\rangle \;\;\rightarrow\;\; \mathbb{W} = \sum_i \overbrace{(n_{i-1}+\delta)\left[\epsilon\sigma_i^+ + \sigma_i^- - \epsilon(1-n_i) - n_i\right]} + (i \leftrightarrow i-1)$$

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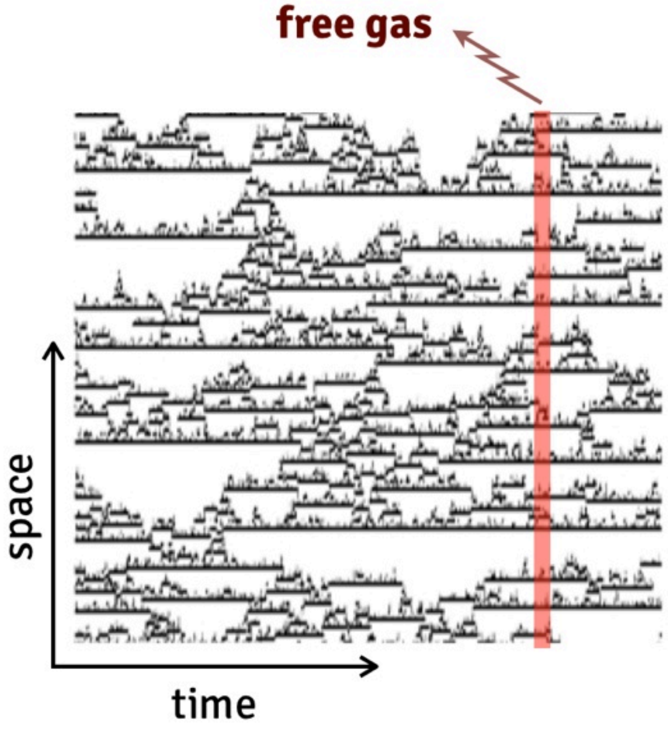

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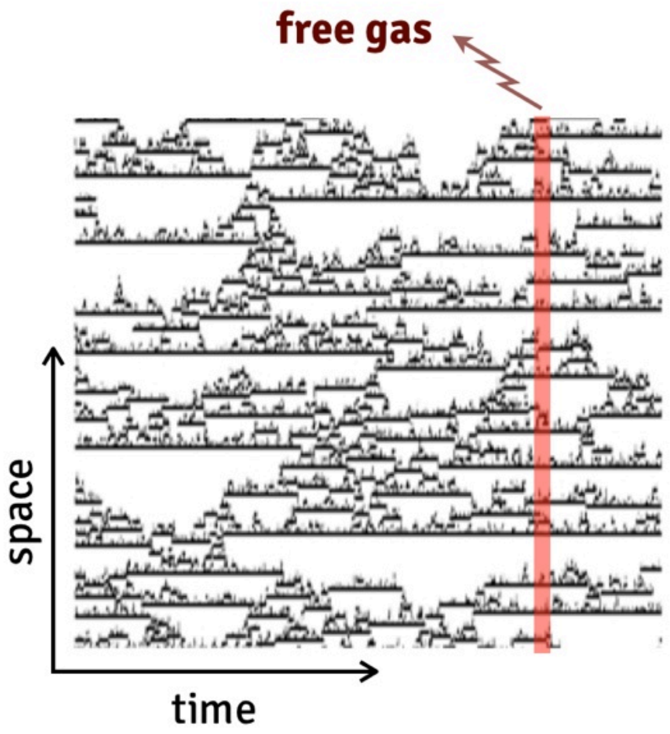

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$$\partial_t |P\rangle = \mathbb{W}|P\rangle \quad \rightarrow \quad \mathbb{W} = \sum_i \overbrace{\overbrace{\underbrace{(n_{i-1}+\delta)}_{\text{constraint = operator valued rate}} \left[\epsilon\sigma_i^+ + \sigma_i^- - \epsilon(1-n_i) - n_i\right]}^{\textbf{East model}} + (i \leftrightarrow i-1)}^{\textbf{FA model}}$$

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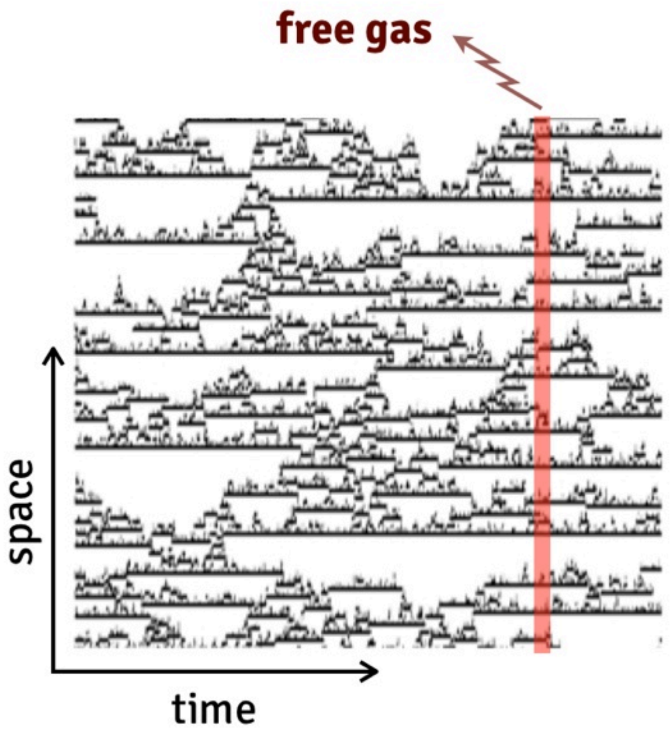

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Dynamics is **more** than statics

Transitions between active (relaxing) and inactive (non-relaxing) **dynamical phases**

Can extend **beyond classical glasses**?

Quantum KCM models of dissipative quantum glasses

{Olmos-Lesanovsky-JPG, **PRL 2012 + New 2014**}

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{Olmos-Lesanovsky-JPG, **PRL 2012 + New 2014**}

Recap: Open quantum systems and Quantum Markov processes

$$H_{\mathrm{T}} = H + \underbrace{H_{\mathrm{B}} + H_{\mathrm{BS}}} \qquad \rho = \mathrm{Tr}_{\mathrm{B}}|\psi_T\rangle\langle\psi_T|$$

quantum master equation:

{Lindblad, Belavkin-Stratonovich}

$$\partial_t\rho = -i[H,\rho] + \overbrace{\sum_\mu L_\mu\rho L_\mu{}^\dagger - 1/2\{L_\mu{}^\dagger L_\mu,\rho\}} \equiv \mathcal{W}(\rho)$$

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stochastic $\mathcal{C}_t$

deterministic $P(\mathcal{C}_t) \ \rightarrow \ \partial_t |P\rangle = \mathbb{W} |P\rangle$

➡

stochastic $|\psi(t)\rangle$

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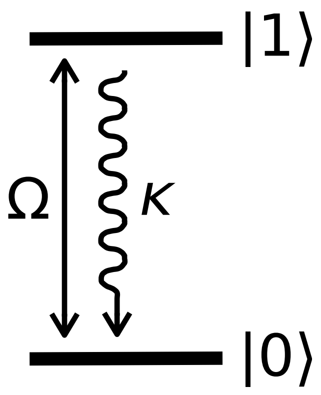
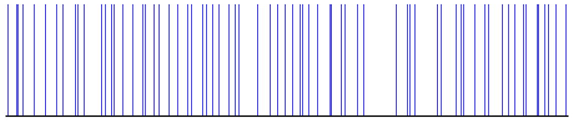
E.g.
laser-driven
2-level system
at T = 0:



$$H = \Omega\,\sigma_x$$

$$L_1 = \sqrt{\kappa}\sigma_-$$



quantum jump trajectory

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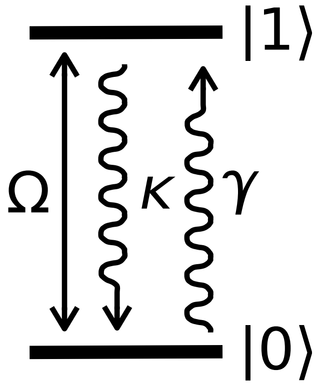
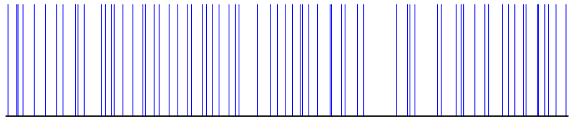
E.g. laser-driven 2-level system at T = 0:



$$H = \Omega\, \sigma_x$$

$$L_1 = \sqrt{\kappa}\sigma_-$$

$$L_2 = \sqrt{\gamma}\sigma_+$$



quantum jump trajectory

NB: classical Master Eq. ⊂ **Quantum Master Eq.** $(H = 0 \;\&\; L_\mu = \text{rank-1} \Rightarrow \rho_{\text{diag}})$

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Quantum facilitated spin models:

(*) constrained dynamics

(**) trivial stationary state

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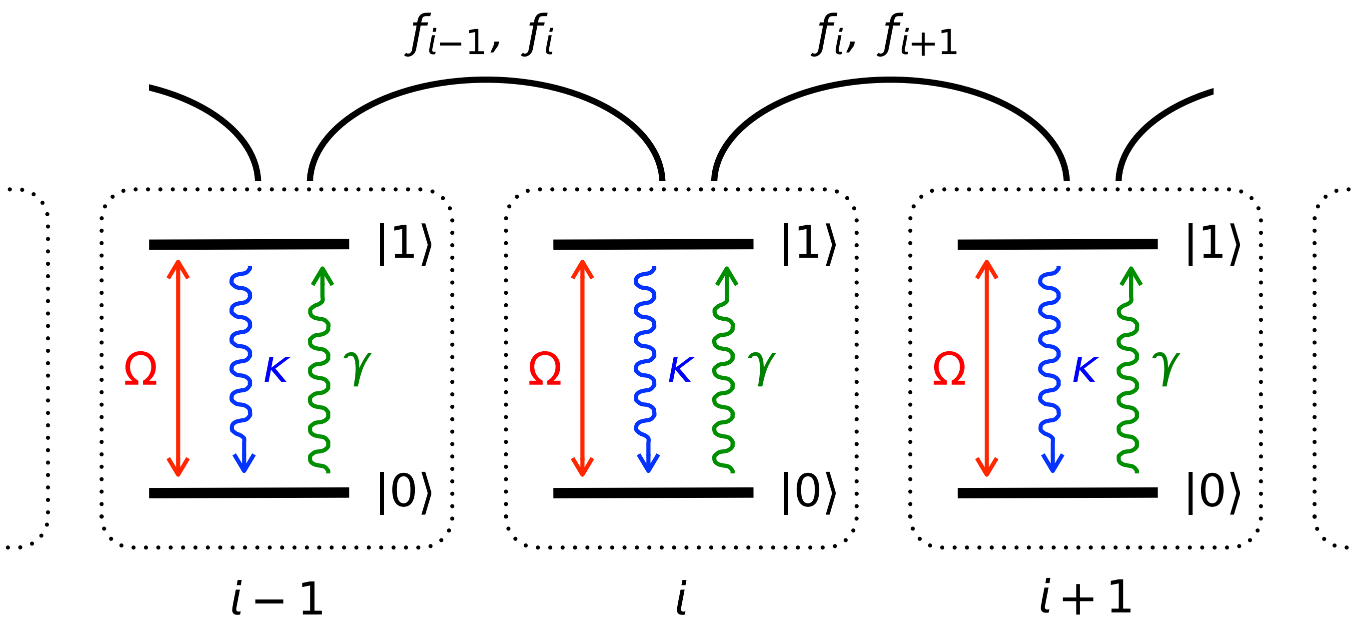

$$\begin{pmatrix} \Omega \to f_i\ \Omega \\ \kappa \to f_i\ \kappa \\ \gamma \to f_i\ \gamma \end{pmatrix} \quad (*)$$

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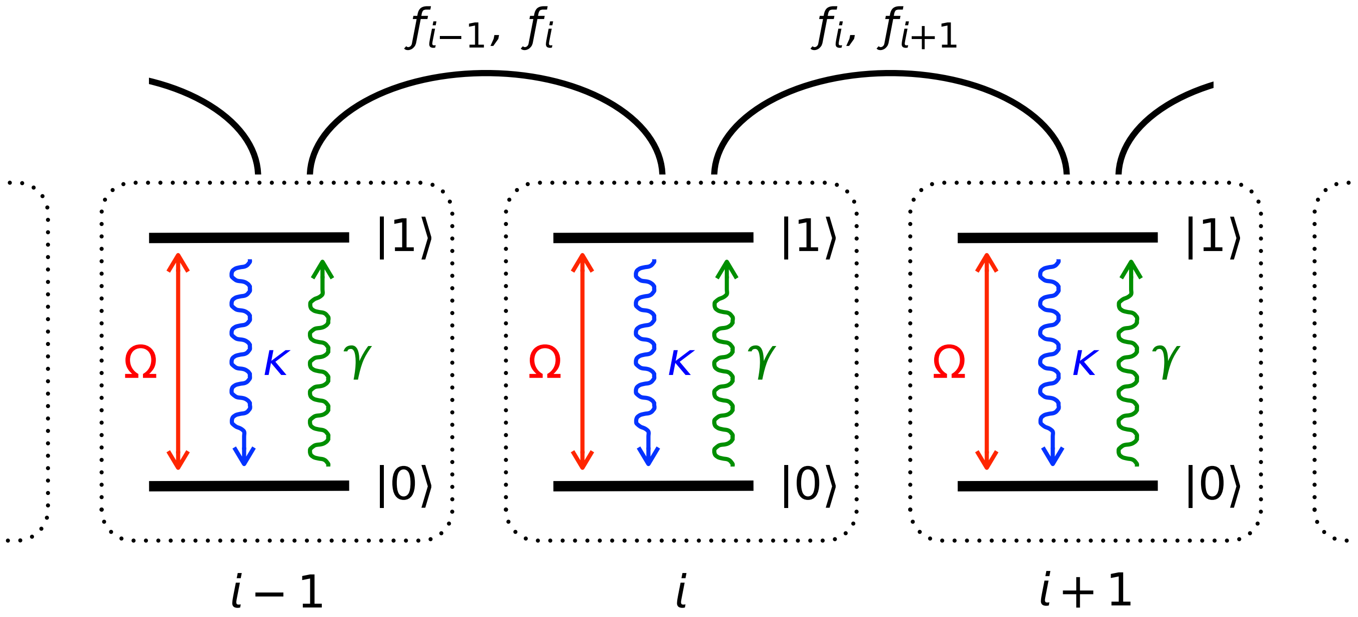

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$$\partial_t\rho = -i[H,\rho] + \sum_\mu L_\mu \rho L_\mu{}^\dagger - 1/2\{L_\mu{}^\dagger L_\mu,\rho\}$$

$$H=\sum_i f_i\ \Omega\ \sigma_i^x$$

$$\begin{cases} L_{i\downarrow} = f_i\ \sqrt{\kappa}\ \sigma_-^i \\ L_{i\uparrow} = f_i\ \sqrt{\gamma}\ \sigma_+^i\end{cases}$$

$$\left(\frac{\gamma}{\kappa}=e^{-1/T}\right)$$

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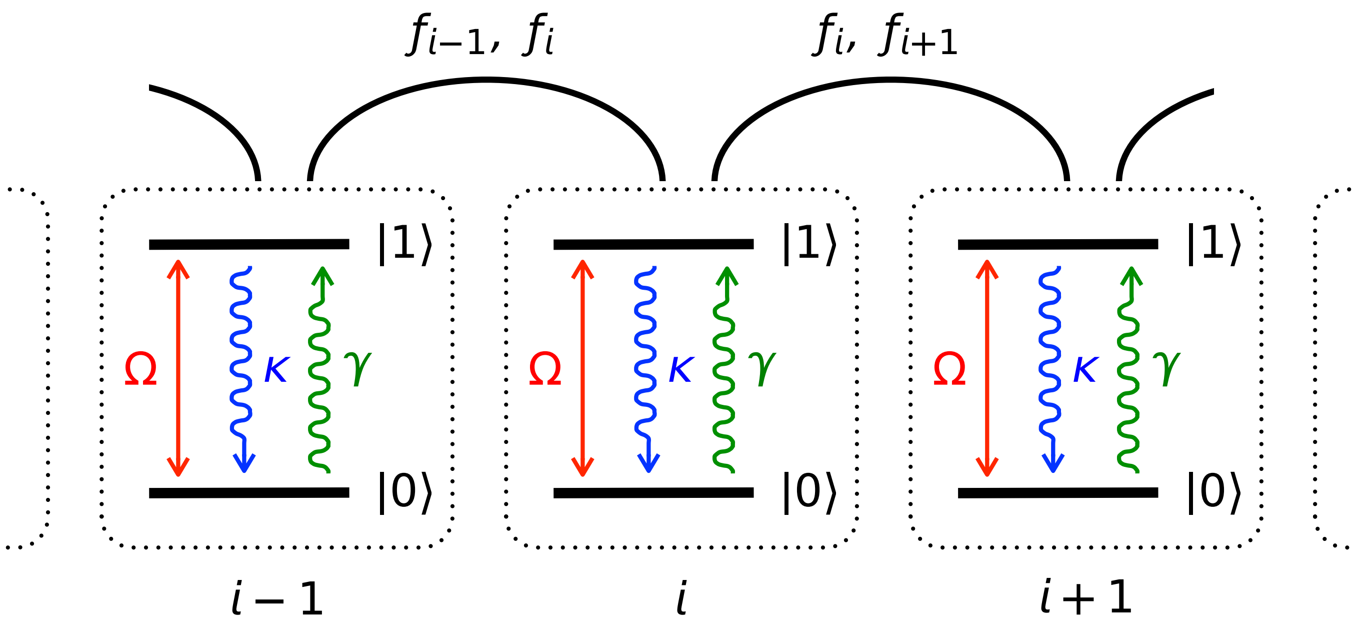

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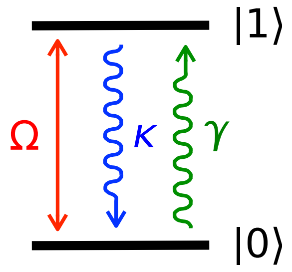
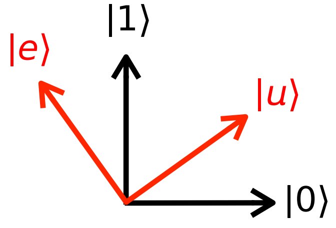


$$\rho^i_{\text{stat.}} = p_u |u_i\rangle\langle u_i| + p_e |e_i\rangle\langle e_i|$$



$$\rho_{\text{stat.}} = \bigotimes_i \rho^i_{\text{stat.}} \qquad (**)$$

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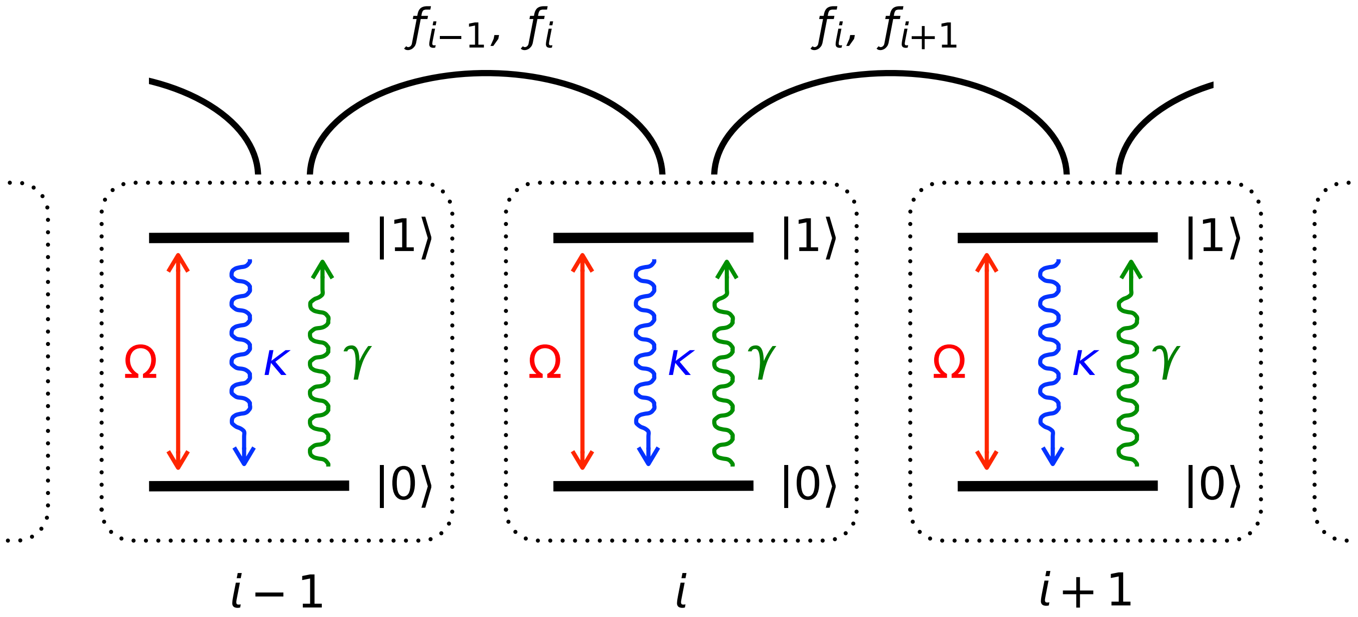

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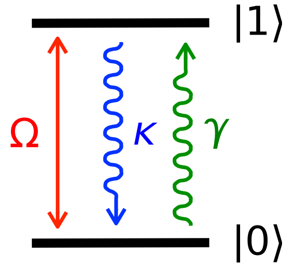
$$H = \sum_i f_i\ \Omega\ \sigma_i^x$$

$$\begin{cases} L_{i\downarrow} = f_i\ \sqrt{\kappa}\ \sigma_-^i \\ L_{i\uparrow} = f_i\ \sqrt{\gamma}\ \sigma_+^i \end{cases} \qquad \left(\frac{\gamma}{\kappa} = e^{-1/T}\right)$$

$$(*)\ \&\ (**) \longrightarrow f_i^\dagger = f_i\,,\quad f_i^2 = f_i\,,\quad [f_i, \rho_{\text{stat.}}] = 0$$ eg. **qEast** $f_{i+1} \equiv |e_i\rangle\langle e_i|$



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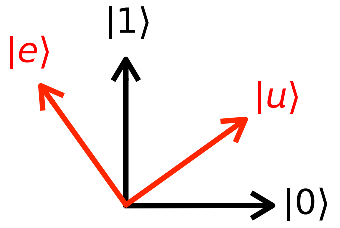

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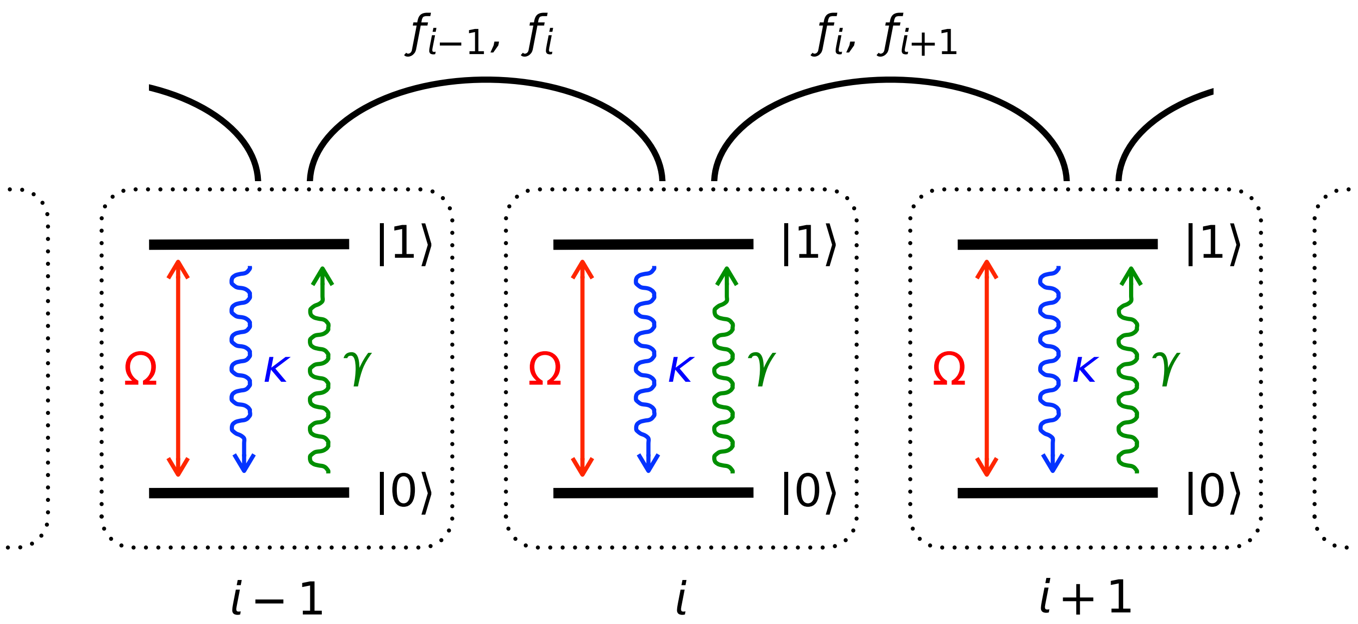

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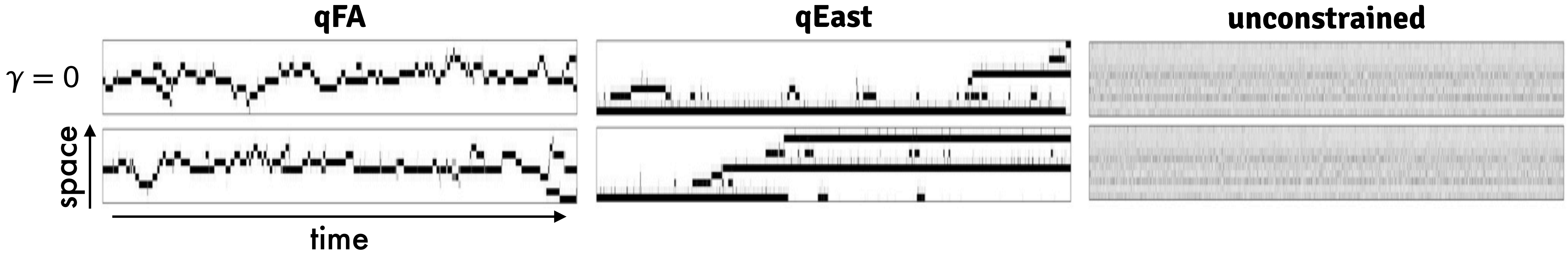

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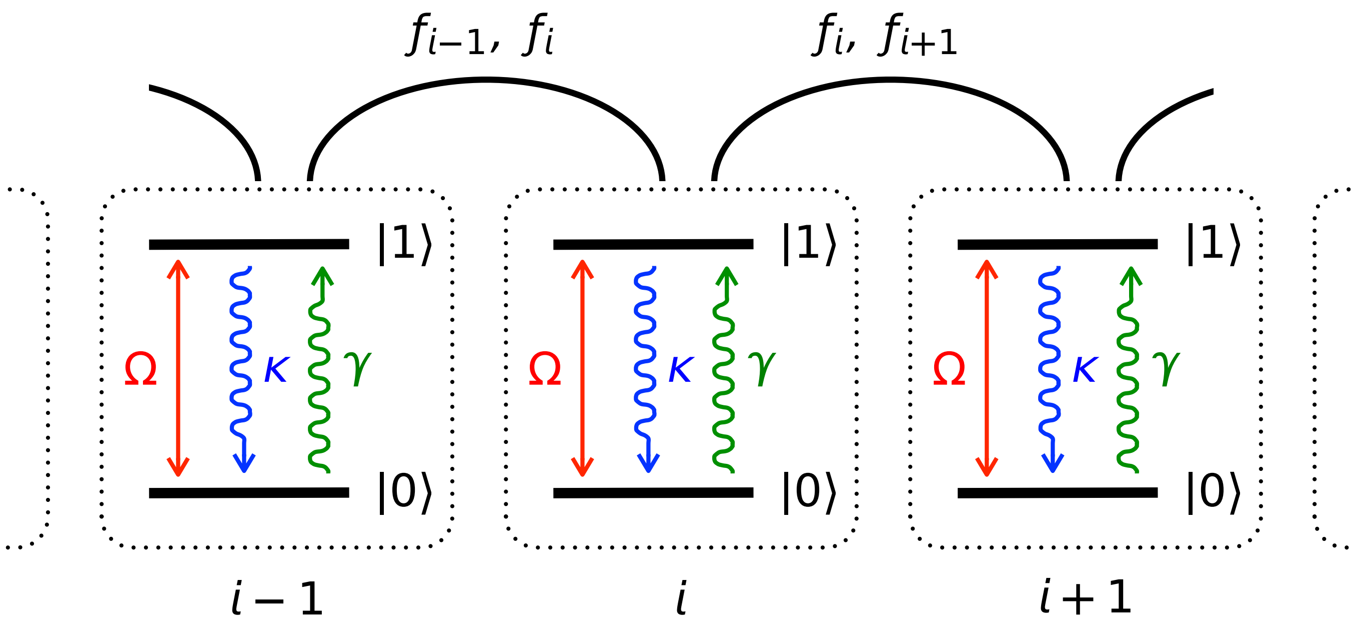

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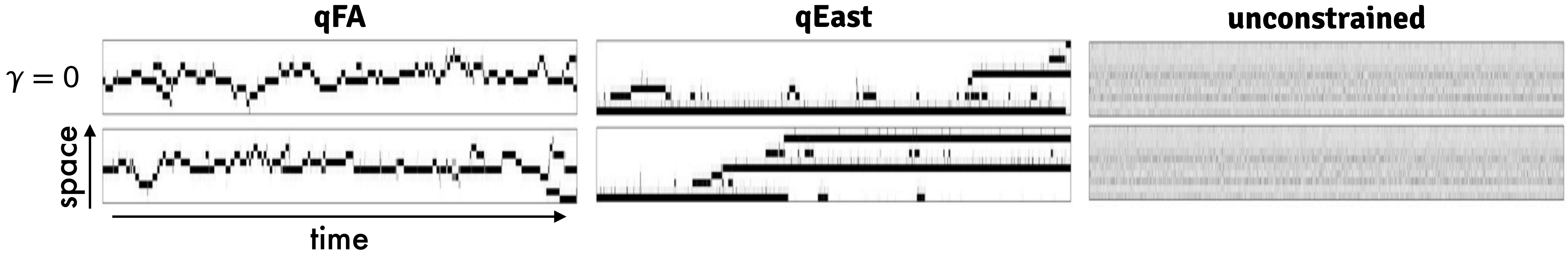
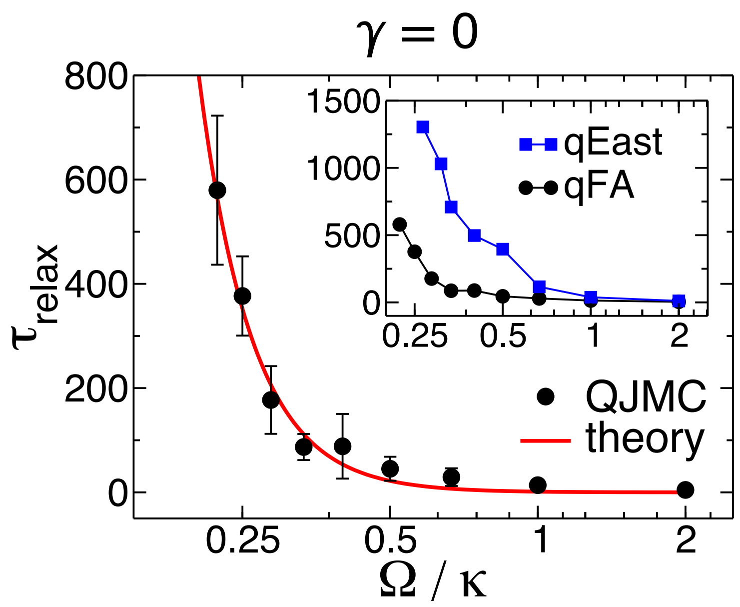
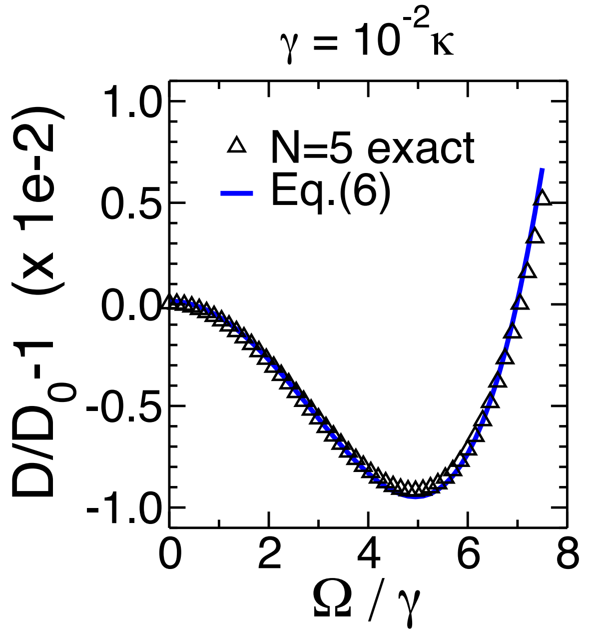


purely quantum relaxation **slowdown at small Ω**



classical vs quantum relaxation **reentrance**

{cf. Markland+ 2011}



Emergence of KCM dynamics in Rydberg systems

{Lesanovsky-JPG, **PRL 2013 + arXiv:1402.2126**}

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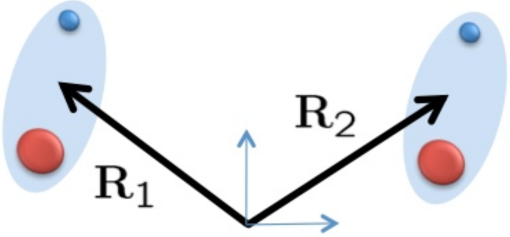
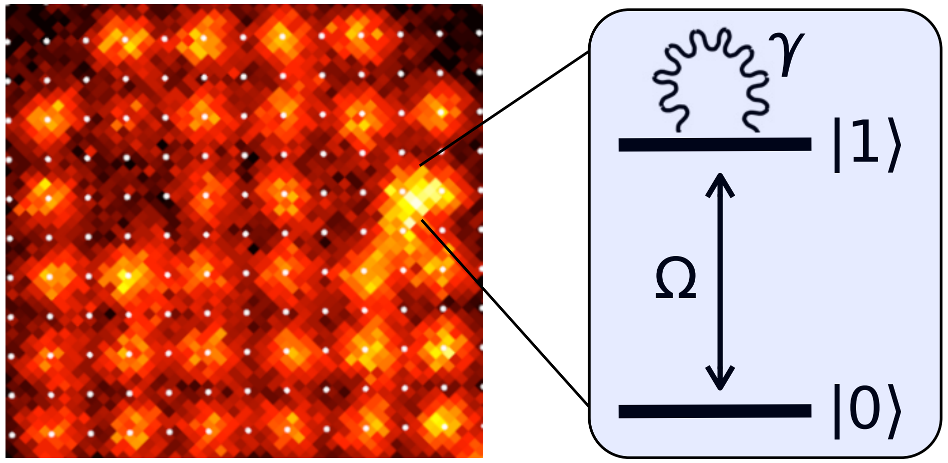
Rydberg atoms s-states
van-der-Waals interaction:

$$V_{\mathrm{vdW}} = \frac{C_6}{|\mathbf{R}_1 - \mathbf{R}_2|^6}$$



{cf. Bloch/Kuhr}



1 atom/site

no hopping

strong dephasing

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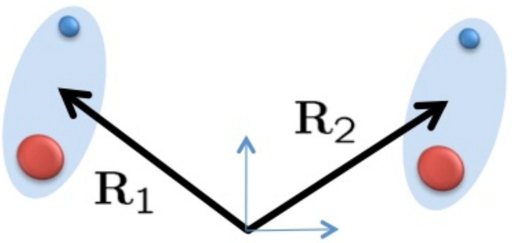

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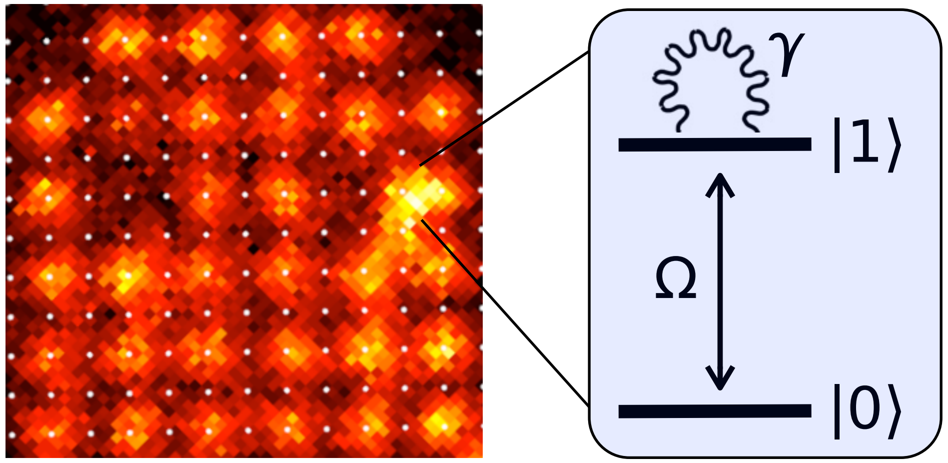

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$$H = \underbrace{\left(\Delta \sum_i n_i + \sum_{ij} V_{ij} n_i n_j\right)}_{\text{classical energy }\textbf{fast}} + \underset{\text{laser }\textbf{slow}}{\Omega \sum_i \sigma_x^i}$$

$$\partial_t \rho = -i\,[H,\rho] + \gamma \overbrace{\sum_i \left(n_i \rho n_i - n_i \rho - \rho n_i\right)}^{\text{dephasing }\textbf{fast}}$$

$$= \mathcal{W}_{\mathrm{fast}}(\rho) + \mathcal{W}_{\mathrm{slow}}(\rho)$$

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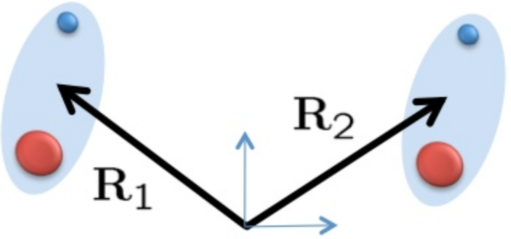

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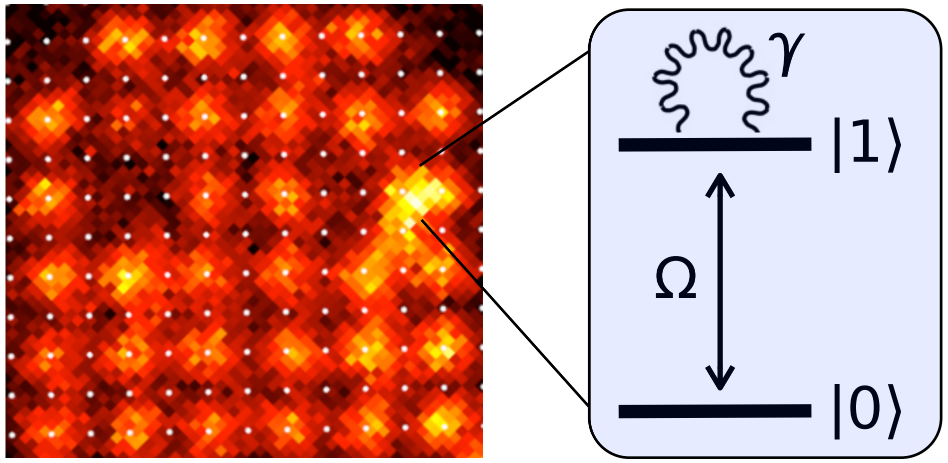

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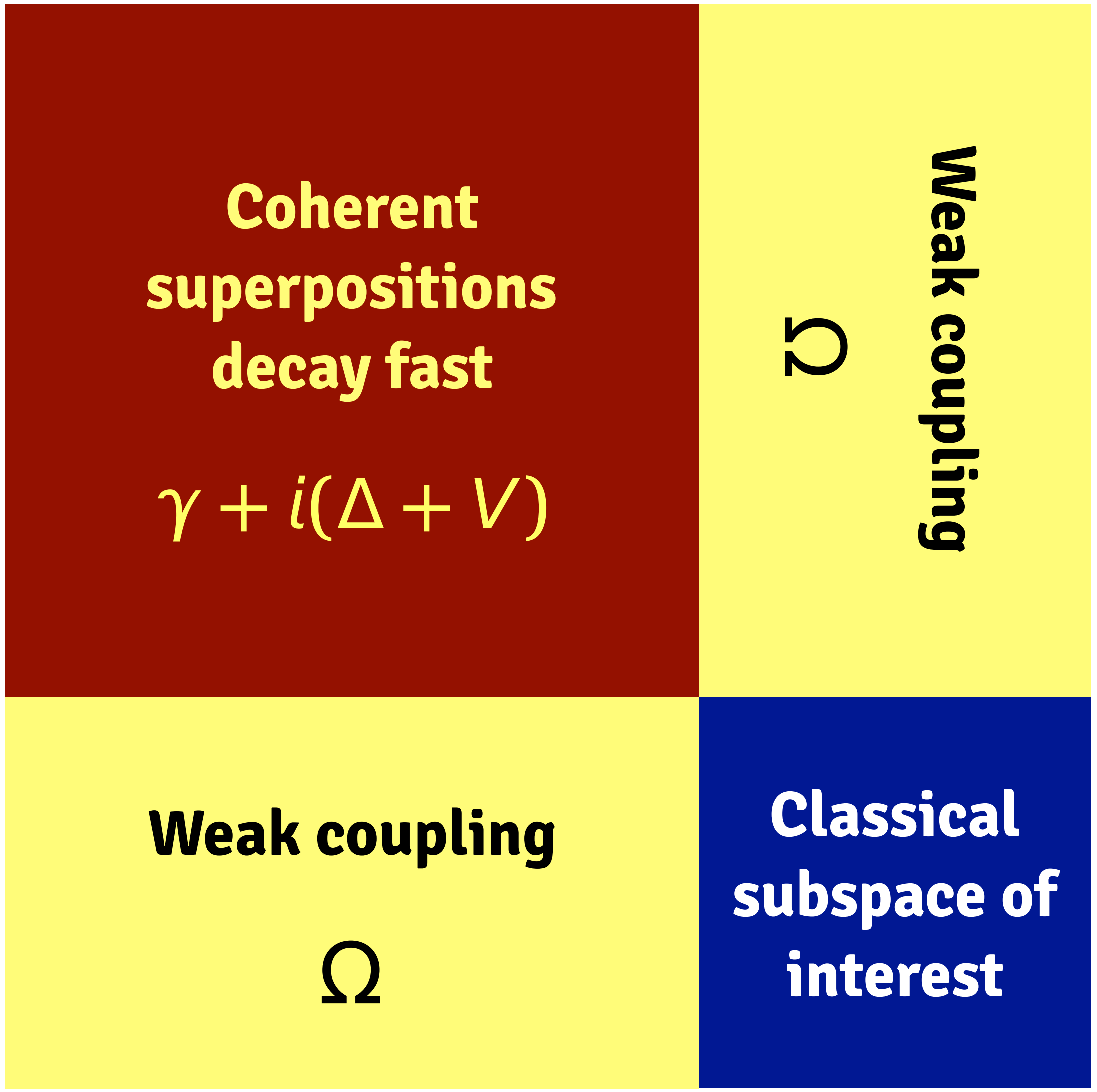


Coherent superpositions decay fast

$\gamma + i(\Delta + V)$

Weak coupling

Ω

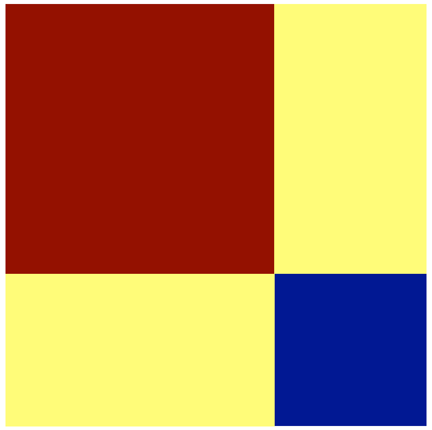
Weak coupling

Ω

Classical subspace of interest

Emergence of KCM dynamics in Rydberg systems

{Lesanovsky-JPG, **PRL 2013 + arXiv:1402.2126**}



$$\partial_t P = \sum_i \underbrace{\left(\frac{1}{1+\left[\Delta + R^{2\alpha}\sum_{i\neq j}\frac{n_m}{|\hat{\mathbf{r}}_i-\hat{\mathbf{r}}_j|^{\alpha}}\right]^2} \right)}_{\text{interactions} \rightarrow \textbf{kinetic constraint}} \left(\sigma_x^i - 1\right) P$$

trivial

$$P_{\text{stat.}} = \mathbb{1}$$

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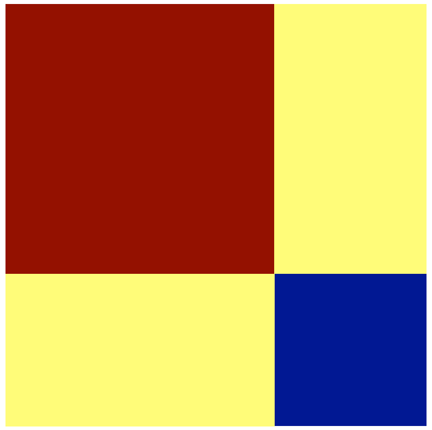

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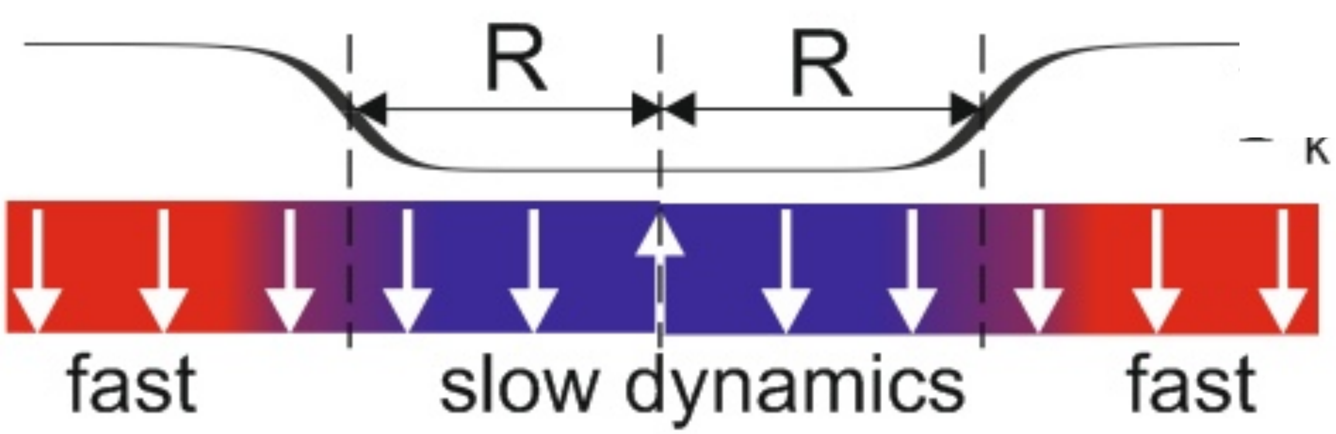
(1)

ON resonance Δ=0

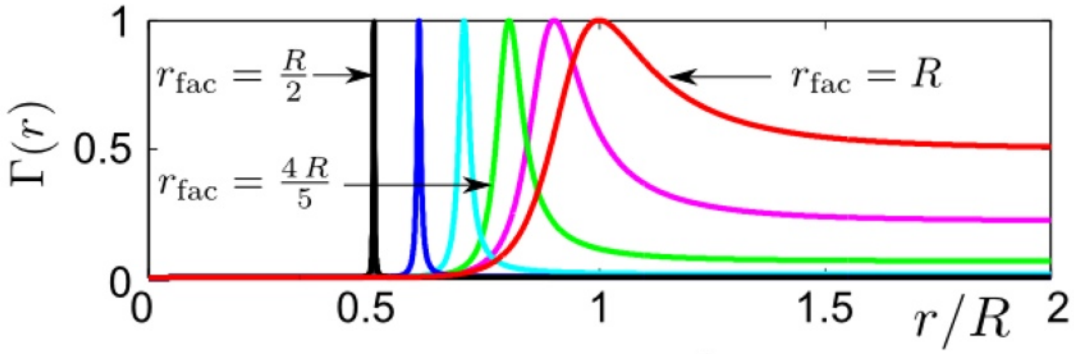
cf. Rydberg “blockade”

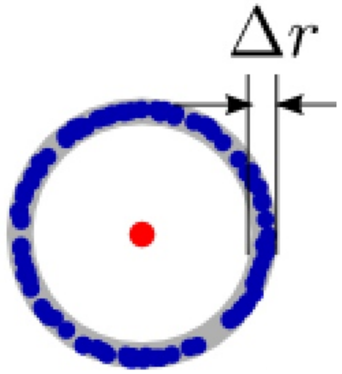


(2)

OFF resonance Δ<0

“facilitation radius”





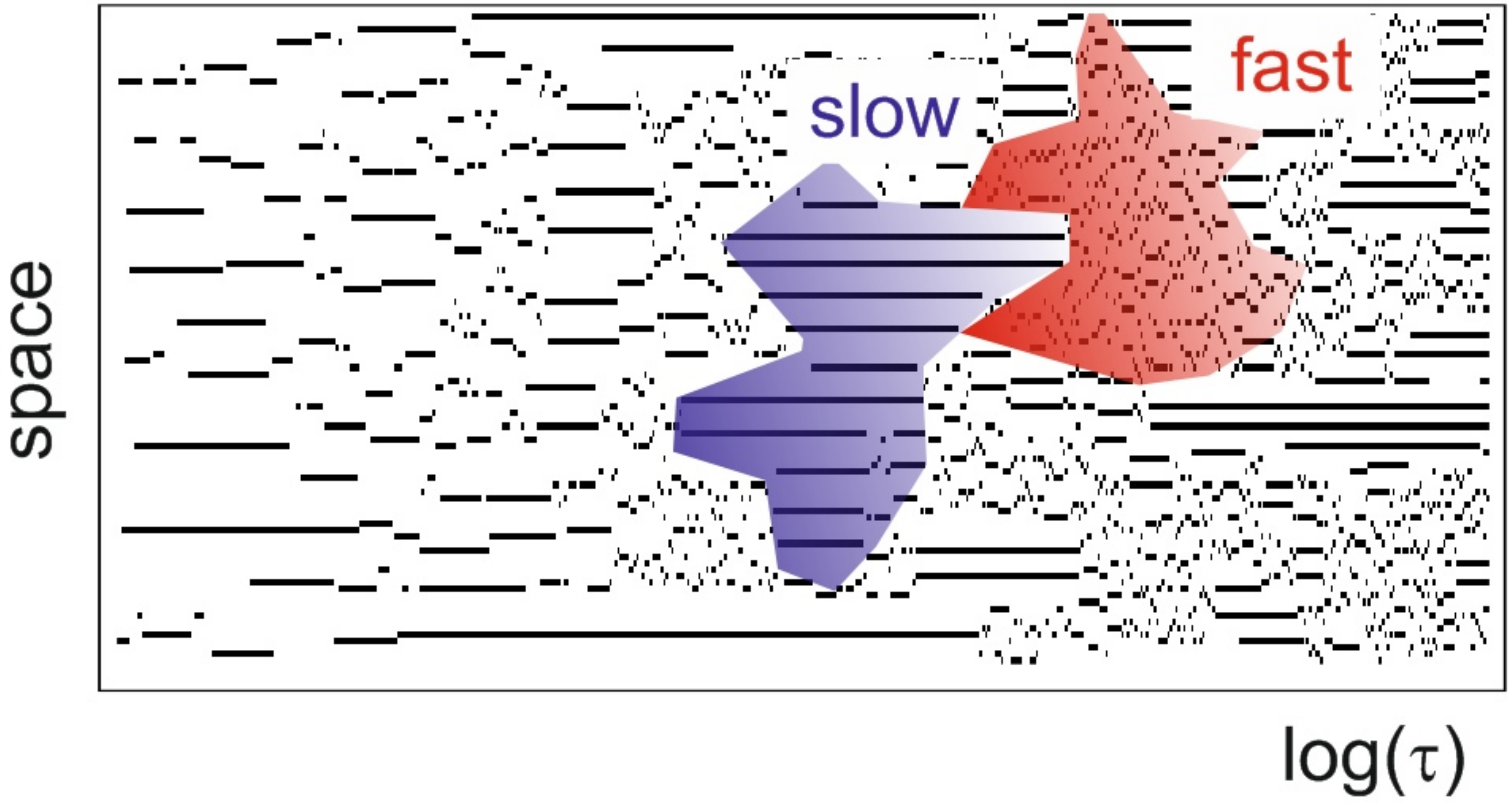
Emergence of KCM dynamics in Rydberg systems

{Lesanovsky-JPG, **PRL 2013 + arXiv:1402.2126**}

(1)
ON resonance
Δ=0

$$\partial_t P = \sum_i \left(\frac{1}{1+\left[\Delta + R^{2\alpha}\sum_{i\neq j}\frac{n_m}{|\hat{\mathbf{r}}_i-\hat{\mathbf{r}}_j|^\alpha}\right]^2} \right) \left(\sigma_x^i - 1\right) P$$

Trajectories (1D): dynamically **heterogeneous** & **glassy**



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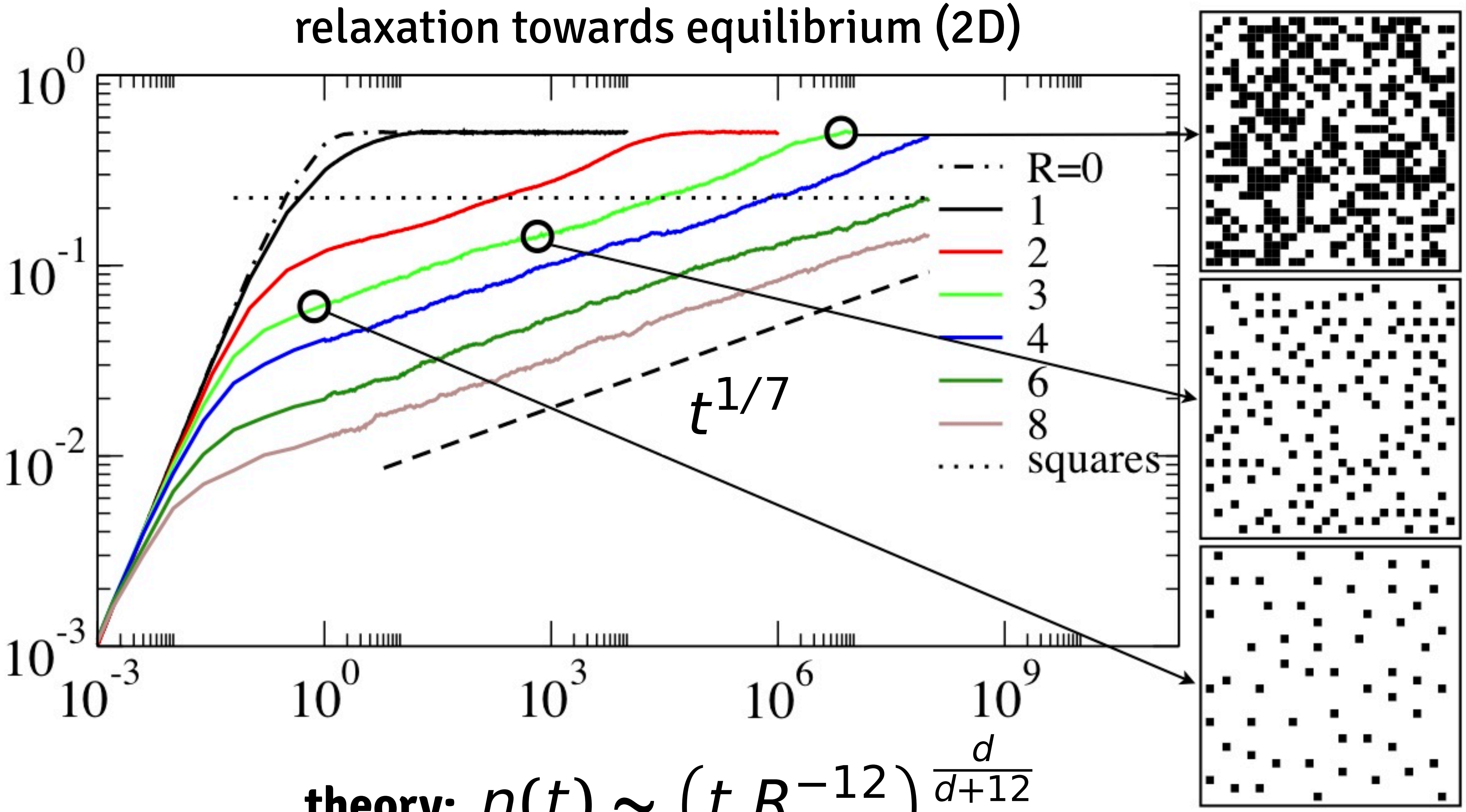
relaxation towards equilibrium (2D)



theory: $n(t) \sim \left(t\, R^{-12}\right)^{\frac{d}{d+12}}$

NB:

trivial at

$t \to \infty$

but via

hyperuniform

states

$\langle N^2\rangle - \langle N\rangle^2 \approx 0$

{cf . Torquato+}

Emergence of KCM dynamics in Rydberg systems

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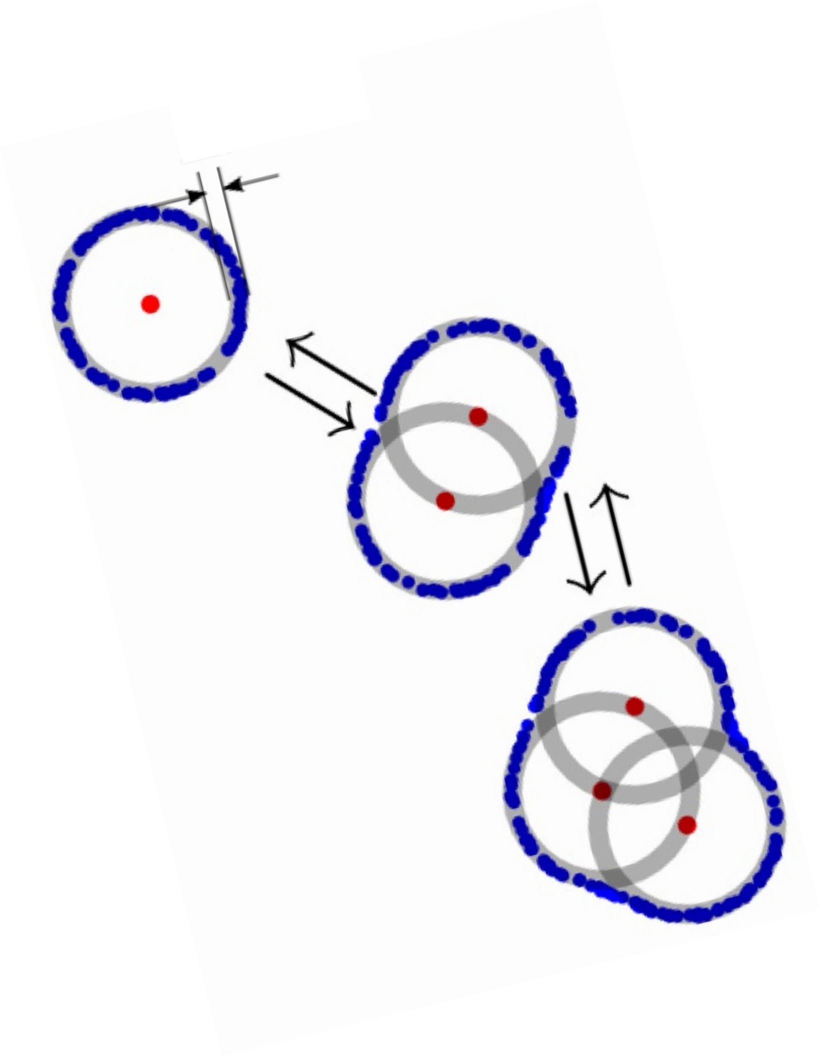

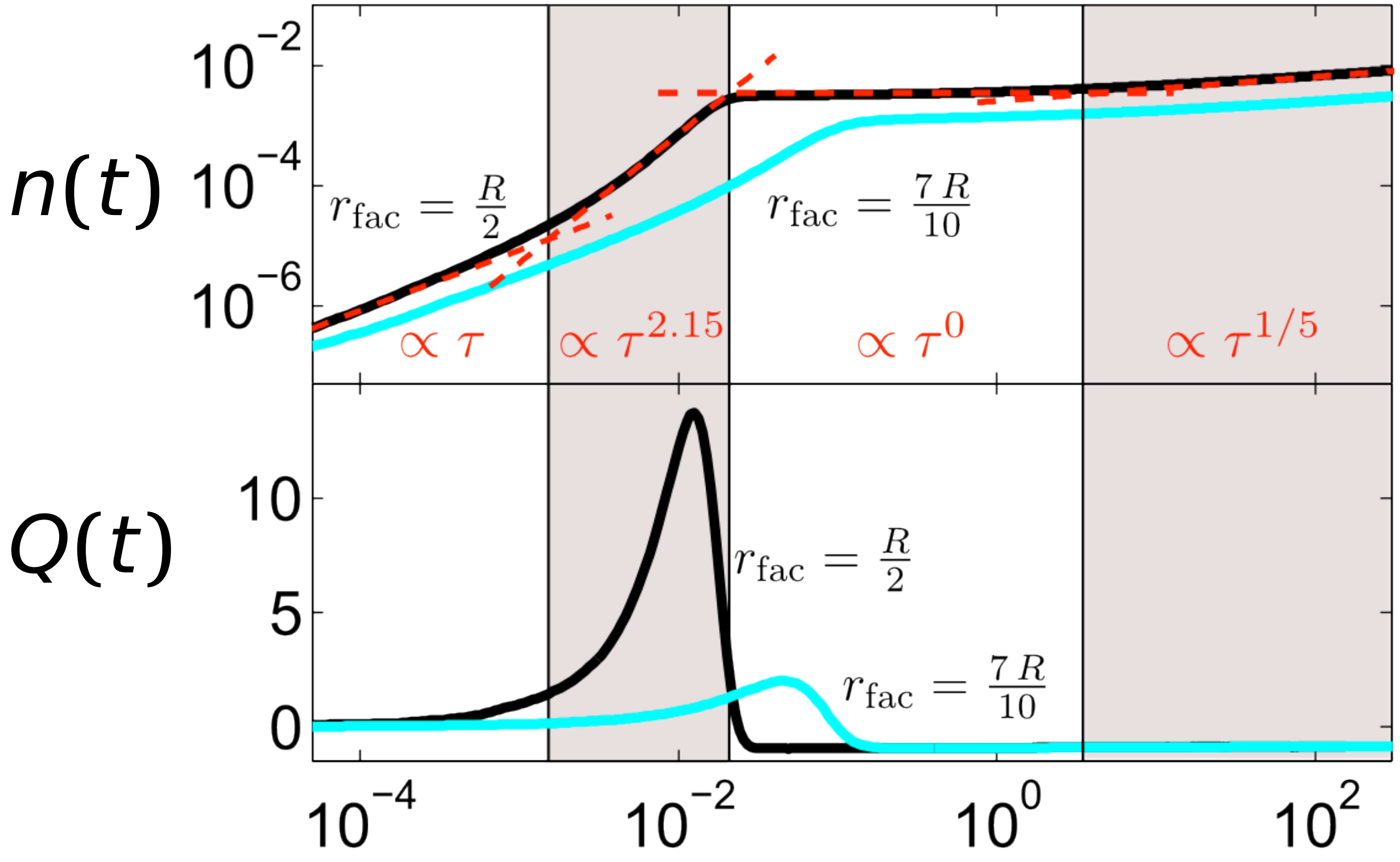

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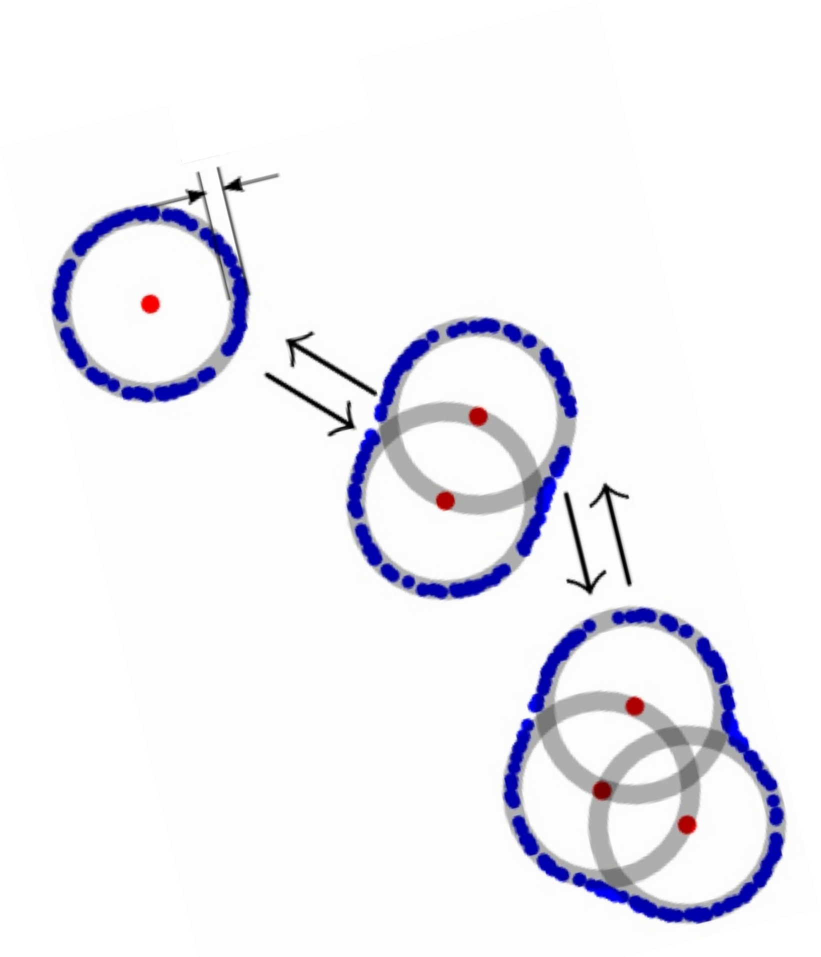
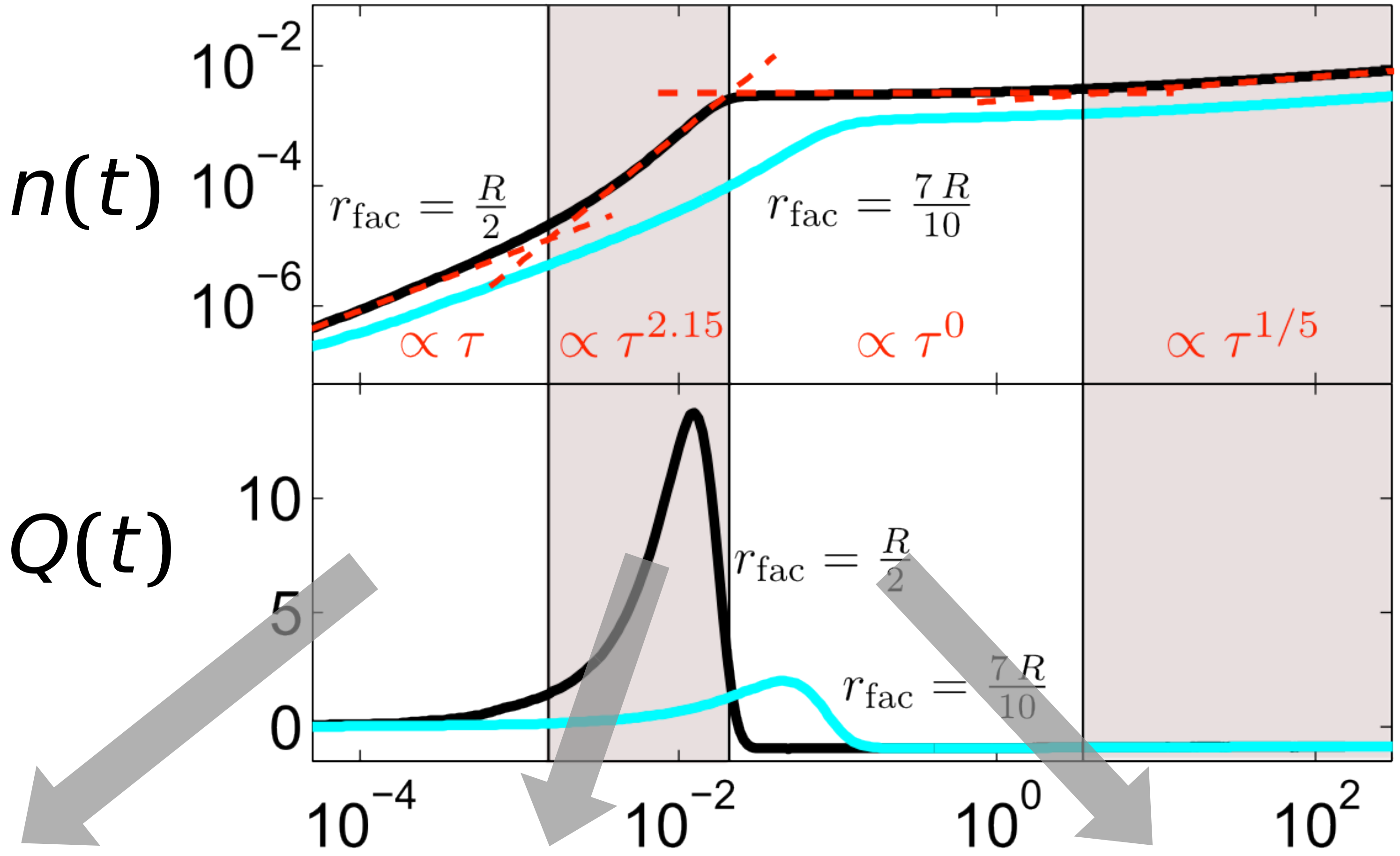
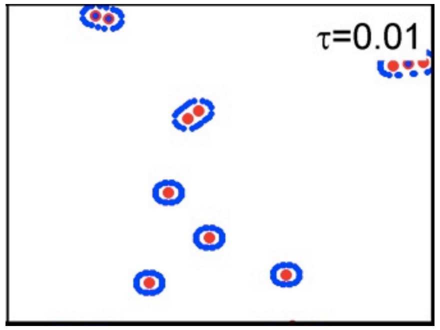
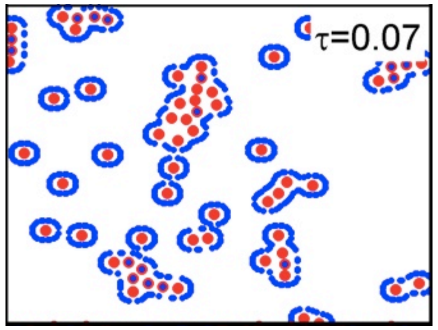




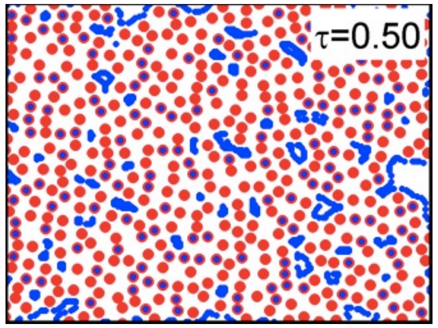
uncorrelated excitation



correlated growth



saturation **(h/u)**



KCMs and many-body localisation in closed quantum systems

{Hickey-Genway-JPG, **arXiv:1405.5780**}

KCMs and many-body localisation in closed quantum systems

{Hickey-Genway-JPG, **arXiv:1405.5780**}

Many-body localisation (**MBL**) transition:

{Altshuler+, Huse+, many others}

- Like Anderson localisation but for **interacting** system
- Singular change **throughout spectrum**
- Eigenstates change from "thermal" (**ETH** {Deutsch, Sdrenicki}) to **MBL**
- Observables **do not relax** in MBL phase
- Often thought of as **"glass transition"** but modelled with **disorder**

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Can KCMs (good models for classical glasses) say anything about quantum MBL?

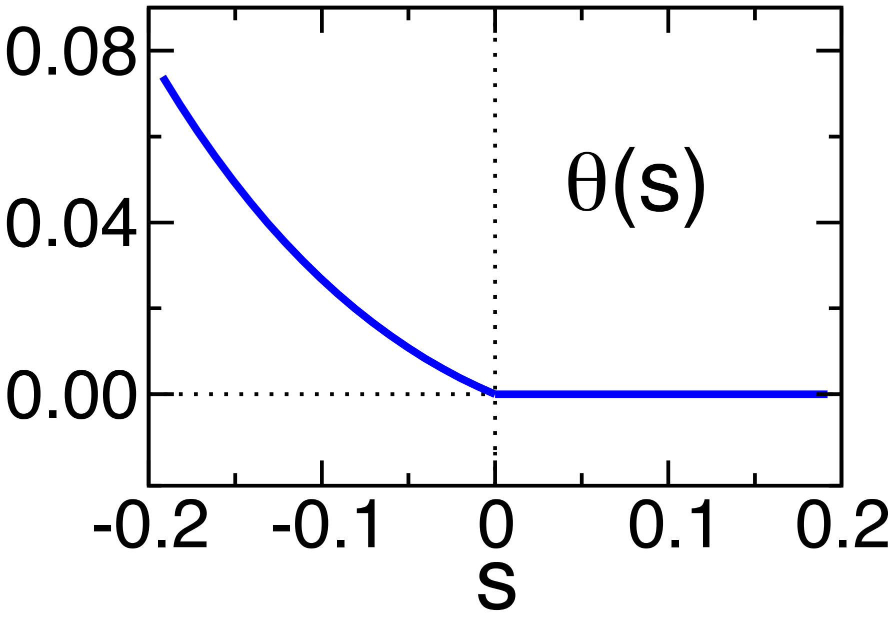
KCMs and many-body localisation in closed quantum systems

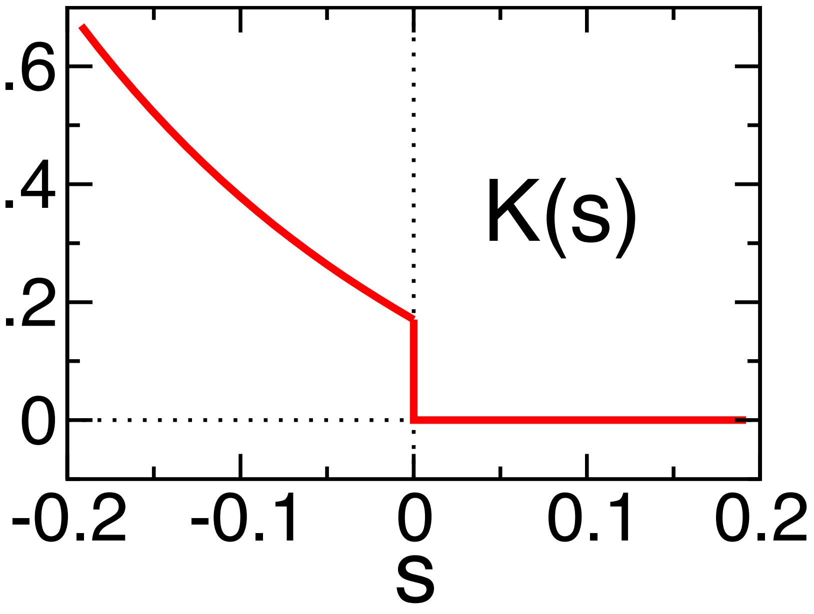
{Hickey-Genway-JPG, **arXiv:1405.5780**}

Recap: active-inactive "space-time" transitions in KCMs (eg. East/FA)

$$\mathbb{W} \;\rightarrow\; \mathbb{W}_s = \sum_i n_{i-1}\left[e^{-s}\left(\epsilon\sigma_i^+ + \sigma_i^-\right) - \epsilon(1-n_i) - n_i\right] + (i \leftrightarrow i-1)$$

Largest e/value = cumulant G.F. for **activity** → **1st order phase transition**





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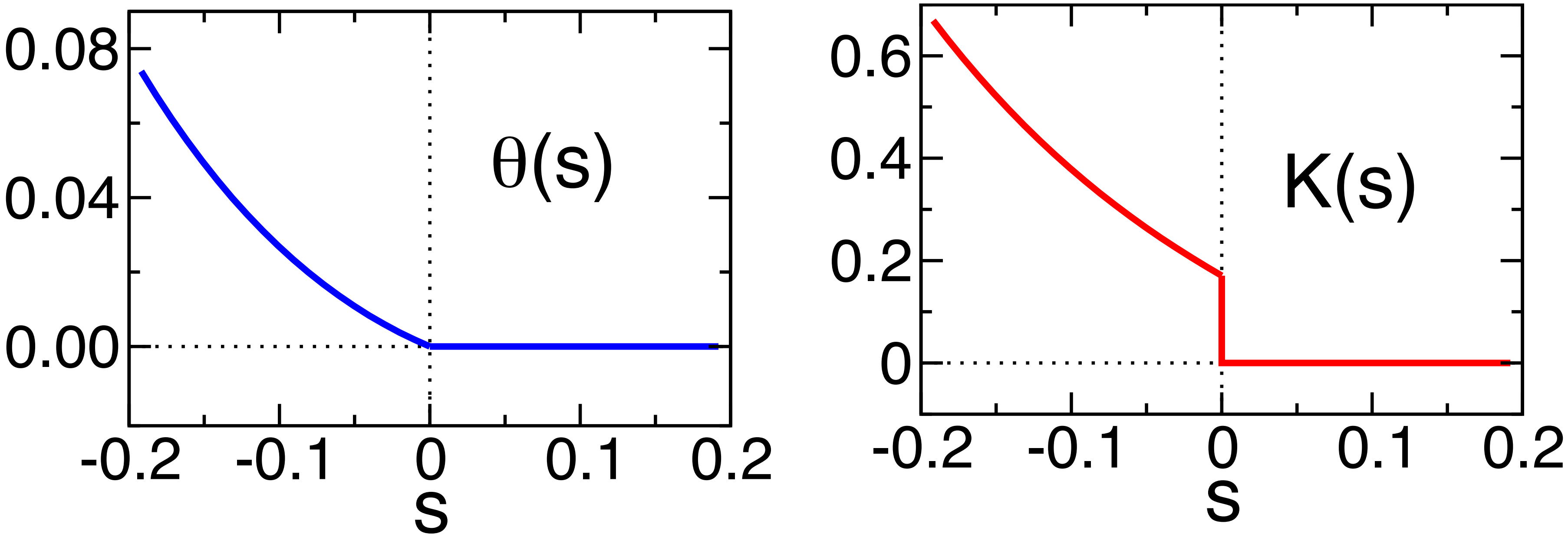

Can transform into **Hermitian** operator through equilibrium distribution

$$\mathbb{H}_s \equiv -\mathbb{P}^{-1} \mathbb{W}_s \mathbb{P} = -\sum_i n_{i-1} \left[e^{-s} \sqrt{\epsilon} \sigma_i^x - \epsilon (1 - n_i) - n_i \right] + (i \leftrightarrow i-1)$$

Consider as **Hamiltonian** and corresponding **quantum unitary dynamics** $|\psi_t\rangle = e^{-it\mathbb{H}_s} |\psi_0\rangle$

KCMs and many-body localisation in closed quantum systems

{Hickey-Genway-JPG, **arXiv:1405.5780**}

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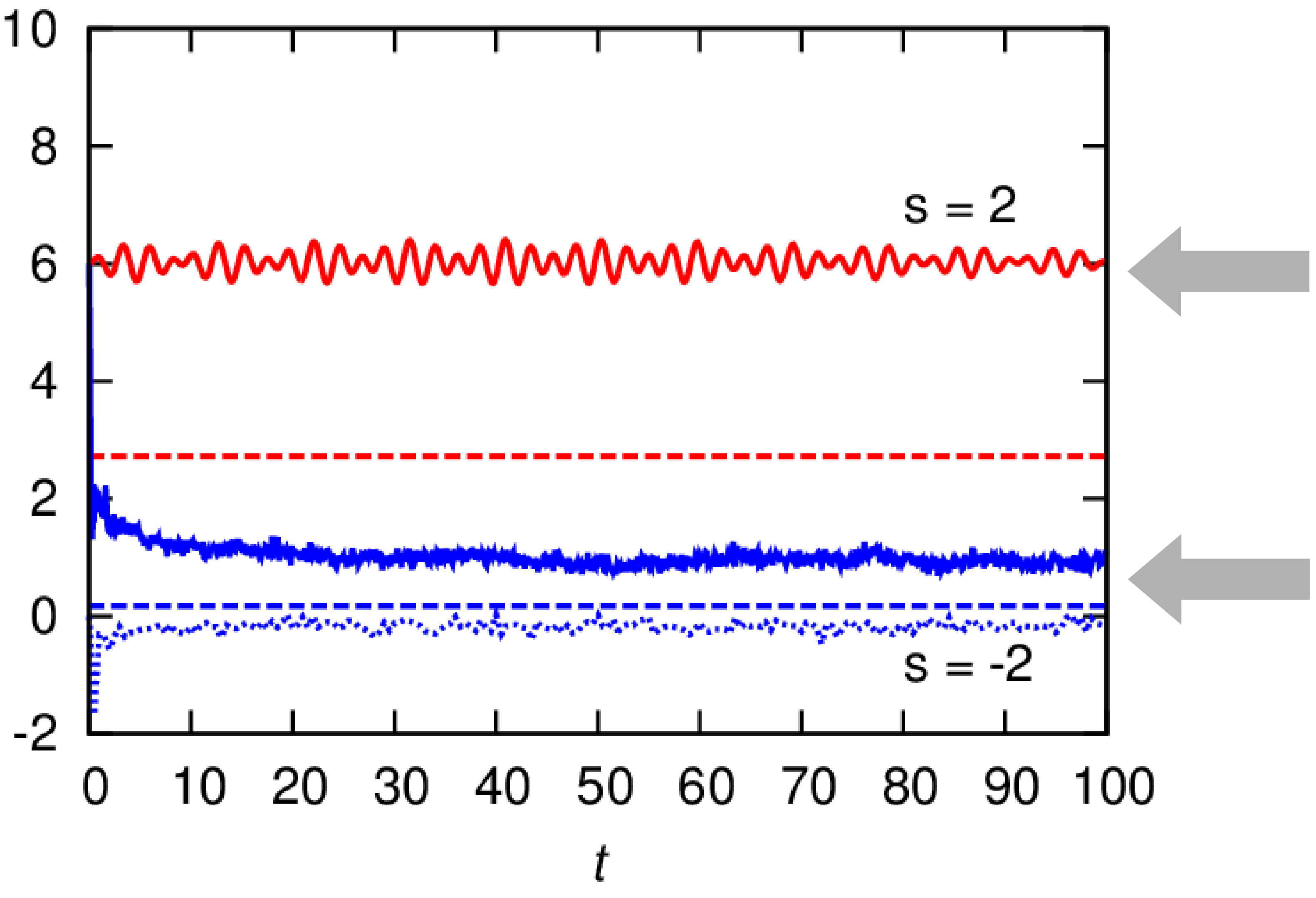
Signatures of **MBL** transition: **(i) relaxation / non-relaxation of observables**

time evolution of magnetisation

$$\langle M \rangle_t = \sum_i \langle \psi_t | \sigma_i^z | \psi_t \rangle$$



does not relax on **inactive** side s>0

relaxes on **active** side s<0

KCMs and many-body localisation in closed quantum systems

{Hickey-Genway-JPG, **arXiv:1405.5780**}

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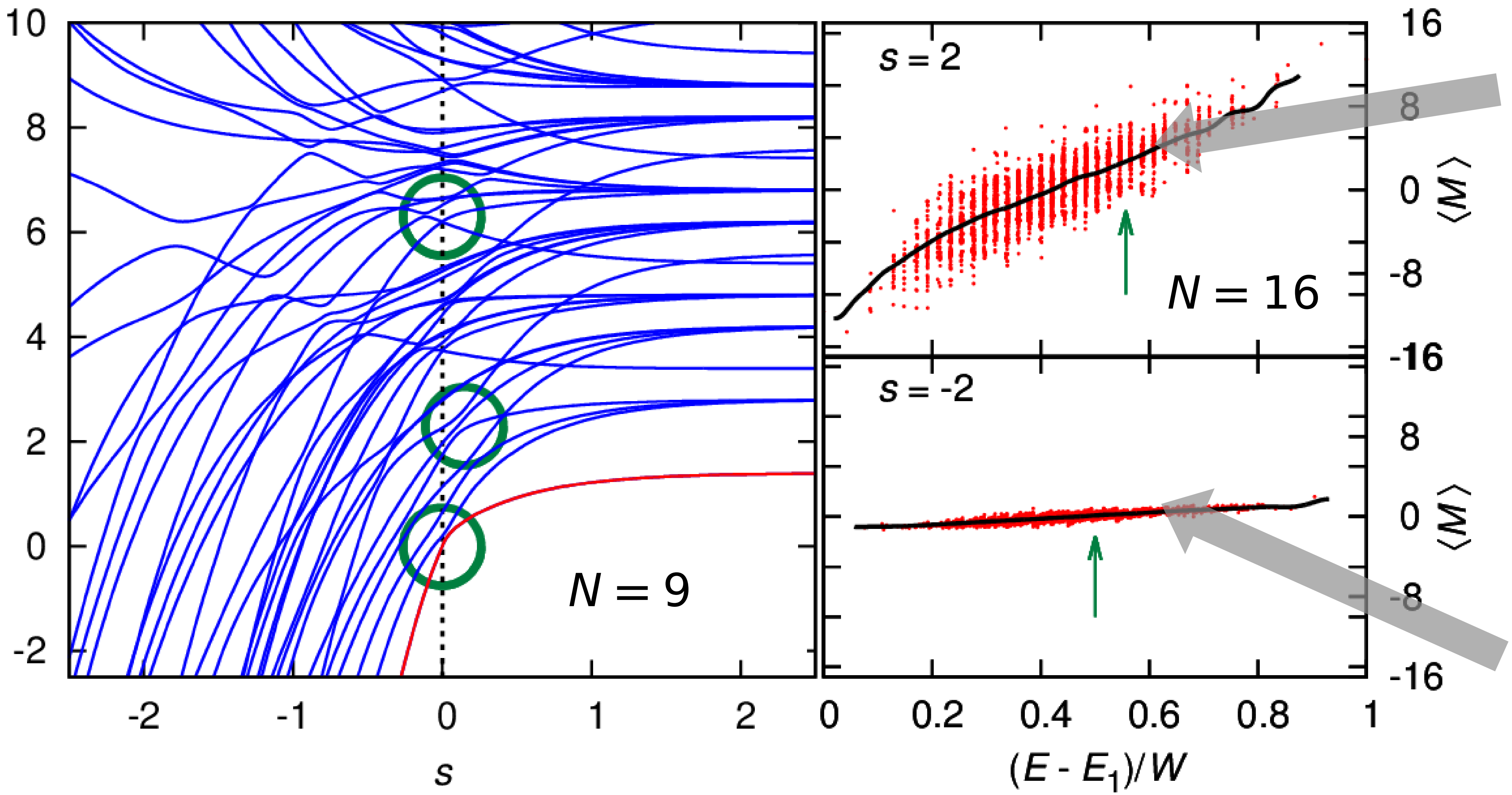
Signatures of **MBL** transition: **(ii) transitions throughout spectrum**



no ETH on **inactive** side s>0

ETH on **active** side s<0

KCMs and many-body localisation in closed quantum systems

{Hickey-Genway-JPG, **arXiv:1405.5780**}

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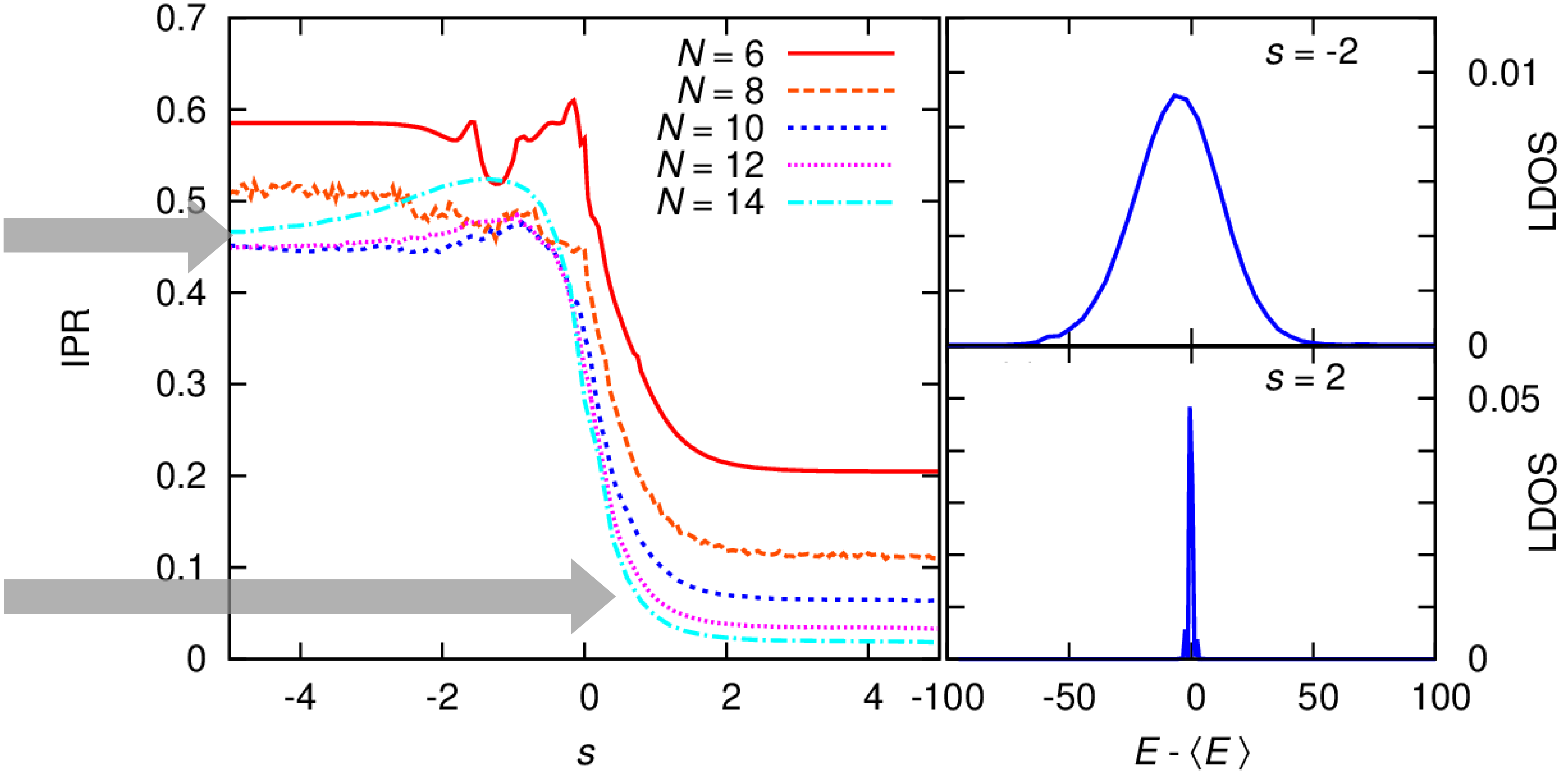
Signatures of MBL transition: **(iii) localisation onto classical basis**

widely
spread
active
s<0

e/states
concentrated
inactive
s>0



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widely spread
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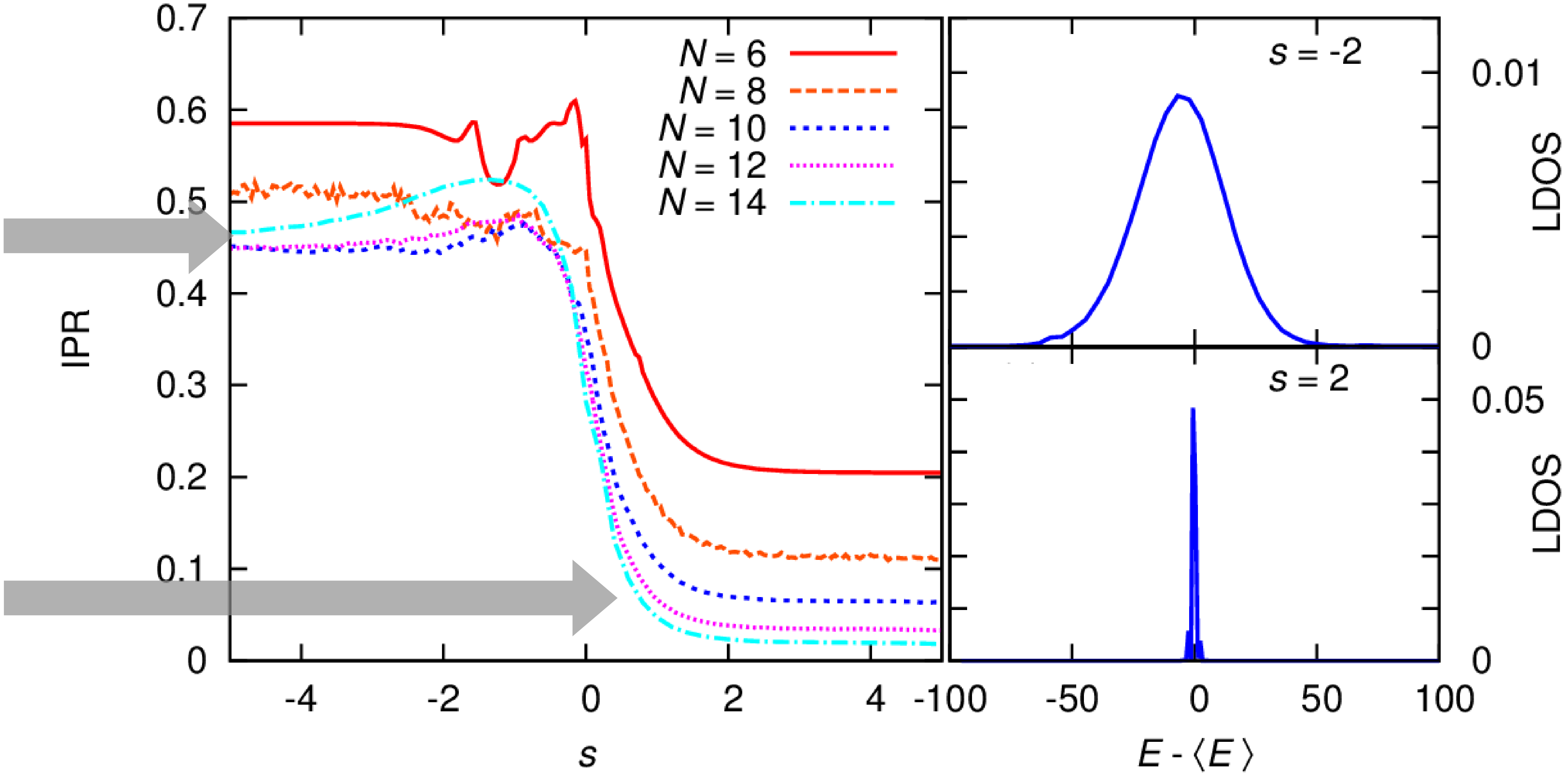

⇒ active-inactive transition if 1st order MBL transition in whole spectrum

first model with **MBL** transition **without disorder**

SUMMARY

KCMs as open quantum glasses

qFA & qEast

interplay between classical & quantum fluctuations

KCMs emergent in atomic Rydberg gases

correlated non-equilibrium structures (e.g. hyperuniform)

recent experimental evidence

KCMs and many-body localisation in quantum systems

s-ensemble active-inactive transition throughout spectrum

MBL models without disorder