The Spin 1 SU(2)-invariant model

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The model

We work on a finite lattice $\Lambda \subset \mathbb{Z}^d$, with a set of edges $\mathcal{E} \subset \Lambda \times \Lambda$. For concreteness we have nearest neighbour edges on

$$\Lambda = \left\{ -\frac{L_1}{2} + 1, ..., \frac{L_1}{2} \right\} \times ... \times \left\{ -\frac{L_d}{2} + 1, ..., \frac{L_d}{2} \right\}.$$

we have the usual spin operators S^1, S^2, S^3. We let $\mathbf{S} = \left(S^1, S^2, S^3 \right)$ and S_x^i be the operator that applies S^i to site $x \in \Lambda$ and leaves other sites unchanged.

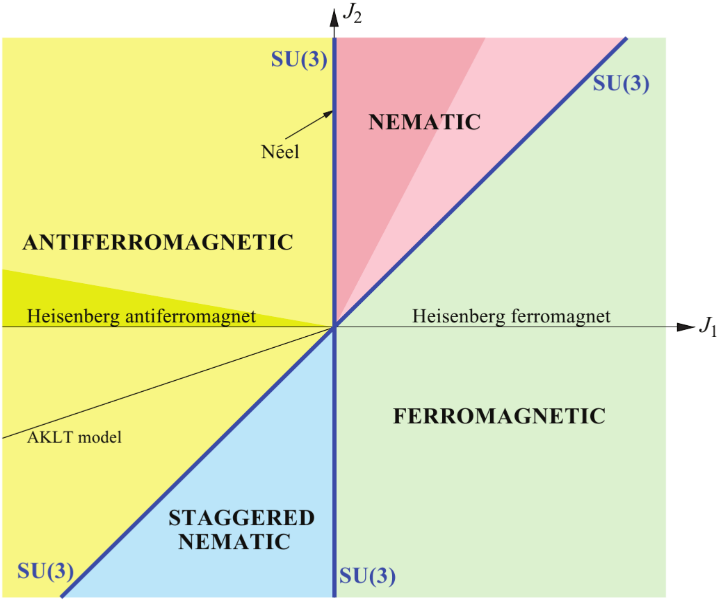
Spin-1: the Phase diagram

For $S = 1$ the most general rotation-invariant interaction is

$$H_\Lambda = -\sum_{\{x,y\}\in\mathcal{E}} \left(J_1 \mathbf{S}_x \cdot \mathbf{S}_y + J_2 \left(\mathbf{S}_x \cdot \mathbf{S}_y \right)^2 \right).$$

The phase diagram has been partially completed, for example for $J_2 = 0$ and $J_1 < 0$ large enough, this is the result of Dyson, Lieb and Simon. However, for some regions very little is known, for example $J_2 < 0$.

Spin-1: the Phase diagram



The case $J_1 = 0, J_2 > 0$

For the case $J_1 = 0$, $J_2 > 0$ the Hamiltonian can be written as

$$H^{nem}_{\Lambda,\mathbf{h}} = -2 \sum_{\{x,y\}\in\mathcal{E}} (\mathbf{S}_x \cdot \mathbf{S}_y)^2 - \sum_{x\in\Lambda} h_x \left((S^3_x)^2 - \frac{2}{3}\mathbb{1} \right)$$

however if we let $U = \prod_{x\in\Lambda_B} e^{i\pi S^2_x}$ and

$$H_{\Lambda,\mathbf{h}} = -2 \sum_{\{x,y\}\in\mathcal{E}} (S^1_x S^1_y - S^2_x S^2_y + S^3_x S^3_y)^2 - \sum_{x\in\Lambda} h_x \left((S^3_x)^2 - \frac{2}{3}\mathbb{1} \right)$$

we have the useful relation $U^{-1} H_{\Lambda,\mathbf{h}} U = H^{nem}_{\Lambda,\mathbf{h}}$. Writing the Hamiltonian in this form allows us to show the model is reflection positive.

Long range order

Theorem

Let $S = 1$. Assume $\mathbf{h} = 0$ and $L_1, ..., L_d$ are even. Then we have the bounds

$$\lim_{\beta\to\infty} \lim_{L_i\to\infty} \frac{1}{|\Lambda|} \sum_{x\in\Lambda} \rho(x) \geq \begin{cases} \frac{2}{9} - \frac{1}{\sqrt{3}} J_d \sqrt{\left\langle S_0^1 S_0^3 S_{e_1}^1 S_{e_1}^3 \right\rangle} \\ \rho(e_1) - \frac{1}{\sqrt{3}} I_d \sqrt{\left\langle S_0^1 S_0^3 S_{e_1}^1 S_{e_1}^3 \right\rangle} \end{cases}.$$

where

$$\rho(x) = \left\langle \left((S_0^3)^2 - \frac{2}{3} \right) \left((S_x^3)^2 - \frac{2}{3} \right) \right\rangle.$$

The two integrals in the theorem are given by

$$
\begin{aligned}
I_d =& \frac{1}{(2\pi)^d} \int_{[-\pi,\pi]^d} \sqrt{\frac{\varepsilon(k+\pi)}{\varepsilon(k)}} \left(\frac{1}{d} \sum_{i=1}^{d} \cos k_i \right)_+ \mathrm{d}k, \\
J_d =& \frac{1}{(2\pi)^d} \int_{[-\pi,\pi]^d} \sqrt{\frac{\varepsilon(k+\pi)}{\varepsilon(k)}} \mathrm{d}k.
\end{aligned} \tag{1}
$$

By relating these correlations to the probability of nearest neighbours being in the same loop in the loop model [Ueltschi '13] we show that one of these bounds is satisfied if $I_d J_d \leq 4/27$, this is satisfied in $d \geq 8$.

references

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