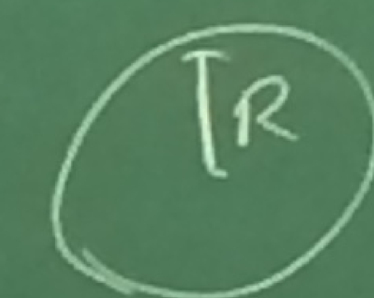
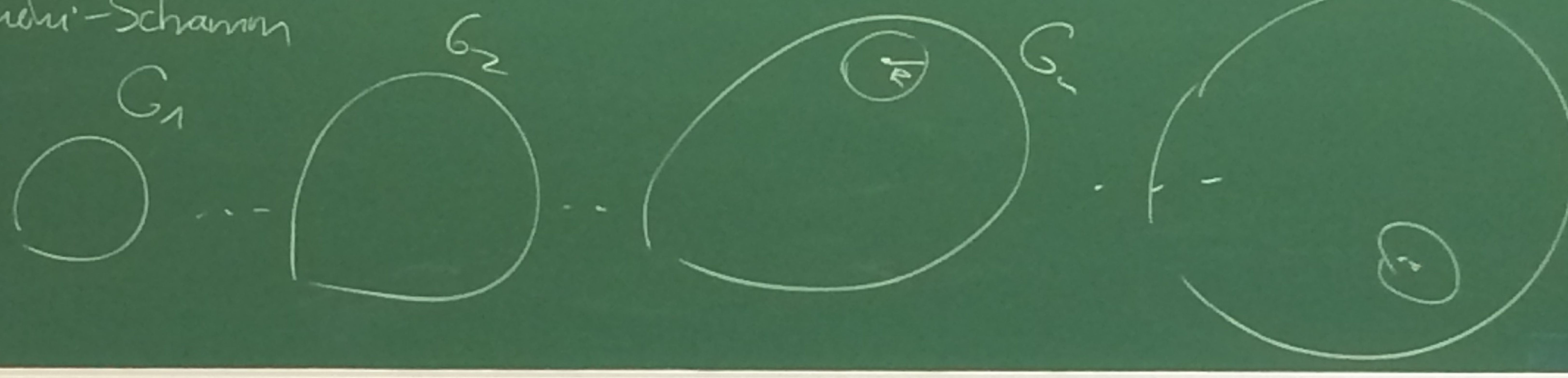
graph : Fix the maximal degree D

$\mathcal{G}_D$: set of graphs $\leftarrow$ . . . $F\mathcal{G}_D$ – ~~finite~~ ch $\mathcal{G}_D$

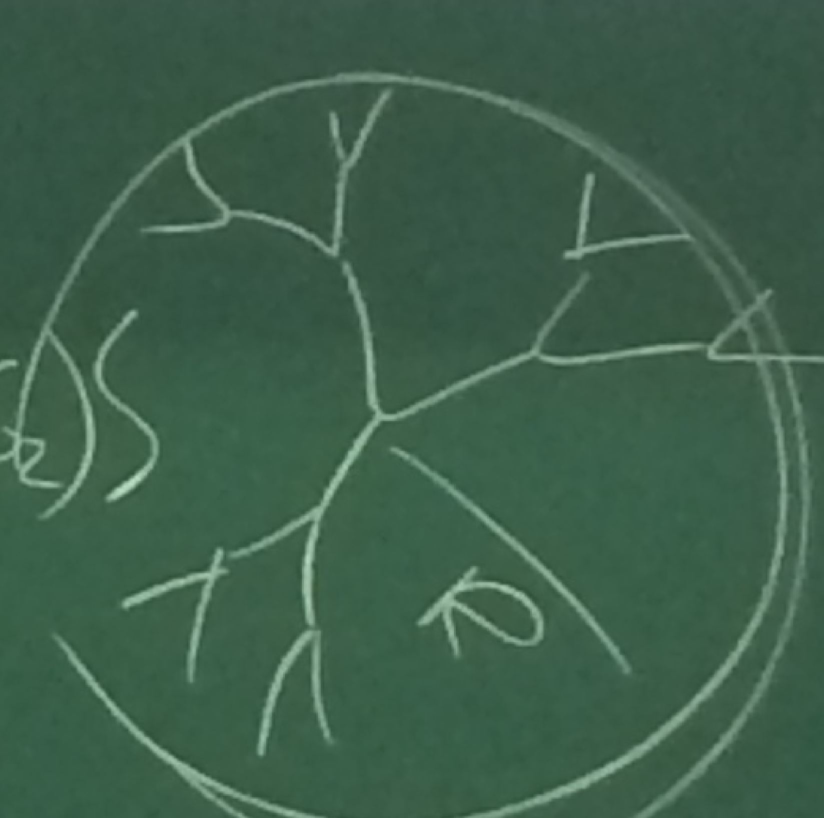
RG_D : spaces of connected rooted graphs



Benjamini–Schramm



$G_1, G_2 \in R\mathcal{G}_D$ $\quad d(G_1,G_2) = \frac{1}{k}$ $\quad k = \max \{B_k(G_1) \simeq B_k(G_2)\}$



If G is finite unrooted graph $\Rightarrow$ λ_G : random rooted graph

Def: G_n BS-converges, if λ_{G_n} weakly converges.

Every sequence has a <u>conv subsequence</u>.

A graph parameter $f : FG_D \to \mathbb{R}$ (finite) is <u>testable</u>, if $f(G_n)$ converges, whenever G_n BS-converges

What is testable? — density of triangles

— [Lyons] tree entropy: $\dfrac{\log \# \text{ of spanning trees of } G}{|G|}$

— {Nguyen-Onak + Elek-Lippner}: $\dfrac{\text{max size of matching}}{|G|}$ is testable

NON-TESTABLE: independence ratio

LINEAR IND RATIO IS TESTABLE

$\dfrac{\dim \ker_p(\mathrm{Adj}(G))}{|G|} = \mu_G(\{0\})$ is testable

OPEN: mod p !!!

REAL STATEMENT: μ_G: uniform measure on eigenvalues of $\mathrm{Adj}(G)$

$\int x^k \, d\mu_{G_n} = \frac{1}{|G|} \sum \lambda_i^k = \frac{1}{|G|} \operatorname{tr}(A^k)$

T: If G_n converges $\Rightarrow$ - μ_{G_n} weakly converges

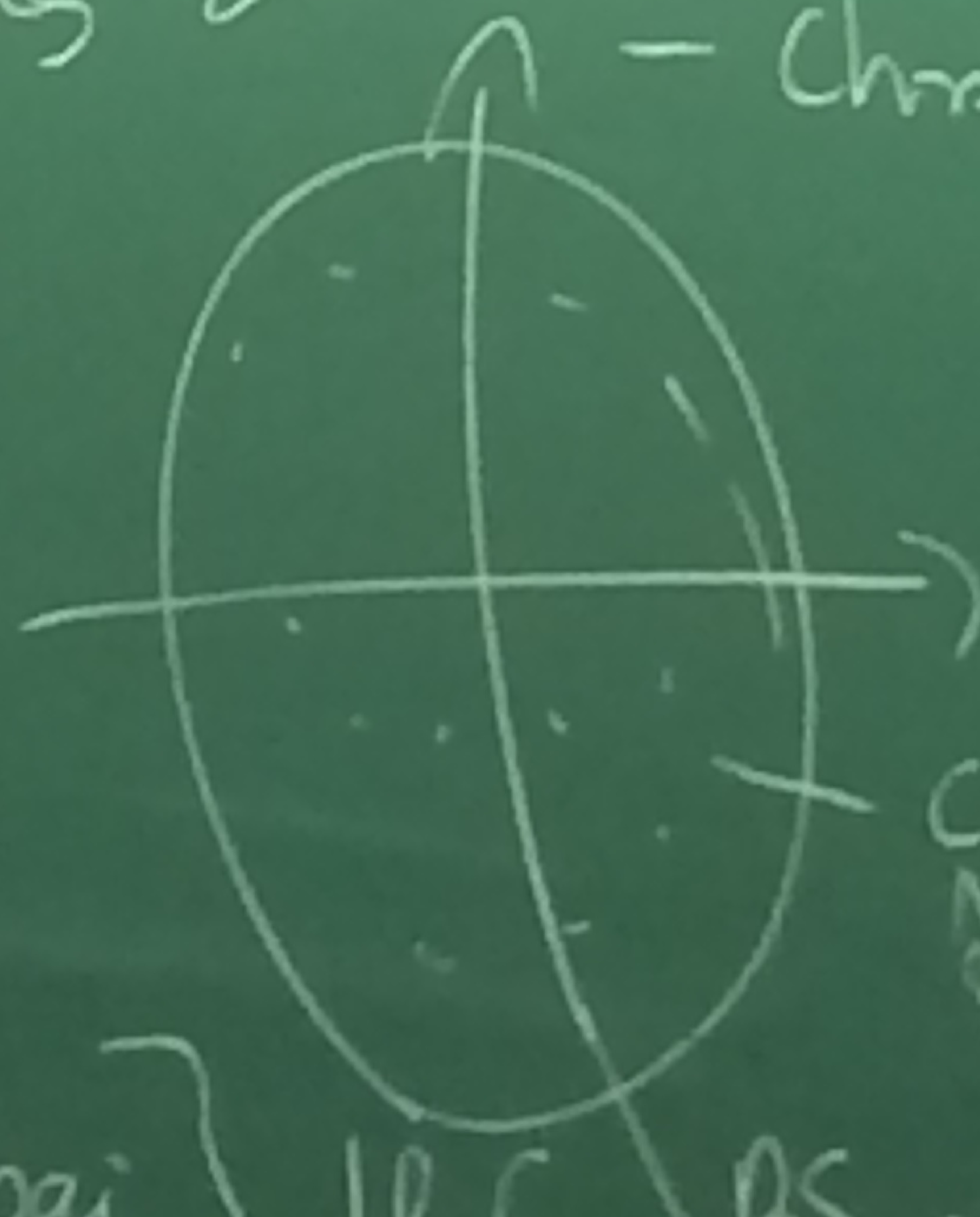
- $\mu_{G_n}(\{x\})$ converges

P: graph polynomial

P · μ_P: uniform measure on the roots of $P(G)$ in $\mathbb{C}$

- char. pols ✓

- Chromatic polynomial



$\exists\ ch_G$ $ch_G(c) = $ # of legal-colorings of G $\forall c$

$C \supset D$ [Sokal]

S_G

T [A-Hubai] If G_n BS-converges $\Rightarrow$ $\int z^k \, d\mu_{G_n}$ converges


T [A-Hubai] If G_n BS-converges ⇒ ∫z^k dμ_G_n converges
T [Csikvári-Frenkel] Generalized for a huge natural class of graph polynomials.
Matching polynomial: Σ (-1)^t m_t(G) x^(n-2t)
m_t(G) # of matchings of t edges in G
m_G - matching measure
BS-continuous
m_Z²
WHAT IS IT?
NO ATOMS
MONOMER-DIMER ENTROPY