Exclusive queueing process:

The dynamics of waiting in line and other jamming phenomena in pedestrian dynamics

Andreas Schadschneider

Institut für Theoretische Physik

Universität zu Köln, Germany

www.thp.uni-koeln.de/~as

Collaborator: **Chikashi Arita** (Saarbrücken)

Introduction

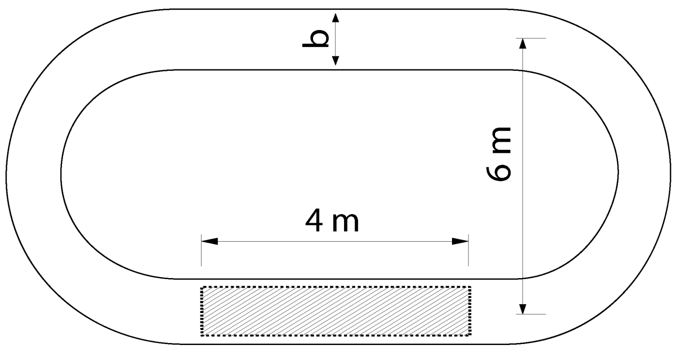
Queues are ubiquitous!

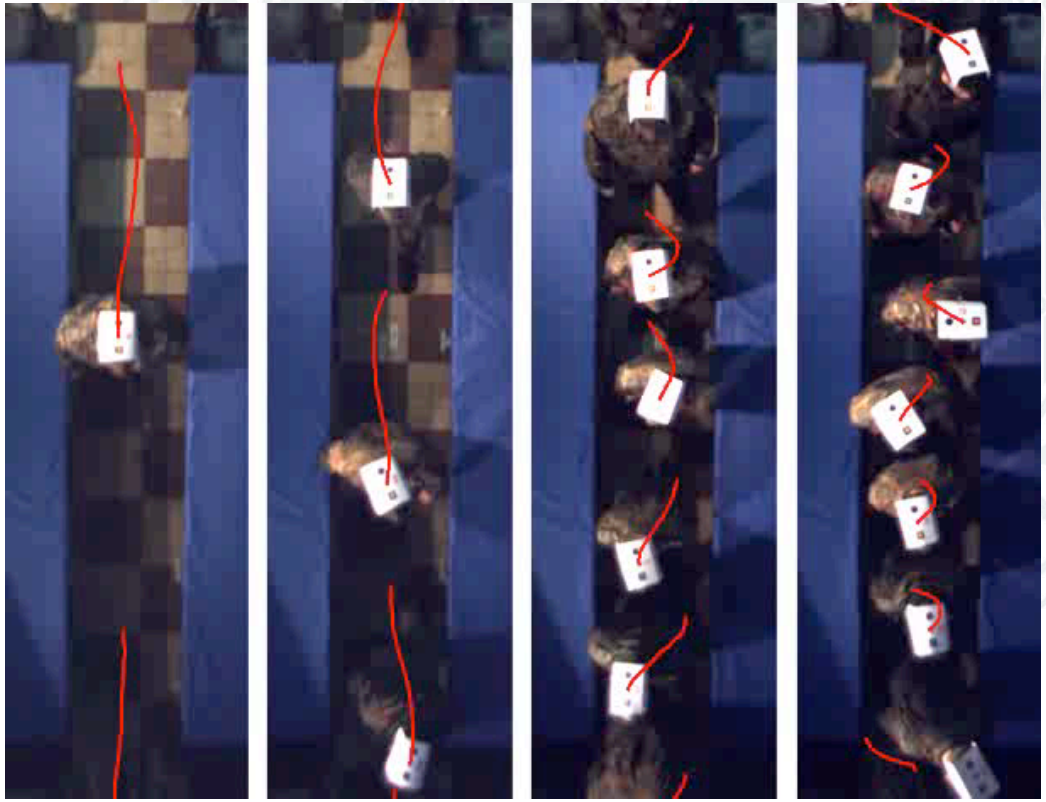
- **customer service**
- **transport**
- **telecommunications**
- **…**

Motivation: pedestrian dynamics, evacuations,…

Pedestrian dynamics

corridor, periodic





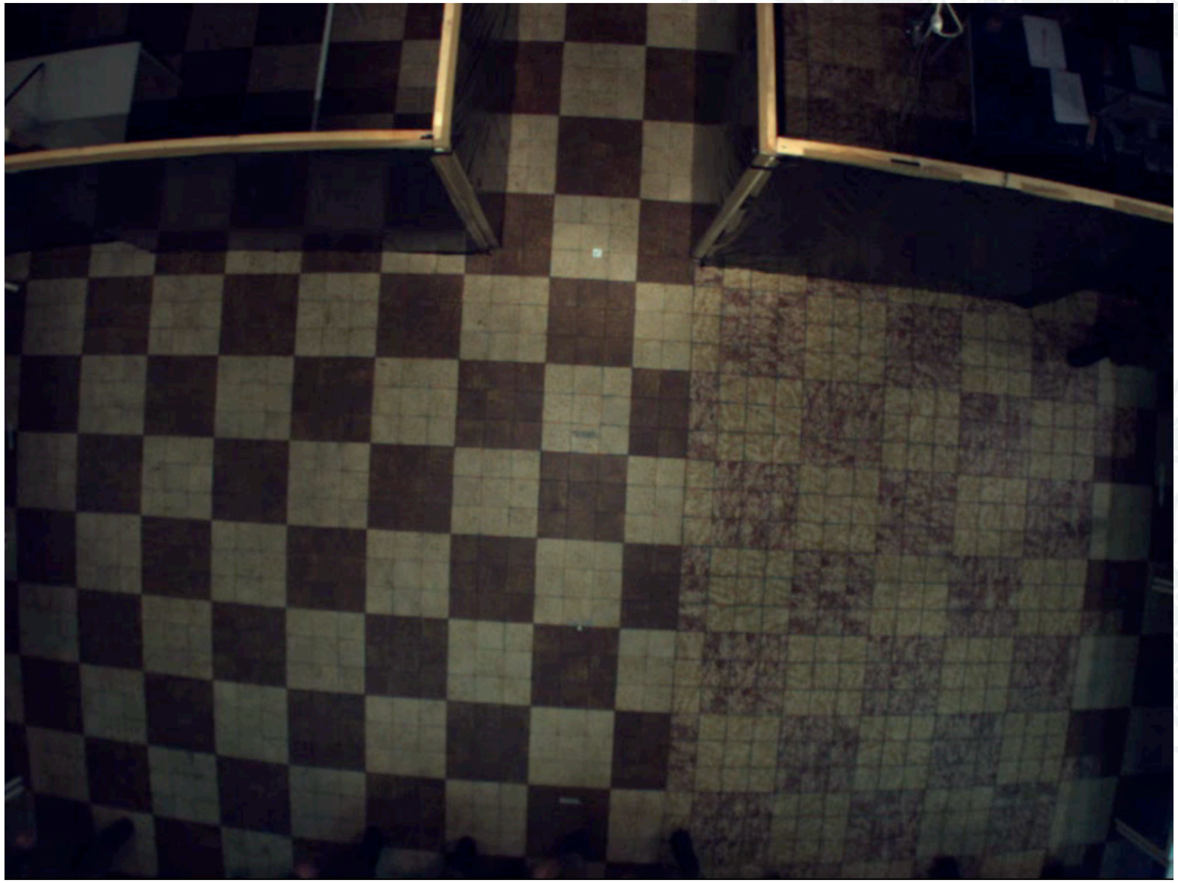
N=14 N=25 N=39 N=56

Wuppertal University

FZ Jülich

Universität zu Köln

Pedestrians at bottlenecks



(University Wuppertal, FZ Jülich)

Classification of queues

queues have different characteristics:

- arrival (input)
- service (output)
- queue discipline
  - first-in-first-out (FIFO)
  - first-in-last-out (FILO)
- number of queues

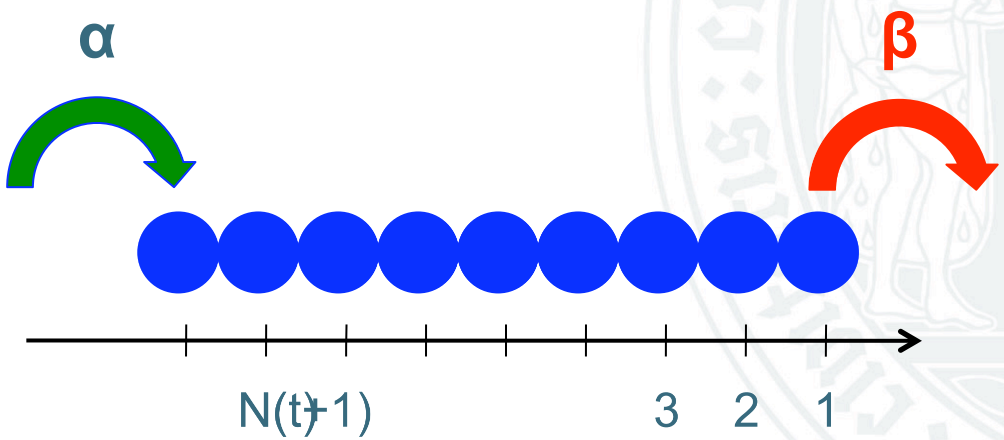
Classification of queues

A/B/C classification according to

- interarrival time distribution A
- service time distribution B
- number of servers (queues) C

e.g. A,B = G (general), D (deterministic), M (Markovian)

Classical queueing models



M/M/1 queue

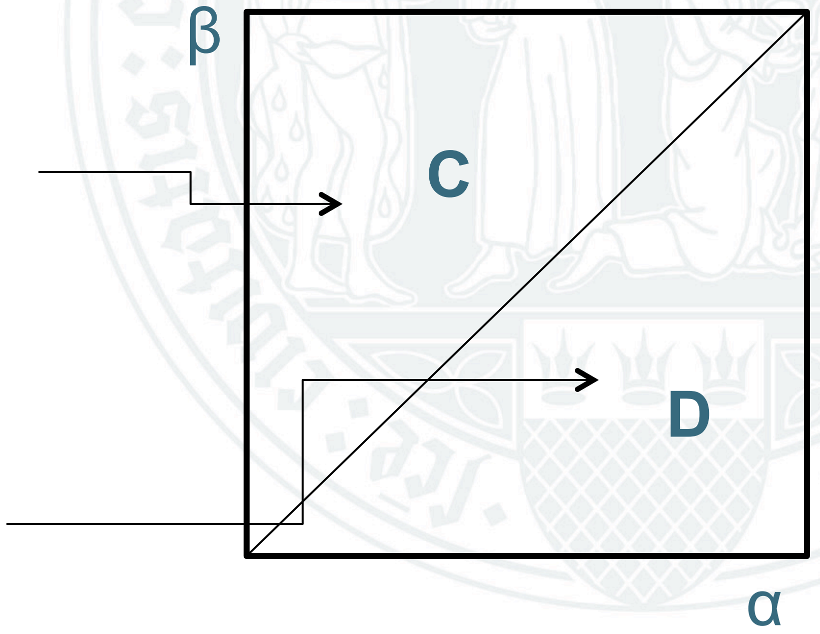
$L(t) = N(t)$

density is constant: $\rho=1$

Phase diagram: M/M/1 queue

convergent phase (α < β)

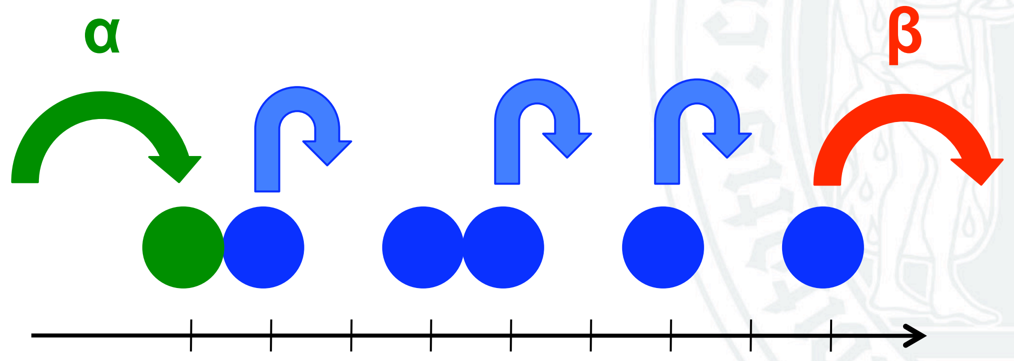
divergent phase (α > β)



Queues with spatial structure

in many real queues the density is not constant!

Exclusive Queueing Process (EQP)



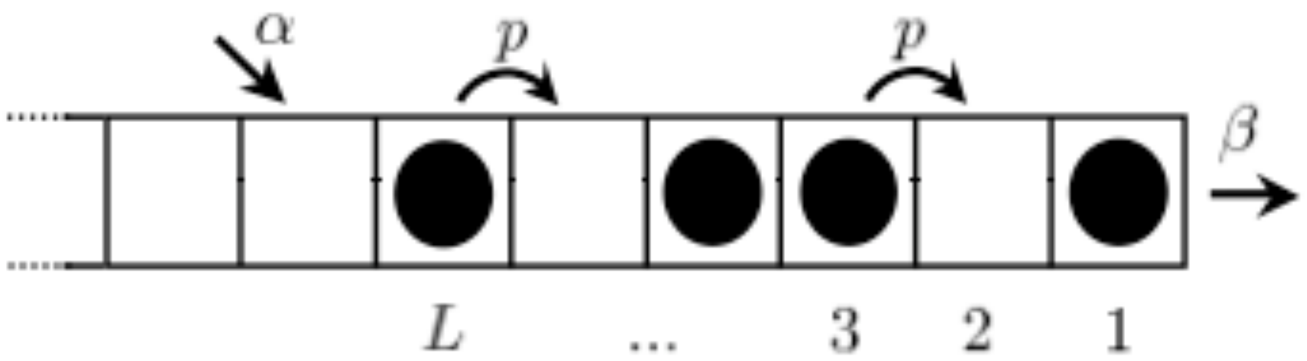
$L(t) \neq N(t)$

density not constant: nontrivial density profile $\rho(x)$

(Arita 2009, Yanagisawa et al. 2010)

Exclusive Queueing Process (EQP)

particle input at site L(t)+1

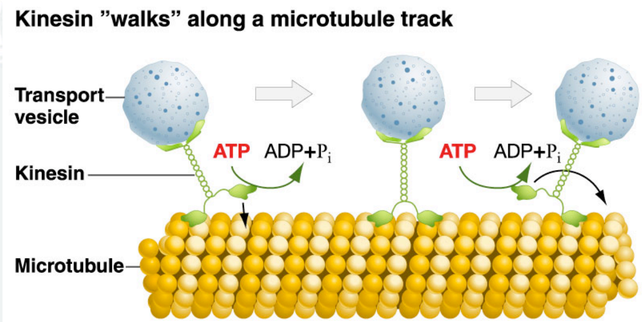


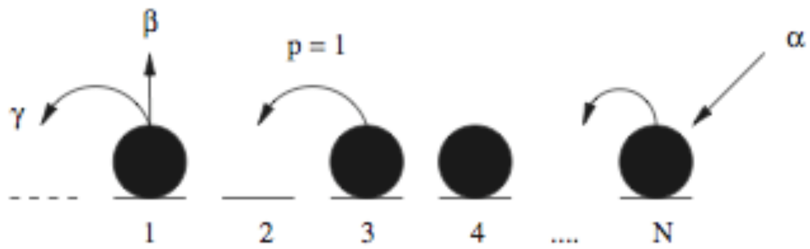
ASEP with variable length
parallel dynamics

Related models

- molecular motors on microtubules
- fungal growth model (Sugden, Evans et al.)

TASEP + growth at output end



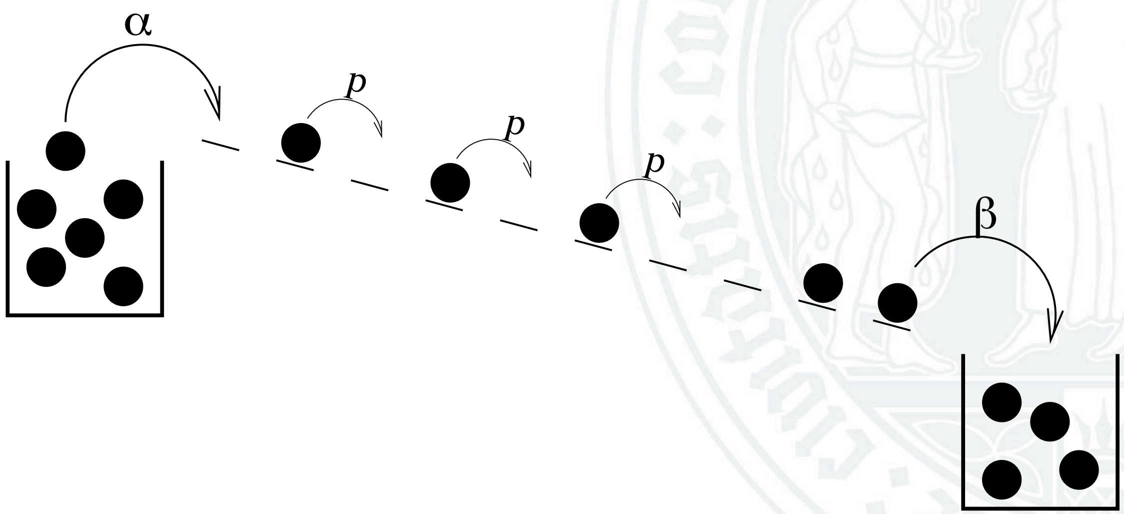


Exclusive Queueing Process (EQP)

2 types of questions:

- related to queueing theory
  - divergent vs. convergent
  - length at time t
- related to ASEP
  - structure of phases
  - density profile

Asymmetric Simple Exclusion Process (ASEP)



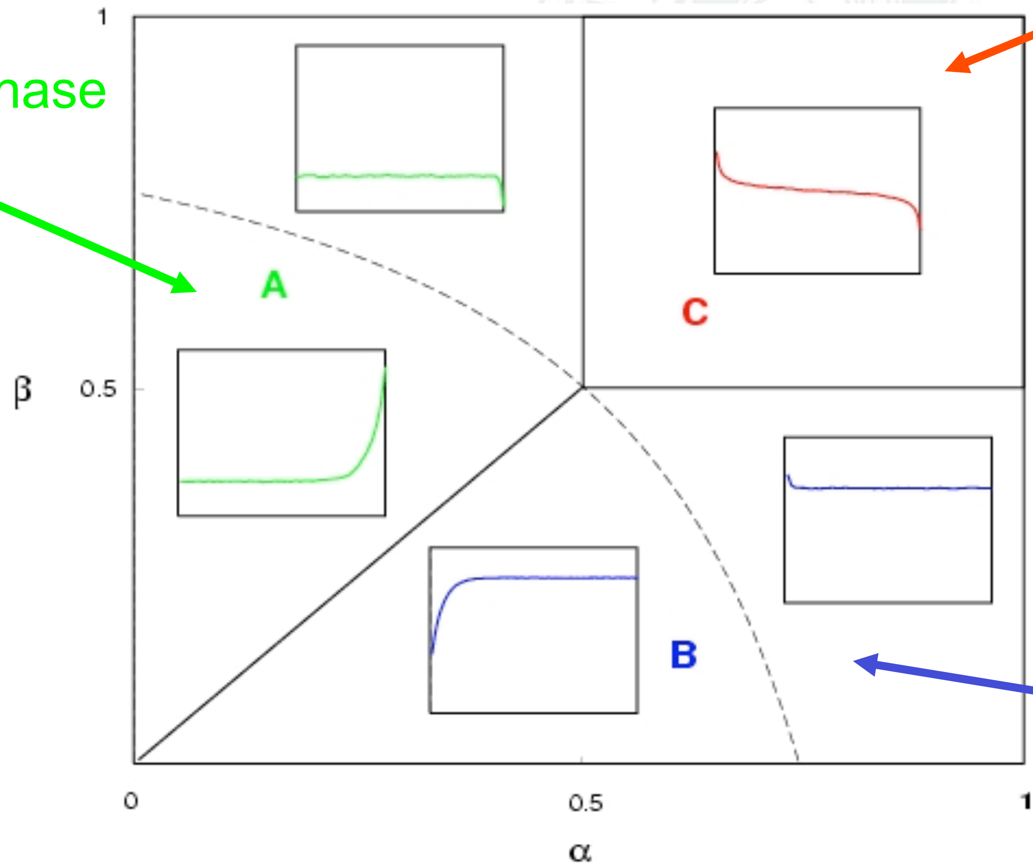
Applications: Protein synthesis, traffic flow, surface growth

boundary induced phase transitions

ASEP: Phase diagram

Low-density phase

$J=J(p,\alpha)$



Maximal current phase

$J=J(p)$

High-density phase

$J=J(p,\beta)$

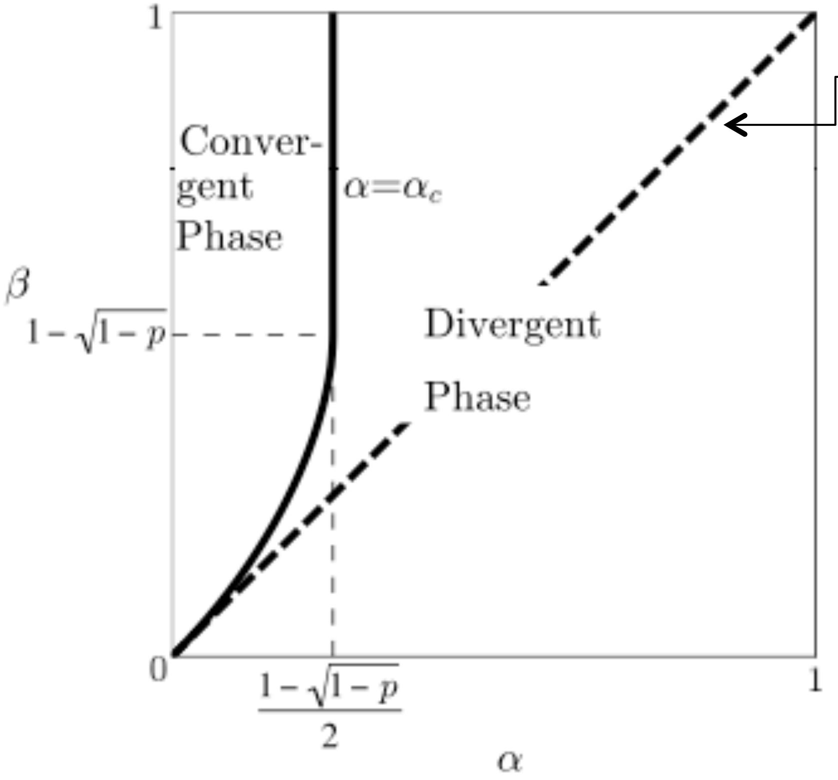
Exclusive queueing process

(Arita & Yanagisawa 2010)

stationary state:

- exact solution by matrix-product Ansatz
- quartic algebra (of parallel TASEP)
- 2 phases: diverging vs. converging
- expectation values, e.g. current J
- continuous time limit
- for p=1: 2d representation for matrices

EQP: Phase diagram I

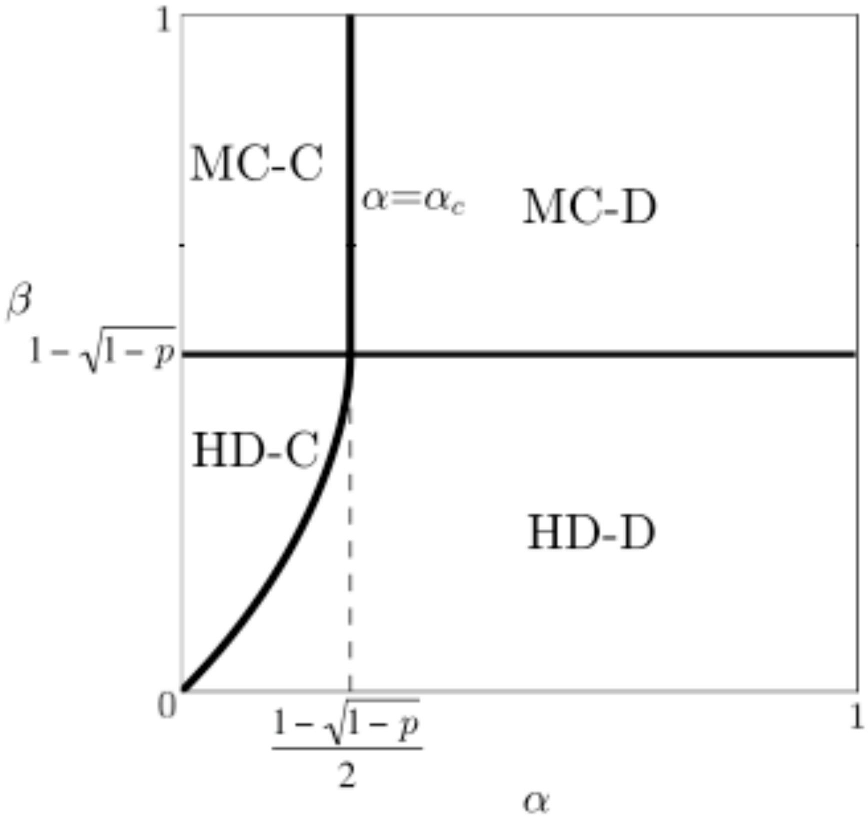


M/M/1 queue

queueing point of view:

convergent phase shrinks

EQP: Phase Diagram II

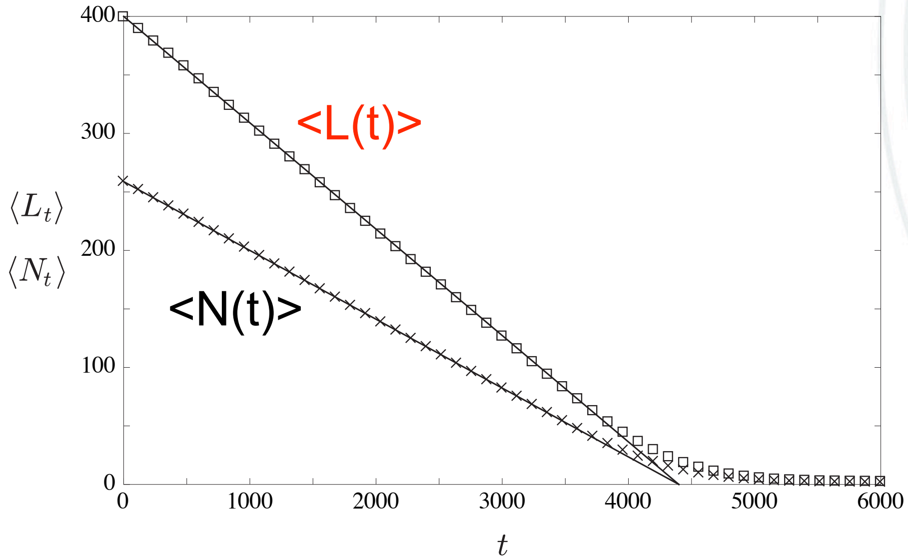


ASEP point of view:

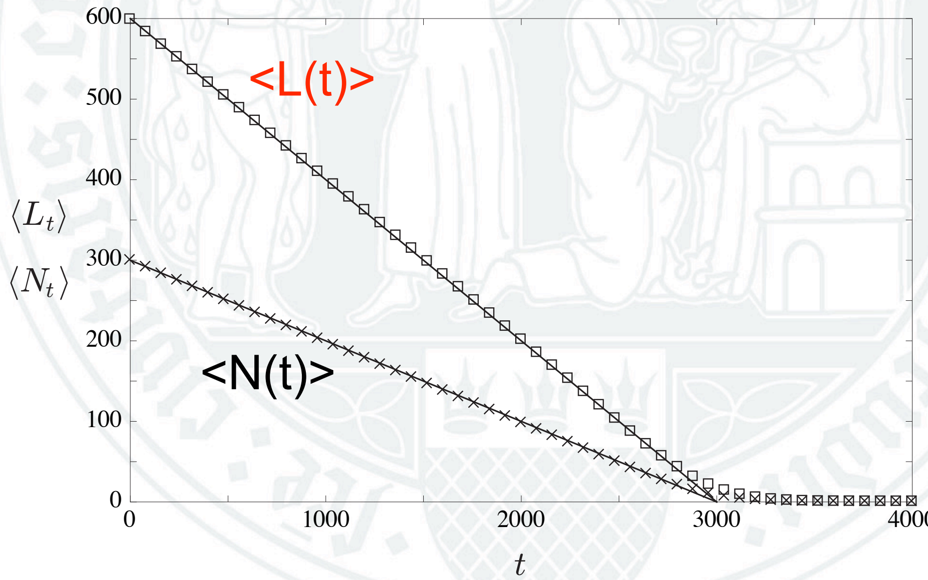
Subphases:

- **maximal current (MC)**
- **high density (HD)**

Dynamics: Convergent Phase

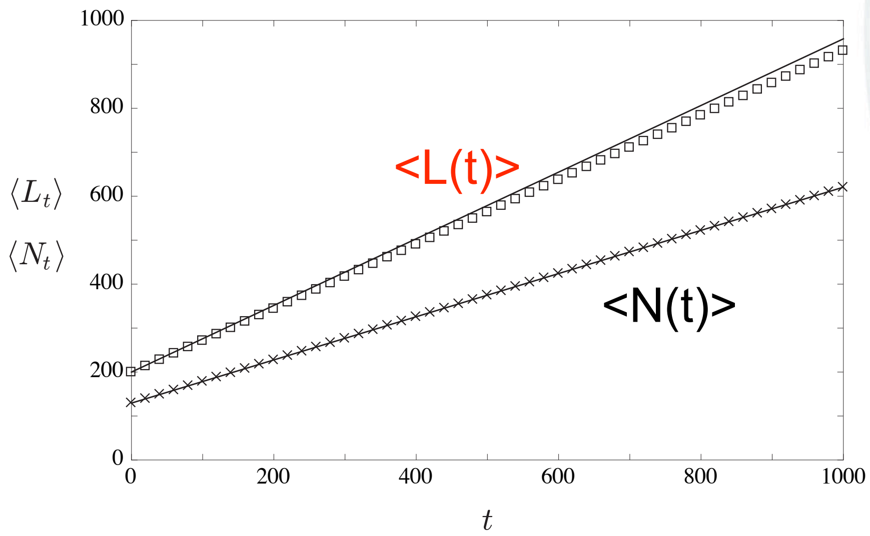


HD-C phase

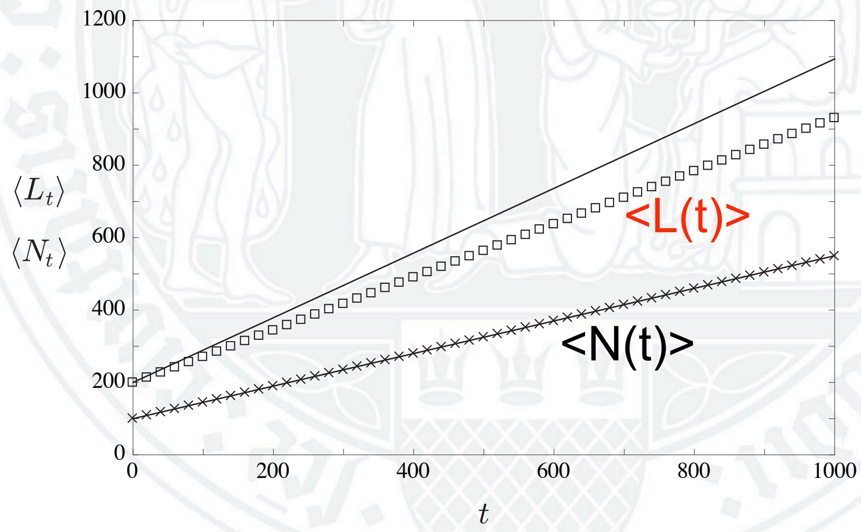


MC-C phase

Dynamics: Divergent Phase



HD-D phase



MC-D phase

Domain wall theory

very successful for ASEP-type models (Kolomeisky et al.)

continuity equation: $$\langle N(t+1)\rangle - \langle N(t)\rangle = J^{in}(t) - J^{out}(t)$$

$J^{in}(t) \to \alpha$

gives: $$\langle N(t)\rangle = (\alpha - J^{out})\cdot t + \langle N_0\rangle$$

DW theory: $$\langle L(t)\rangle = \frac{\alpha - J^{out}}{\rho}\cdot t + \langle L_0\rangle$$

Domain wall theory

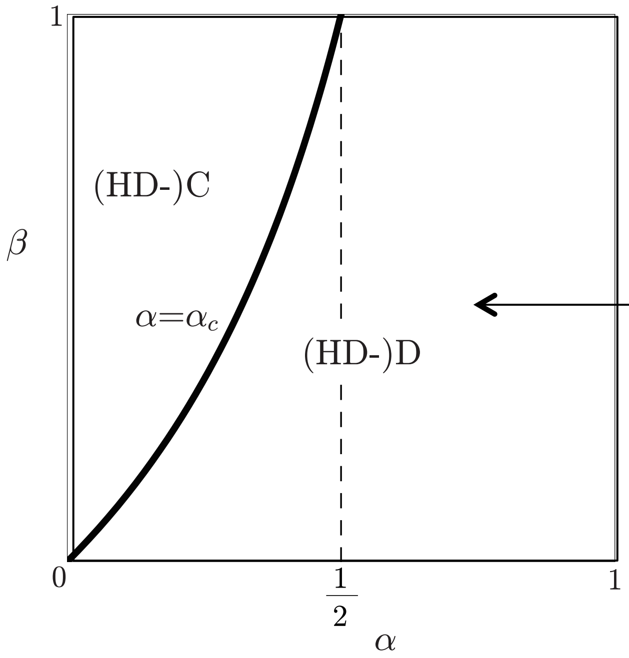
domain wall theory works quantitatively for

- convergent phase
- <N(t)> in the divergent phase

and qualitatively for

- <L(t)> in the divergent phase

Exact results for p=1



generating function technique:

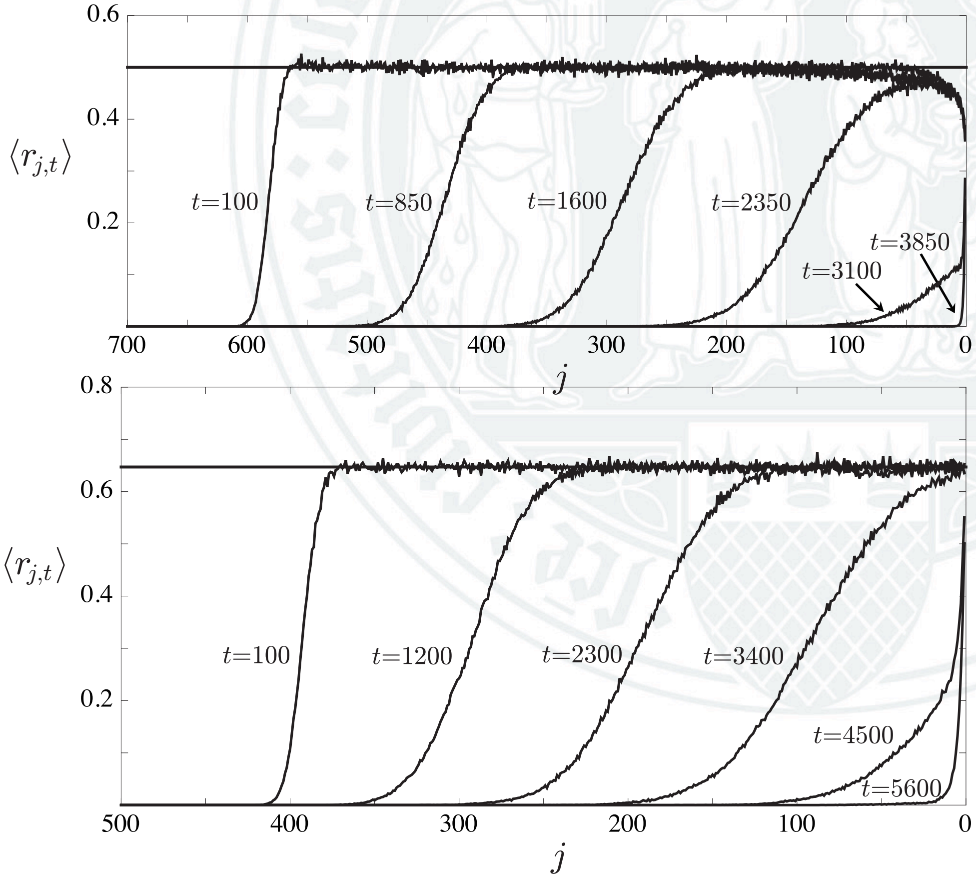
$$\langle N(t)\rangle = \left(\alpha - \frac{\beta}{1+\beta}\right)\cdot t + o(t)$$

$$\langle L(t)\rangle = (\alpha(1+\beta)-\beta)\cdot t + o(t)$$

Density profiles: Convergent phase

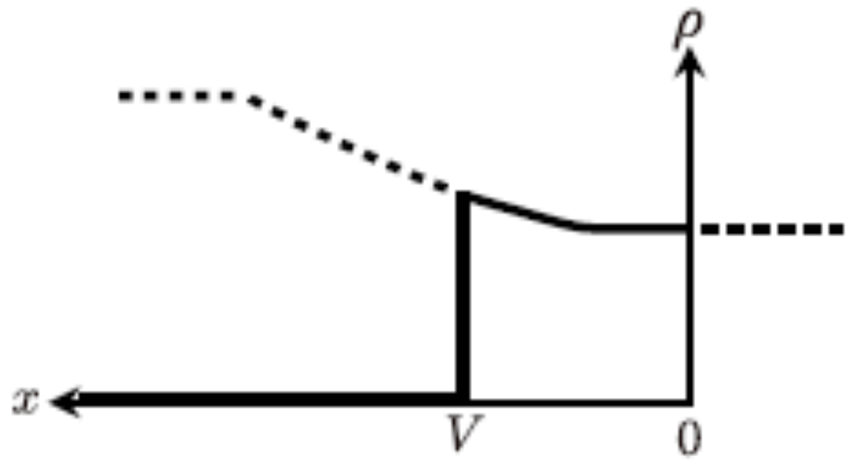
maximum current phase

high density phase



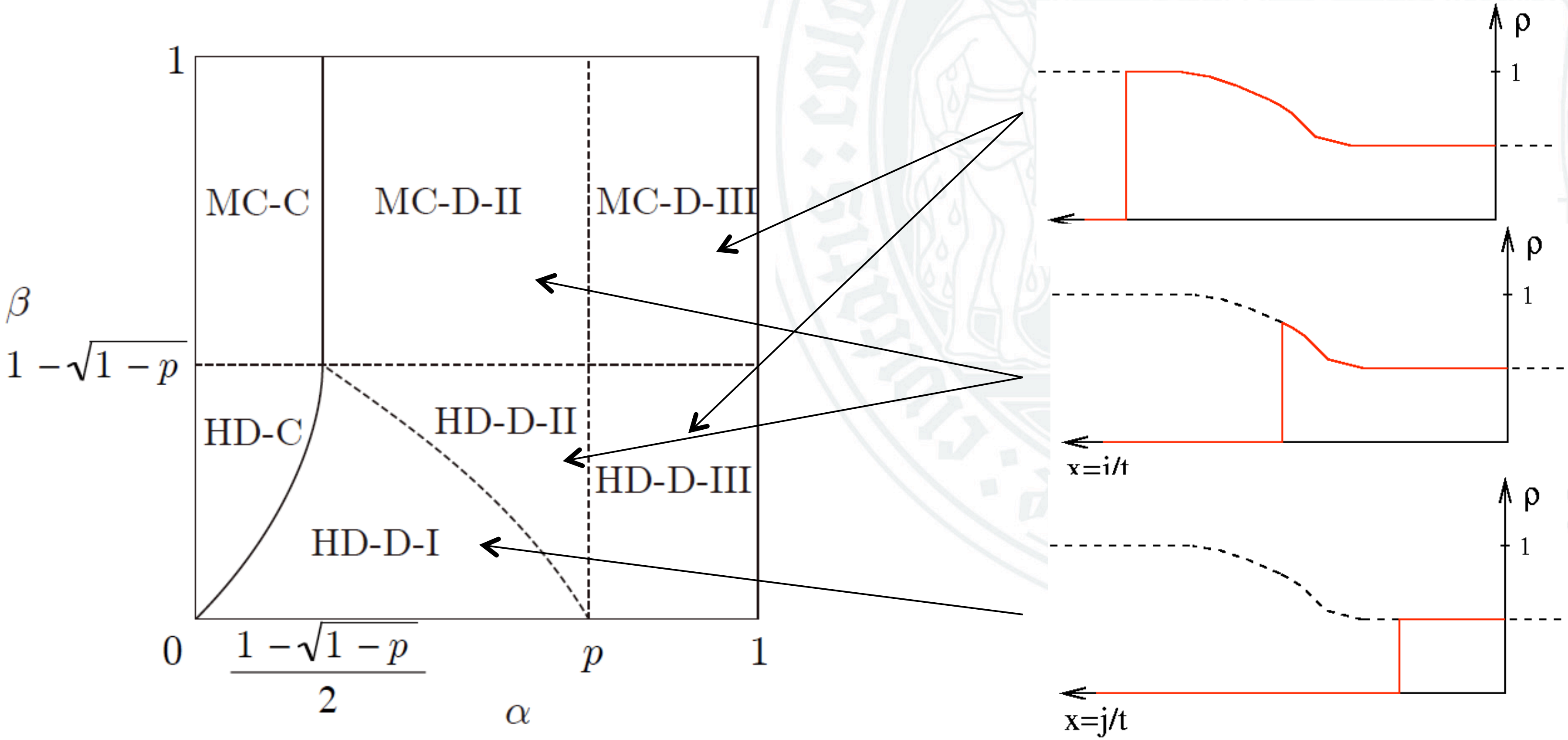
Density profiles: Divergent phase

rarefaction wave:
"cut" by server (x=0)
and leftmost customer
(depending on α)

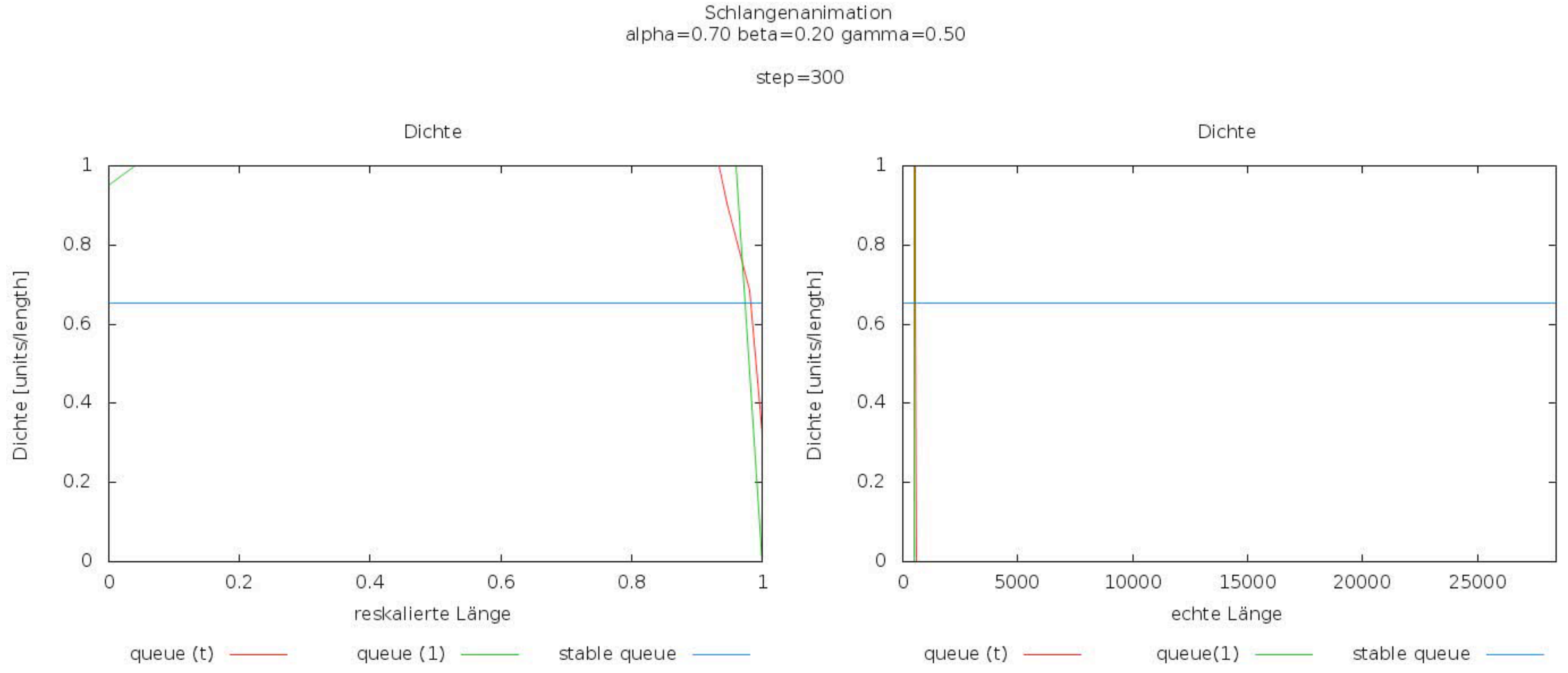


rescaled position: x = j/t

Phase diagram II: parallel update



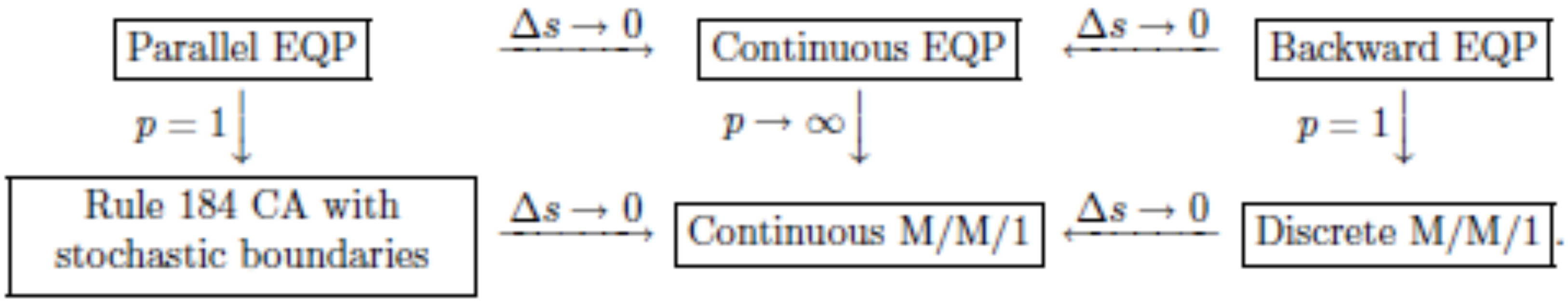
Density profiles



(C. Behlau)

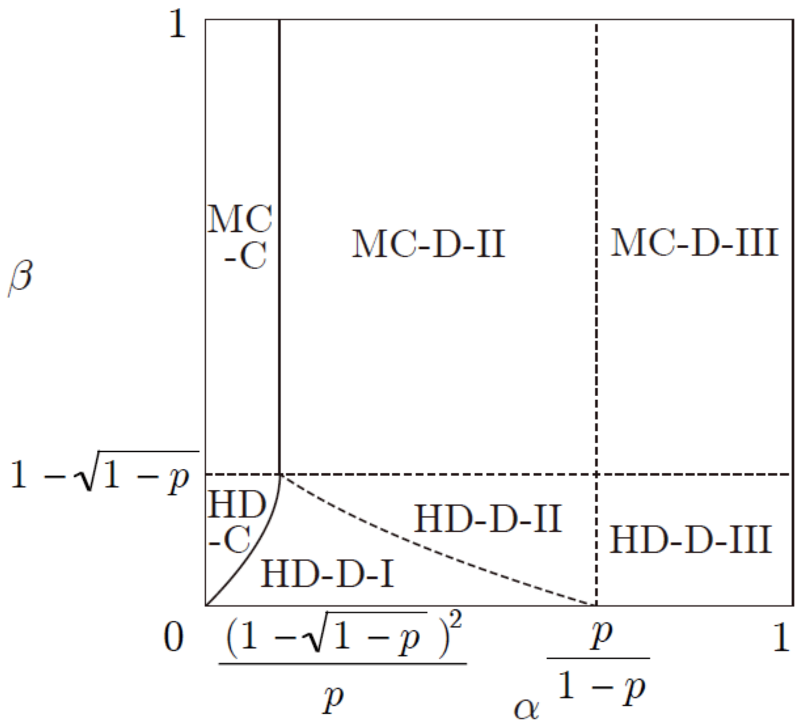
Other updates

- parallel (synchronous) update
- random-sequential update (continuous time)
- backward-sequential update

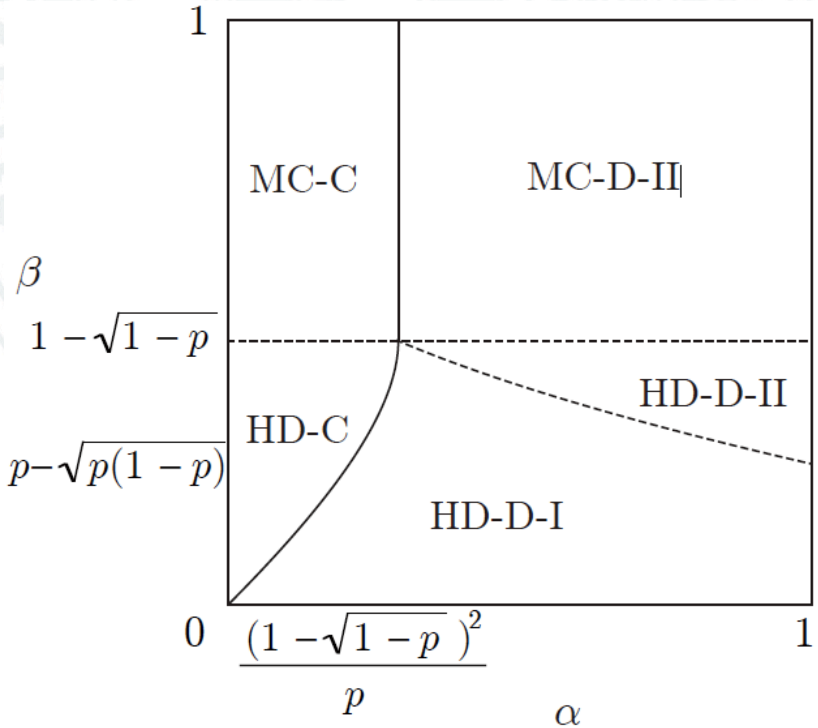


Phase diagram II: backward update

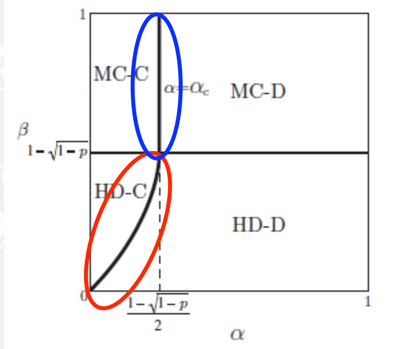
p < 1/2



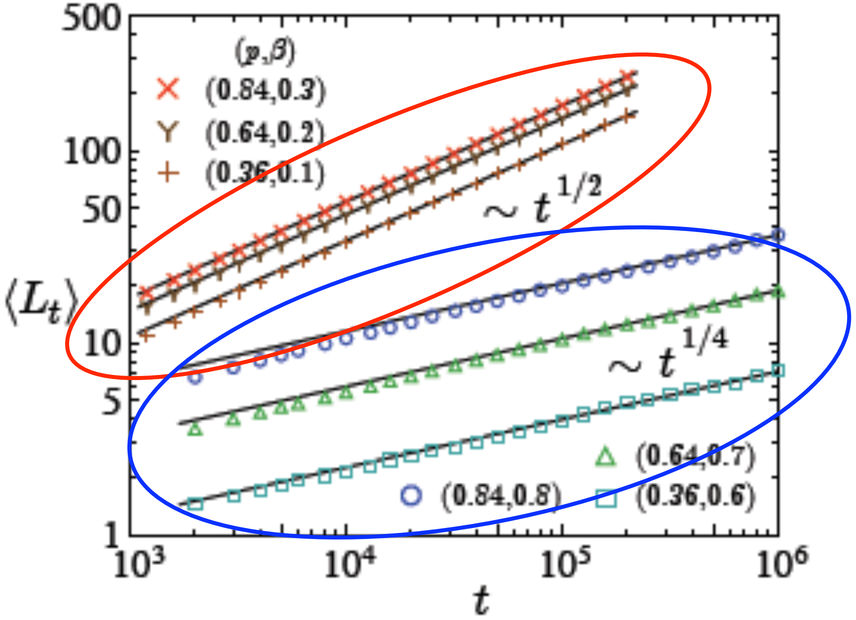
p > 1/2



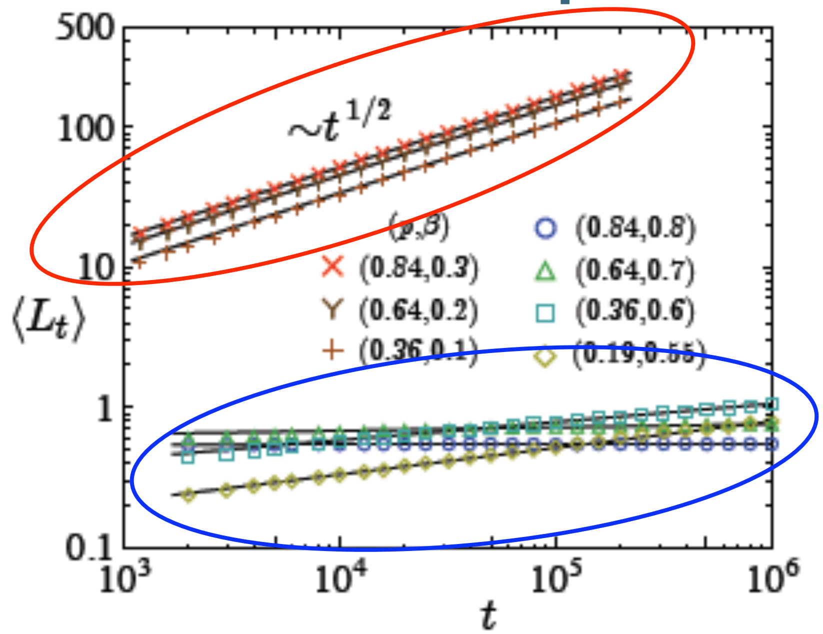
Critical line



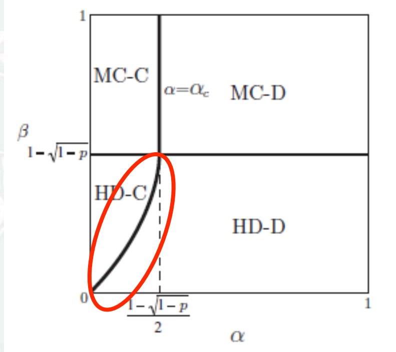
parallel update



backwards update

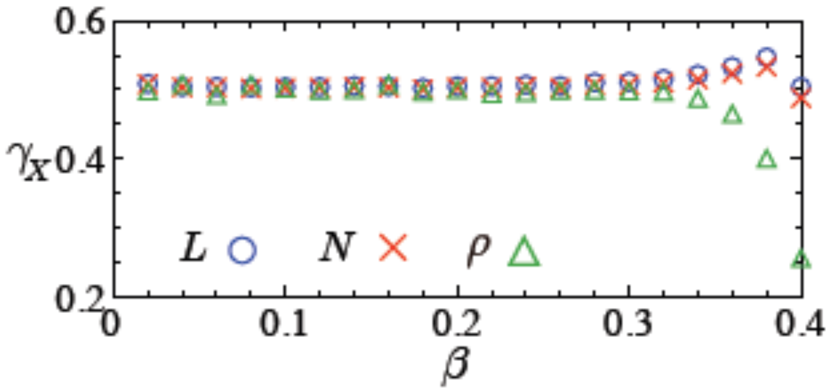


Critical line: exponents

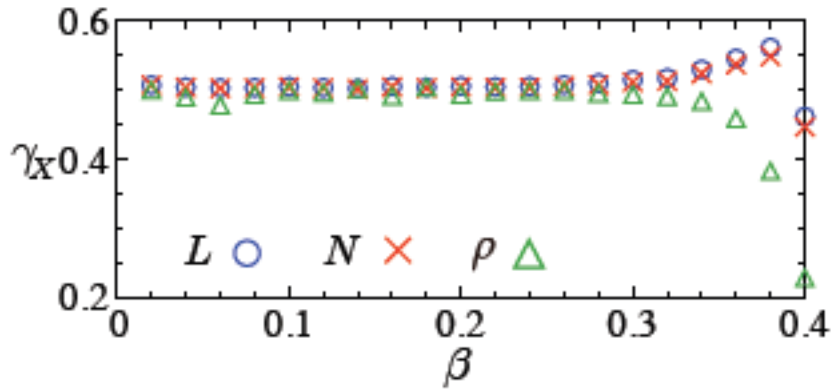


on curved part of critical line

parallel update

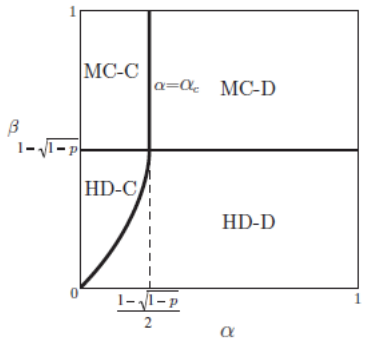


backwards update

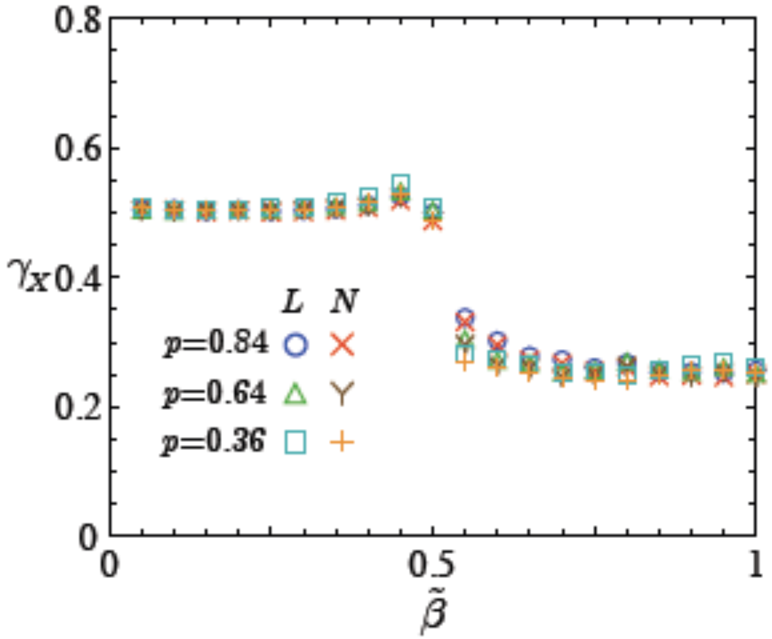


$\gamma = \frac{1}{2}$: diffusive behavior

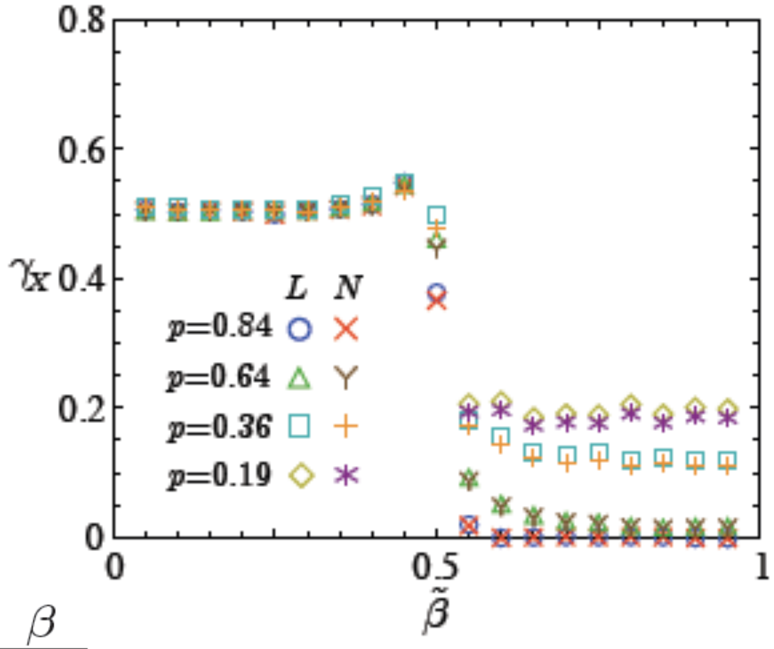
Critical line: exponents



parallel update

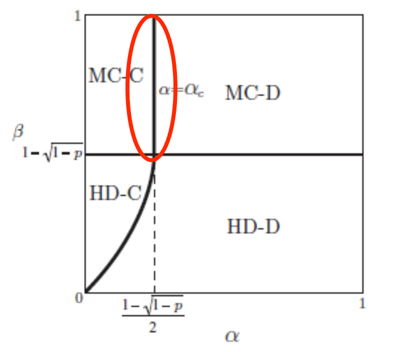


backwards update

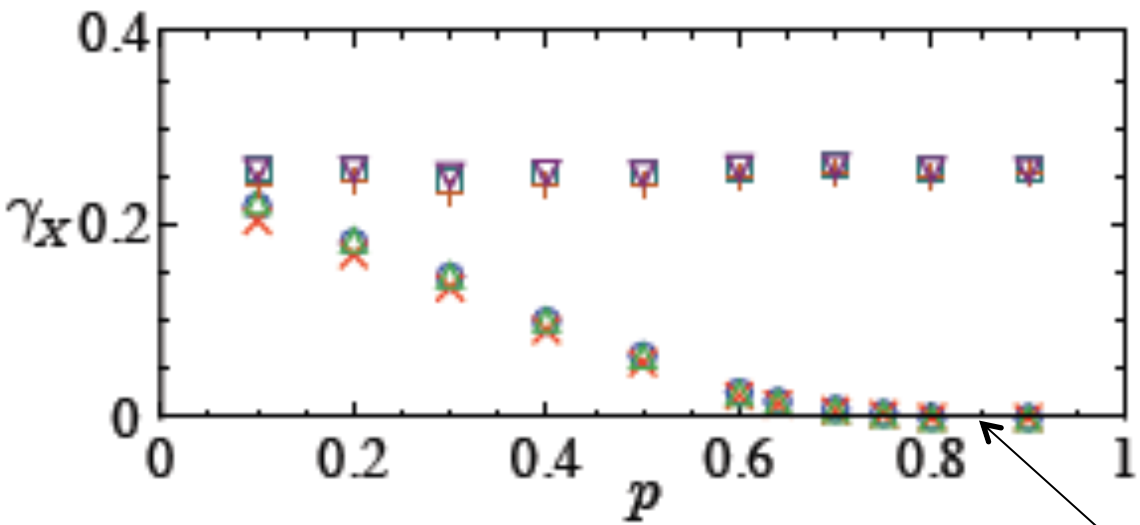


$$\tilde{\beta} = \frac{\beta}{2\beta_c}$$

Critical line: exponents



backwards update: straight line



first-order transition

Critical line: Update dependence

parallel update: universal behavior

- exponents independent of model parameters

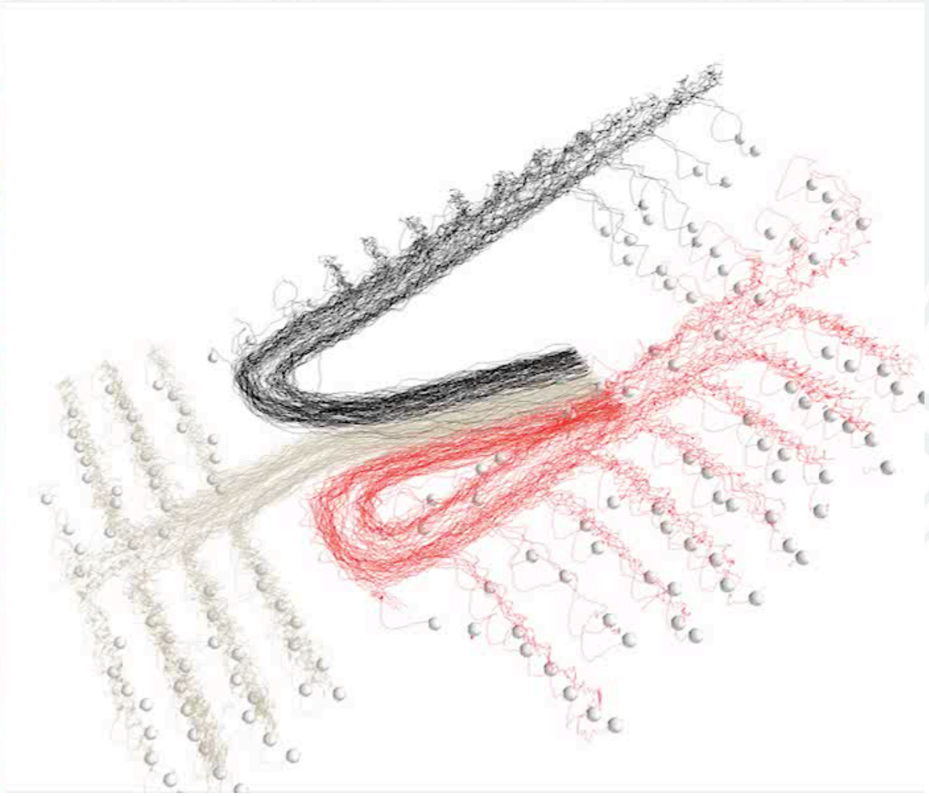
backward update: non-universal behavior

- exponents depend on p
- order of phase transition p-dependent

Generalizations

- length-dependent α
- disorder: p_j
- gap-dependent p
- groups (= larger particles)
- interacting queues

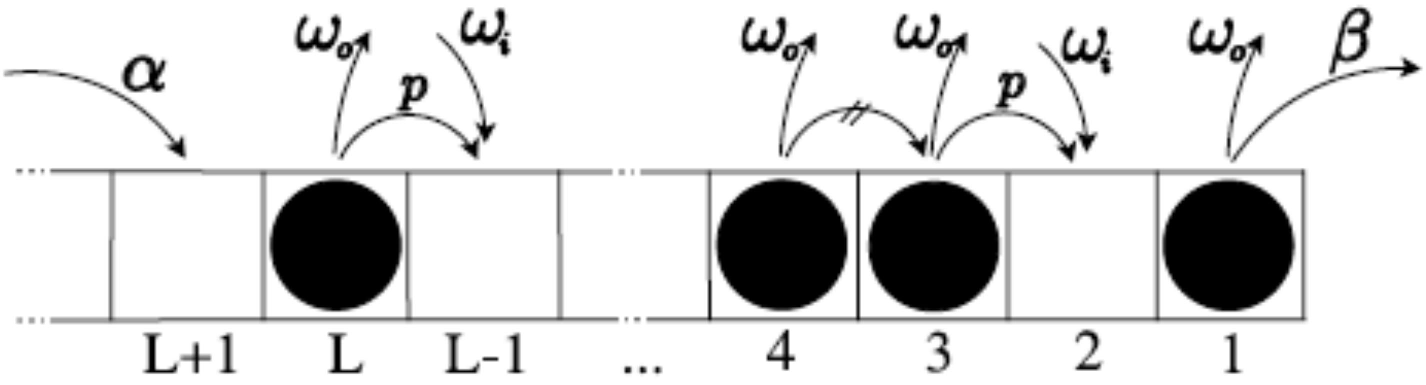
Interacting Queues



Esprit Arena (Düsseldorf)

(M. Boltes, FZ Jülich)

EQP with Langmuir kinetics

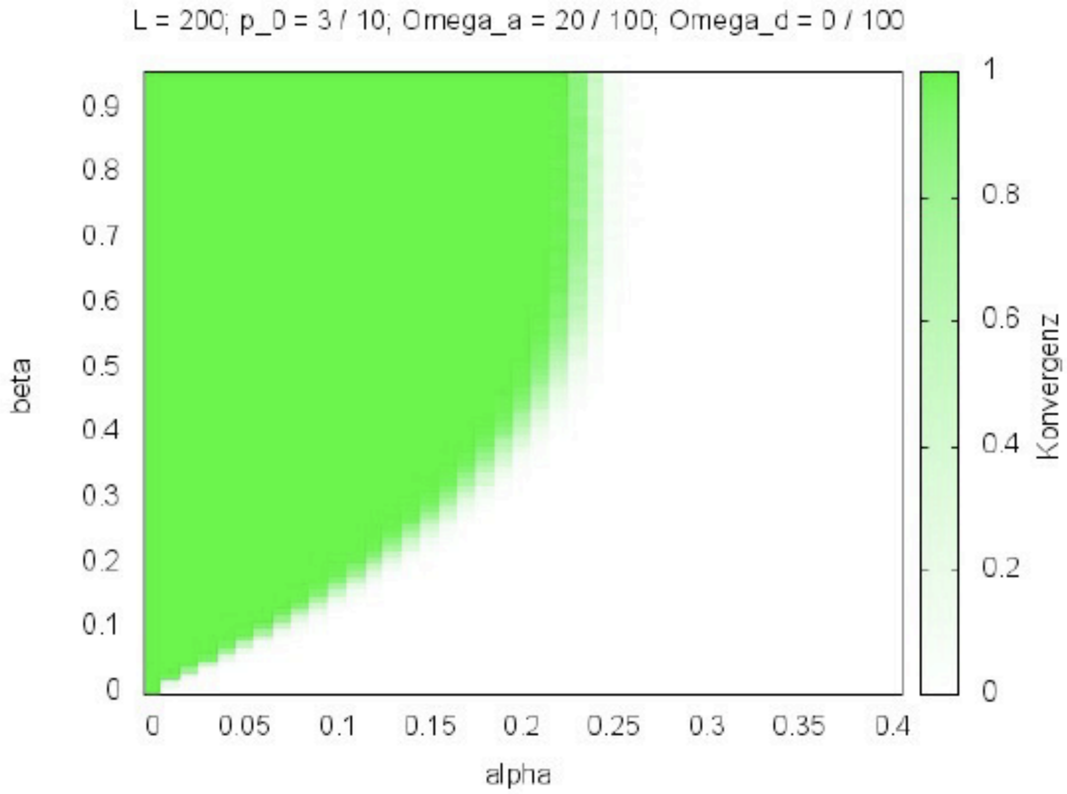


ω_i, ω_o length-dependent ~1/L(t)

EQP with Langmuir kinetics

- **nonergodic behavior:**

  not all samples are convergent or divergent



relative convergence

(Schultens, Borghardt, Arita, AS: work in progress)

Noninteracting deterministic queues!

www.flabber.nl

Summary

- EQP = queueing process with spatial structure
- equivalent to TASEP of variable length
- rich phase diagram
  - 2 phases: divergent + convergent
  - 2 subphases based on dynamics: HD + MC
  - further subphases: number of plateaus
- non-universal critical behavior

Thank you for your attention!!

A. Schadschneider, D. Chowdhury, K. Nishinari:

Stochastic Transport in Complex Systems
- From Molecules to Vehicles -

Elsevier (2010)