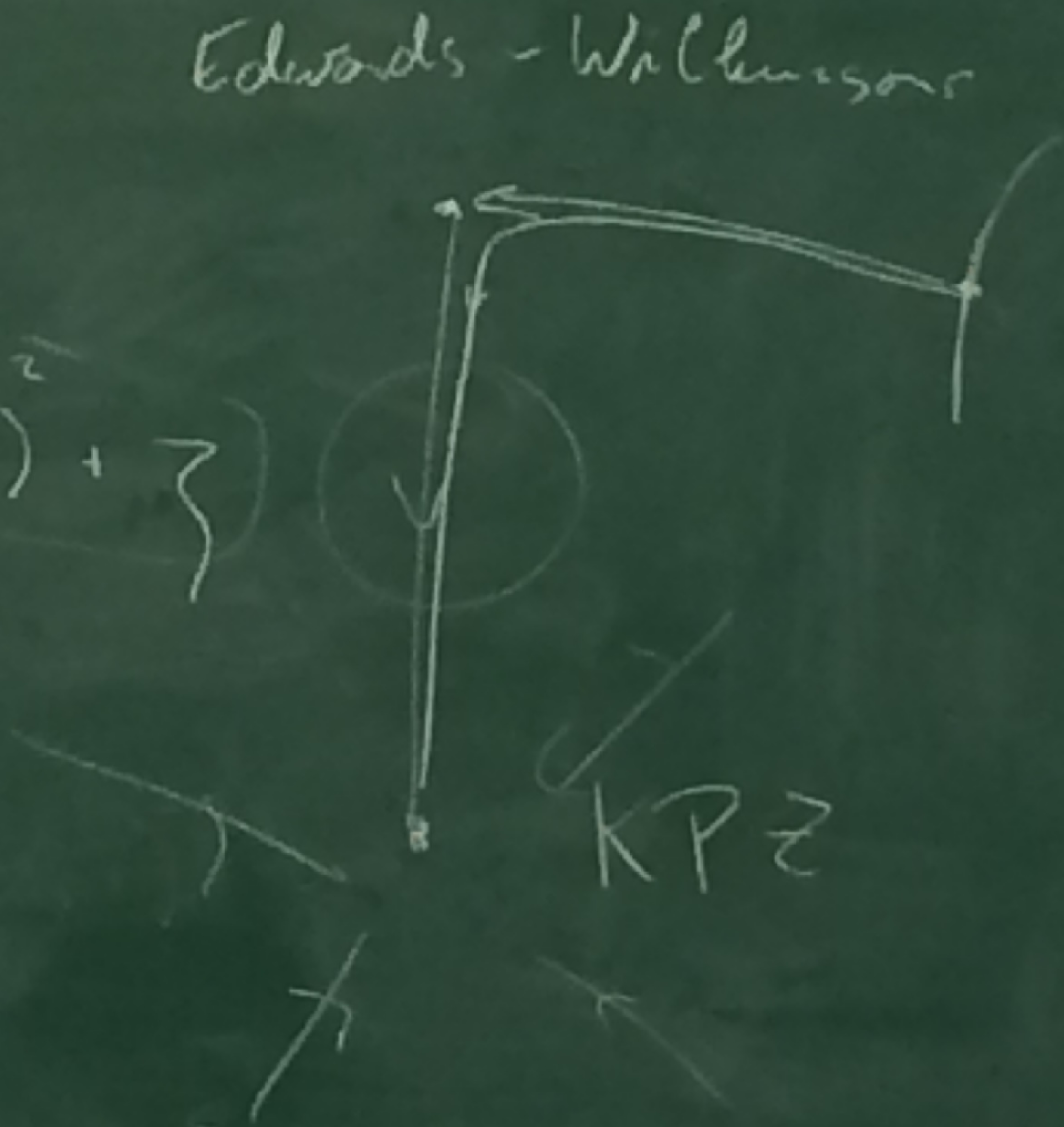
$$\boxed{\partial_t h = \partial_x^2 h + \lambda(\partial_x h)^2 + \xi}$$



$$\partial_t h = \partial_x^2 h + \xi$$

$$h \sim \varepsilon h(\varepsilon^2 x, \varepsilon^4 t)$$

$$\varepsilon h(\varepsilon^2 x, \varepsilon^3 t)$$

With J Quastel

stationary Gaussian process

$$\partial h = \partial^2 h + \sqrt{\varepsilon}P(\partial h) + \eta$$


p
q
Hopf-Cole
Z = exp(h)
∂_t Z = ∂_x^2 Z + Z ξ
LAZY
DAYS

With J. Quastel

$$\partial_t h_\varepsilon = \partial_x^2 h_\varepsilon + \sqrt{\varepsilon}\, P(\partial_x h_\varepsilon) + \eta(x,t)$$

stationary Gaussian process

P even polynomial

Thm $\varepsilon^{1/2} h_\varepsilon(\varepsilon x, \varepsilon^2 t) - \overset{\sim 1/\varepsilon}{C_\varepsilon}\, t \Longrightarrow h(x,t)$

solves $(KPZ)_\lambda$

for some λ

$$P(\partial_x h) \sim a_1(\partial_x h)^2 + a_2(\partial_x h)^4$$

$$\partial_t h = \partial_x^2 h + a_1(\partial_x h)^2 + a_2\,\varepsilon(\partial_x h)^4 + \xi_\varepsilon$$

$\mathcal{N}(0,1)$

$$\lambda = a_1 + 6a_2 = \frac{1}{2}\int P''(x)\,\mu(dx)$$

$$C_\varepsilon = \frac{1}{\varepsilon}\int P(x)\,\mu(dx) + O(1)$$

$$P(\partial_x h) = (\partial_x h)^4$$

$$\partial_t h = \partial_x^2 h + \varepsilon (\partial_x h)^4 + \zeta_\varepsilon$$

$$\partial_t \Phi = \partial_x^2 \Phi + \xi$$

$$h(x,t) - h(y,s) \approx \underline{\left(\Phi(x,t) - \Phi(y,s)\right) 1}$$

$$\partial_t h = \partial_x^2 h + \underbrace{\lambda(\partial_x h)^2} + \xi - C_\varepsilon$$

$$\begin{tikzcd}
\partial_t h = \partial_x^2 h + \lambda(\partial_x h)^2 + \xi - C_\varepsilon \arrow[r, phantom, ">"] & ? \arrow[loop above, "R"] \arrow[r] & H \arrow[d] \\
\mathrm{RHS} \arrow[loop below, "R"] \arrow[r, phantom, ">"] & \xi_\varepsilon \arrow[u] \arrow[r] & h
\end{tikzcd}$$

$$\partial_t \Phi = \partial_x^2 \Phi + \xi$$

$$h(x,t) - h(y,s) \approx \underline{(\Phi(x,t) - \Phi(y,s))} \cdot 1$$

$$\partial_t h = \partial_x^2 h + \underbrace{\lambda(\partial_x h_\varepsilon)^2}_{-\frac{3}{2}} + \xi - C_\varepsilon \approx$$

$$\begin{tikzcd} \bigcirc \arrow[loop above, "c \circ R"] \arrow[r] & H \arrow[d] \\ \mathrm{RHS} \approx \xi_\varepsilon \arrow[loop below, "R"] \arrow[u, "\mathcal{L}_\alpha"] \arrow[r] & h \end{tikzcd}$$

$$P(\partial_x h) = (\partial_x h)^4$$

$$\partial_t h = \partial_x^2 h + \varepsilon(\partial_x h)^4 + \zeta_\varepsilon$$

$$\partial_t H = \partial_x^2 H + a(\partial_x H)^2 + \varepsilon(\partial_x H)^4 + \zeta$$

$$\partial_t h = \partial_x^2 h + a\left((\partial_x h)^2 - c\right) + \alpha\left((\partial_x h)^4 - 6c(\partial_x h)^2 + 3c^2\right) + \zeta_\varepsilon$$

$$M_{1/\varepsilon} \mathcal{L}_\varepsilon(\zeta_\varepsilon)$$

$$M_{1/\varepsilon} \mathcal{L}_0(\zeta_\varepsilon)$$

$a = 6, \quad \alpha = \varepsilon, \quad c = 1/\varepsilon$

$a = 6, \quad \alpha = 0, \quad c = 1/\varepsilon$