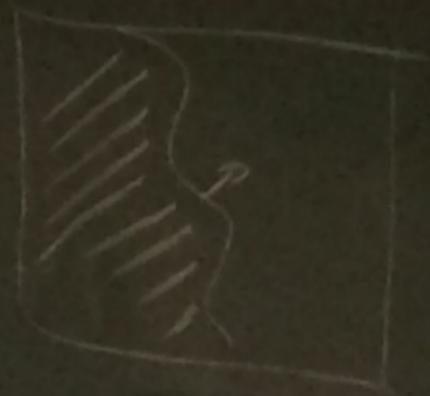


$$\partial A := \{ (a, b) \mid a \in A, b \in A^c \}$$

$A$  co-connected :=  $A, A^c$  connected

$A, B$  boundary disjoint :=  $\partial A \cap \partial B = \emptyset$

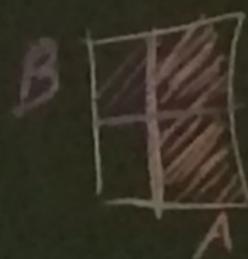
no  $(v_{00}, v_{01}, v_{11}, v_{10})$  4-cycle s.t.  $v_{ij} \in A \Leftrightarrow i=j=1, v_{ij} \in B \Leftrightarrow j=1$



$$A+z := \{ a+z \mid a \in A \}$$

$$T_A := \{ A+nz \mid z \in \mathbb{Z}^d \}$$

$A$  toric :=  $A$  co-connected and  $A, B$  boundary disjoint for all  $B \in T_A$



Thm: if  $A, B$  co-connected, boundary

disjoint  $\Rightarrow$

one of the following holds

1.  $A \cap B = \emptyset$
2.  $A^c \cap B^c = \emptyset$
3.  $A \supset B$  or  $B \supset A$

THM I: If  $A$  is toric,  $|T_A| \neq 1$ , then exactly one of the following holds:

(or i) 1.  $\forall A_1, A_2 \in T_A \quad A_1 \cap A_2 = \emptyset$

(or ii) 2.  $\forall A_1, A_2 \in T_A \quad A_1^c \cap A_2^c = \emptyset$

(or iii) 3.  $T_A$  is totally ordered by inclusion, discrete and there is  $\Delta$  s.t.  $\forall B_1, B_2 \in T_A, B_1 \subsetneq B_2$  implies  $B_1 + \Delta \subseteq B_2$ .