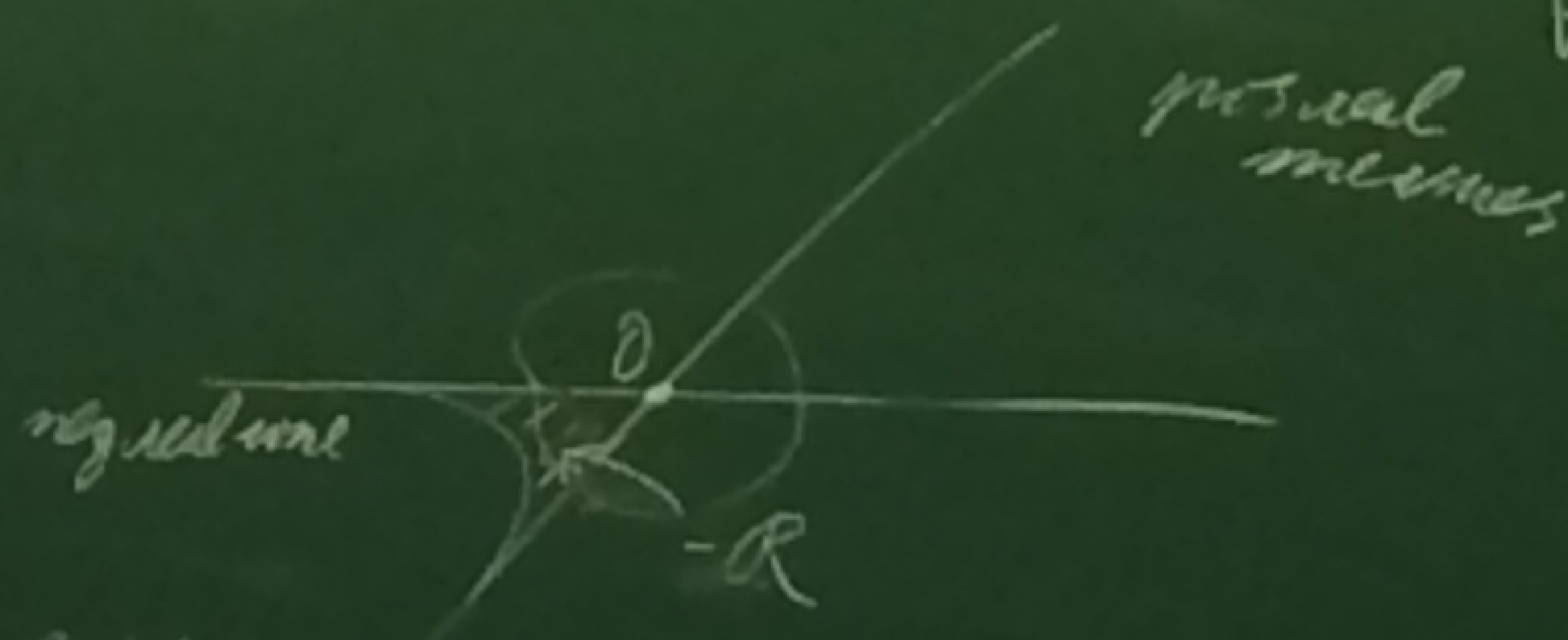
Non-physical singularity in 1D hardsphae

X csms, d metric

M loc bdd complex measure

Λ bdd Borel

$$Z(\Lambda, M) = \sum_{n=0}^{\infty} \frac{1}{n!} \int_{\Lambda^n} \underbrace{S(x_1 \; x_n)}_{\forall i,j:\ d(x_i,x_j) \geq 1} \prod_{i=1}^{n} M(dx_i)$$



$R := \{ M \underset{\text{pos}}{\text{real}} : \forall \Lambda \text{ bdd}: Z(\Lambda, -M) \geq 0 \}$

$\mathring{R} =$

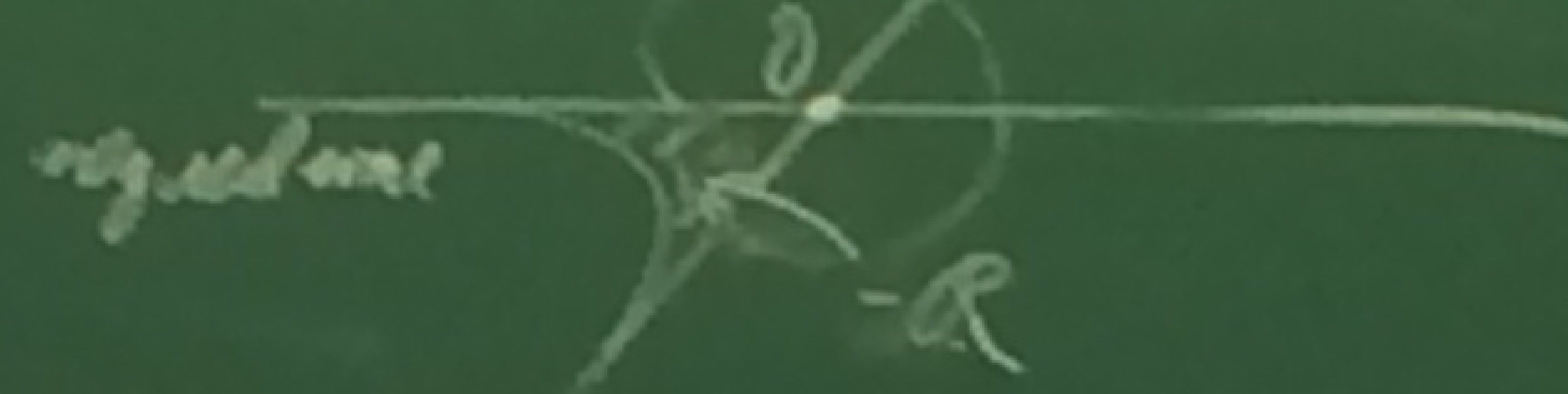
$>$

complex M

$Z(\Lambda, M)$

$\lim Z(\Lambda, M)$

$Z(\Lambda, 0) = 1$



$$\mathcal{R} := \{ M \ \underset{\text{pos}}{\text{real}} : \forall \Lambda \text{ bdd}: Z(\Lambda, -M) \geq 0 \}$$

$$\mathring{\mathcal{R}} = \qquad\qquad\qquad\qquad >$$

complex M:

$$\frac{Z(\Lambda, M)}{Z(\Lambda', M)} , \qquad \frac{\log Z(\Lambda, M)}{M(\Lambda)} \qquad\qquad Z(\Lambda, 0) = 1$$

Sufficient conditions for $|M|$ to lie in $\mathcal{R}$:
- Dobrushin
- FP
- Ruelle $\mathbb{R}^d$

X, d, M non-neg real: ξ PP st
- simple
- intensity M $\qquad \Lambda \to \mathbb{E}(\xi(\Lambda)) = M(\Lambda)$
- $\forall x \in X, \forall \Lambda : d(x, \Lambda) \geq 1$

$$P_x(\xi(\Lambda) = 0) = P(\xi(\Lambda) = 0)$$

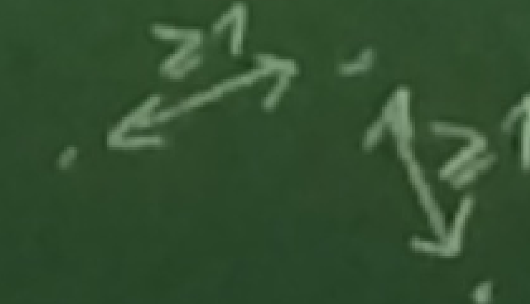
Thm (Cont LLL):

$M \in \mathring{R}$: $\forall\, \xi$ lhs : $\forall \Lambda$ bdd Borel : $P(\xi(\Lambda)=0) \geq Z(\Lambda,-M) > 0$

$M \in M \setminus R$ $\exists\, \xi$ lhs : $\exists \Lambda$: $P(\xi(\Lambda)=0) = 0$

Thm (Shearer's PP, 84):

$M \in R \iff \exists$ PP η lhs $\oplus$ $P(S(\eta)) = 1$



$\oplus$ $P(\eta(\Lambda)=0) = Z(\Lambda,-M)$

Thm ($X=Y$): LSS 98

$M \in \mathring{R}$. $\exists\, \vec{c} < \vec{1}$: $\forall\, \xi$ lhs : $\xi \overset{st}{\leq}$ Bernoulli product ($\vec{c}$)

$\overset{\shortparallel}{\vec{c}(\vec{m})}$

$\leq E(f(\vec{c}))$

$\mathbb{Z}_k$ $\leq k$ $M = S\#$

Thm $(X = \mathcal{G})$: LSS 98

(m)-will $\in \mathring{R}$: $\exists\, \vec{c} < \vec{1}$: $\forall\, \xi$ lhs : $\xi \overset{st}{\leq}$ Bernoulli product $(\vec{c})$

$\vec{c} = \vec{c}(\vec{m})$

$\leq \mathbb{E}(f(\vec{c}))$

•) $\mathbb{Z}_k$  $\leq k$, $M = \rho\,\#$

$(\mathbb{Z}, (k+1)|.|)$

$\rho^* := \sup\{\rho : \forall n : Z([0,n], -M) > 0\}$

•) $(\mathbb{R}, |.|)$ $\quad M = \rho\,\mathrm{Leb}$

$\mathbb{R}^d$: $\rho^*_d \geq \dfrac{1}{e\,\mathrm{Vol}(B_1)}$

k

cont

Dob $\quad \rho^*_k \geq \dfrac{(2k)^{2k}}{(2k+1)^{2k+1}} \sim \dfrac{1}{e\,2k}$

Ruelle: $\rho^* \geq \dfrac{1}{2e}$ — CE

canonical

FP $\quad \rho^*_k \geq$

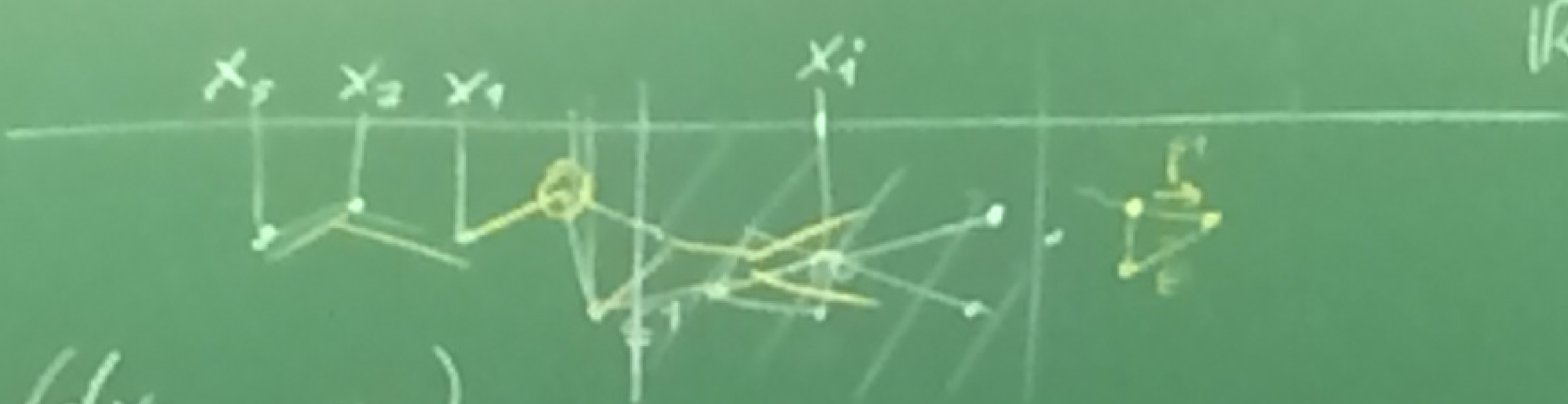
FPS. $\rho^* \geq \dfrac{1}{2+\sqrt{2}}$

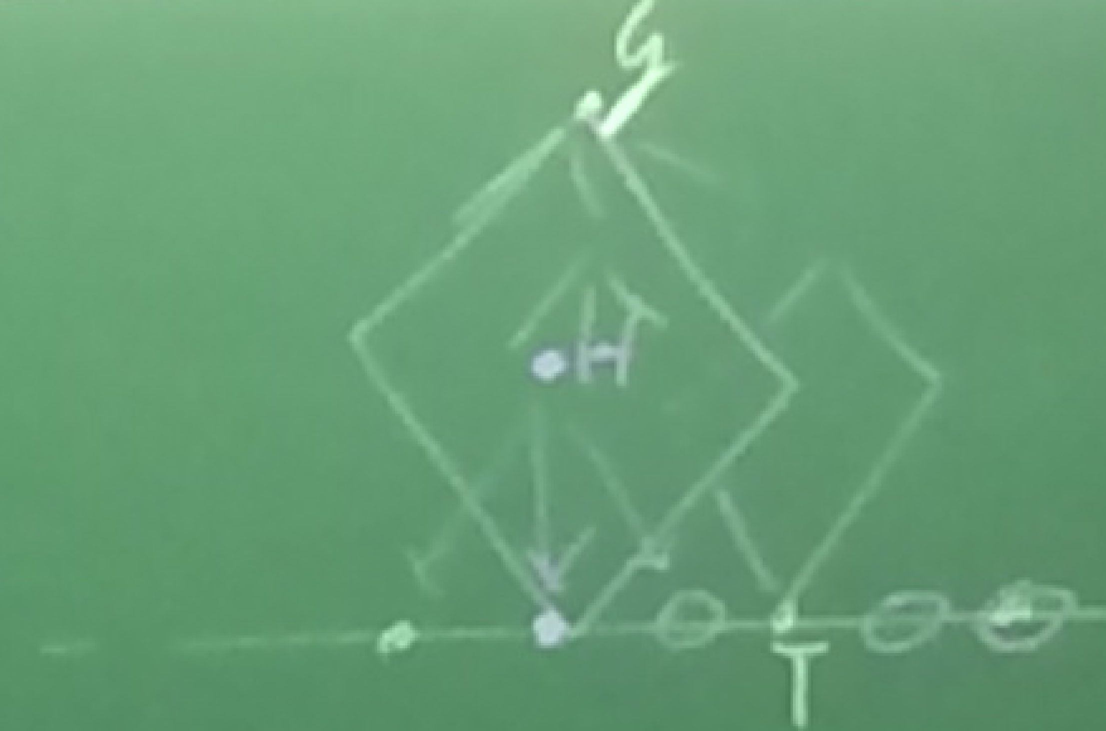
LSS $\quad \rho^*_k = \dfrac{k^k}{(k+1)^{k+1}} \sim \dfrac{1}{ek}$

CE

Tonks gas: $\rho^* = \dfrac{1}{e}$

Dob

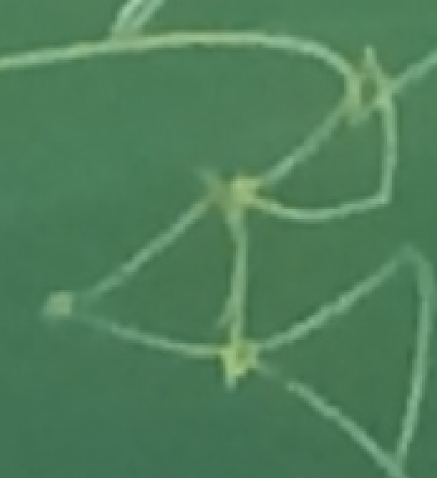


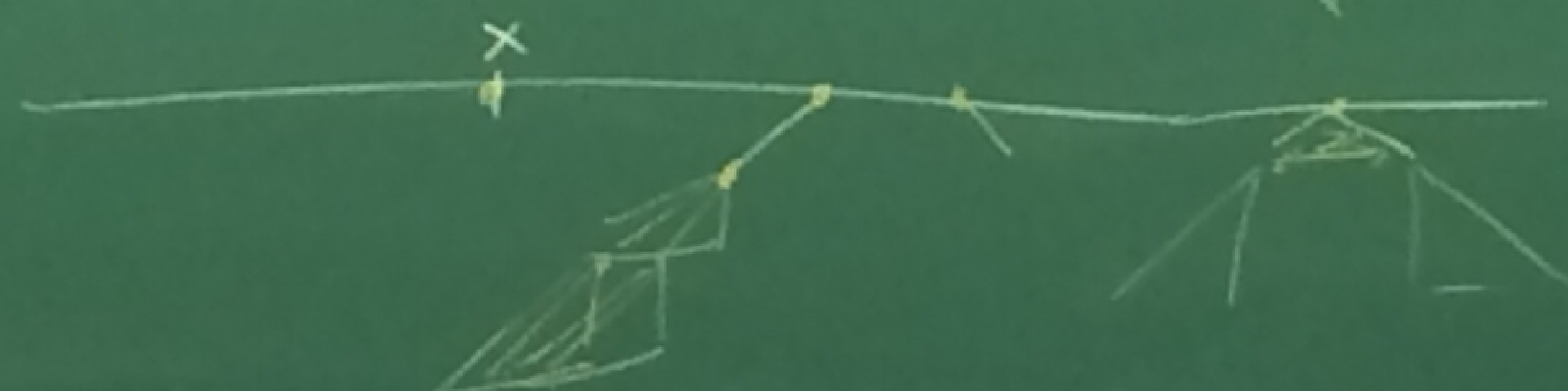


$\mathcal{G}(x_1, \dots x_n)$

$$\sum (-1)^{|E(H)|} \overset{\text{Penrose}}{=} \underline{(-1)^{n-1}}\, |\{T \subseteq \mathcal{G} : S(T) = S\}|$$

$\mathcal{T}_{\mathcal{G}} = \{H \text{ connected spanning} \subseteq \mathcal{G}\}$

$\leq$ 



$$F(\delta) = \delta\left(1 + 2W(-\delta) + \frac{W(-\delta)^2}{2}\right)$$

$$F(\delta) = \underline{W(-\delta)}$$

$r > 0.\quad z(r) = Z([0,r], -\varrho\,\mathrm{Leb})_S$