25th British Combinatorial Conference

University of Warwick

6th – 10th July 2015

Contents

The week at a glance

	Monday	Tuesday	Wednesday	Thursday	Friday
08:00 - 08:50	Registration & Welcome				
08:50 - 09:00		Welcome			
09:00 - 09:55	**Kalai** MS.02	**Gerke** MS.02	**Ruškuc** MS.02	**Xing** MS.02	**McGuire** MS.02
10:00 - 10:30	Coffee break				
10:30 - 10:50	Parallel sessions	Parallel sessions	**Norin** MS.02	Parallel sessions	Parallel sessions
10:55 - 11:15					
11:20 - 11:40					
11:45 - 12:05			Lunch		
12:05 - 02:00	Lunch			Lunch	
02:00 - 02:55	**Conlon** MS.02	**Łuczak** MS.02	**Excursion to Warwick Castle** Departure of buses to Warwick Castle at 12:50	**Bodirsky** MS.02	**END**
03:00 - 03:30	Coffee break			Coffee break	
03:30 - 03:50	Parallel sessions	Parallel sessions		Parallel sessions	
03:55 - 04:15					
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05:10 - 05:30		BCC Business Meeting MS.02			
05:35 - 06:00	Open problem session 1, MS.02			Open problem session 2, MS.02	
06:00 - 06:30					
06:30 - 07:00					
07:00 - . . .	**Welcome Reception** Bar Fusion	**BCC concert** Chancellors Suite at 07:30		**Conference Dinner** Panorama Suite	

The programme in detail

Monday

08:00 - 08:40	Registration
08:40 - 09:00	Welcome, `MS.02` **Vadim Lozin**
09:00 - 09:55	**Gil Kalai** (Chair: Vadim Lozin), `MS.02` *Some old and new problems in combinatorial geometry I: Around Borsuk's problem*
10:00 - 10:30	Coffee break
10:30 - 12:05	Contributed talks (4 slots) in 4 parallel sessions Rooms `MS.02`, `MS.03`, `MS.04`, `B3.02`
12:05 - 02:00	Lunch
02:00 - 02:55	**David Conlon** (Chair: Daniel Král'), `MS.02` *Recent developments in graph Ramsey theory*
03:00 - 03:30	Coffee break
03:30 - 05:30	Contributed talks (5 slots) in 4 parallel sessions Rooms `MS.02`, `MS.03`, `MS.04`, `B3.02`
05:35- 06:00	Open problem session 1 (Chair: Peter Cameron), `MS.02`
07:00 - . . .	Welcome Reception, Bar Fusion

Monday morning contributed talks

Chair: D. B. Penman, MS.02

10:30 - 10:50	**P. J. Cameron** Remembering Donald Preece
10:55 - 11:15	**K. Tyros** General stability and exactness theorems
11:20 - 11:40	**J. Sliacan** Flagmatic and Turán densities
11:45 - 12:05	**S. Nakamura** The number of contractible edges in a 4-connected graph having a small number of edges not contained in triangles

Chair: M. Ellingham, MS.03

10:30 - 10:50	**K. K. Dabrowski** Clique-width of restricted graph classes
10:55 - 11:15	**V. Zamaraev** Well-quasi-ordering does not imply bounded clique-width
11:20 - 11:40	**A. Collins** Clique-width, linear clique-width and well-quasi-ordering
11:45 - 12:05	**M. M. Ferrari** On the partition graph of a positive integer

Chair: K. Staden, MS.04

10:30 - 10:50	**M. Bachratý** Approaching the Moore bound for diameter 3 by Cayley graphs
10:55 - 11:15	**R. F. Bailey** On the metric dimension of imprimitive distance-regular graphs, II
11:20 - 11:40	**G. Erskine** New constructions for large circulant graphs of given degree and diameter
11:45 - 12:05	**J. B. Gauci** Diameter vulnerability of the generalized Petersen graphs

Chair: J. W. P. Hirschfeld, `B3.02`

10:30 - 10:50	**S. R. Blackburn** Probabilistic existence results for separable codes
10:55 - 11:15	**M. B. Paterson** Characterisations of optimal algebraic manipulation detection codes
11:20 - 11:40	**K. Shiromoto** On covering dimension of linear codes and matroids
11:45 - 12:05	**B. S. Webb** Subsystems of Netto triple systems

Monday afternoon contributed talks

Chair: Ł. Grabowski, MS.02

03:30 - 03:50	**R. Mycroft** Hamilton cycles in quasirandom hypergraphs
03:55 - 04:15	**A. Żak** Hamilton saturated hypergraphs of essentially minimum size
04:20 - 04:40	**A. Nicholas Day** Saturated graphs of prescribed minimum degree
04:45 - 05:05	**K. Staden** On a degree sequence analogue of Pósa's conjecture
05:10 - 05:30	**L. Yepremyan** The Local Stability Method

Chair: P. Hu, MS.03

03:30 - 03:50	**D. Bevan** The growth of permutations avoiding 1324
03:55 - 04:15	**P. G. Tsikouras** Equivalence classes of Dyck paths modulo udu
04:20 - 04:40	**N. Korpelainen** On the enumeration of juggling patterns and related combinatorial objects
04:45 - 05:05	**B. Granovsky** Developments in the Khintchine-Meinardus method for asymptotic enumeration
05:10 - 05:30	**O. Bagdasar** On the enumeration of integer tuples having the same lcm

Chair: I. M. Wanless, `MS.04`

03:30 - 03:50	**J. W. P. Hirschfeld** Open problems in finite projective spaces
03:55 - 04:15	**J. P. McSorley** A new technique for finding small Kirkman Covering and Packing Designs; a KCD(11), a canonical KCD(13), and more examples
04:20 - 04:40	**J. T. LeGrow** Cycle extensions in 0-block-intersection graphs of balanced incomplete block designs
04:45 - 05:05	**D. A. Pike** Equitably Coloured BIBDs
05:10 - 05:30	**S. Huczynska** External difference families

Chair: K. K. Dabrowski, `B3.02`

03:30 - 03:50	**L. DeBiasio** Covers of (pseudo)random graphs by monochromatic subgraphs
03:55 - 04:15	**T. Pinto** Directed paths in the cube
04:20 - 04:40	**R. R. Lewis** The degree-diameter problem for circulant graphs of arbitrary diameter up to degree 10
04:45 - 05:05	**V. Falgas-Ravry** Full subgraphs of a graph
05:10 - 05:30	**R. Simanjuntak** Strong Oriented Graphs with Largest Directed Metric Dimension

Tuesday

08:50 - 09:00	Welcome, `MS.02` **Vadim Lozin**
09:00 - 09:55	**Stefanie Gerke** (Chair: Vadim Lozin), `MS.02` *Controllability and matchings in random bipartite graphs*
10:00 - 10:30	Coffee break
10:30 - 12:05	Contributed talks (4 slots) in 4 parallel sessions Rooms `MS.02`, `MS.03`, `MS.04`, `B3.02`
12:05 - 02:00	Lunch
02:00 - 02:55	**Tomasz Łuczak** (Chair: Artur Czumaj), `MS.02` *Randomly generated groups*
03:00 - 03:30	Coffee break
03:30 - 04:40	Contributed talks (3 slots) in 4 parallel sessions Rooms `MS.02`, `MS.03`, `MS.04`, `B3.02`
05:00 - 06:00	BCC Business Meeting, `MS.02`
06:30	BCC concert, Chancellors Suite

Tuesday morning contributed talks

Chair: A. Treglown, `MS.02`

10:30 - 10:50	**S. Noble** Delta-matroids, ribbon graphs and connectivity
10:55 - 11:15	**A. Liebenau** First order convergence of matroids
11:20 - 11:40	**E. Csóka** Generalized solution for the Herman Protocol Conjecture
11:45 - 12:05	**P. Borg** Cross-intersecting families

Chair: P. Allen, `MS.03`

10:30 - 10:50	**J. Böttcher** Chromatic thresholds in random graphs
10:55 - 11:15	**T. A. McCourt** Face 2-coloured triangulations and directed Eulerian digraphs on the sphere
11:20 - 11:40	**J. van den Heuvel** Generalised colouring numbers of graphs
11:45 - 12:05	**C. Çiftçi** Local connective chromatic number of a graph

Chair: V. Falgas-Ravry, `MS.04`

10:30 - 10:50	**B. Atay** Exponential domination number of cycle related graphs
10:55 - 11:15	**K. Meeks** Graph modification problems in epidemiology
11:20 - 11:40	**G. Boruzanli** On reliability of the generalized Petersen graphs
11:45 - 12:05	**G. Hurlbert** On computing graph pebbling numbers

Chair: T. S. Griggs, `B3.02`

10:30 - 10:50	**R. J. Stones** Counting partial Latin rectangles	
10:55 - 11:15	**N. Cavenagh** Which Latin square is the loneliest?	
11:20 - 11:40	**I. M. Wanless** Symmetries of Latin Squares	
11:45 - 12:05	**A. J. W. Hilton** Completing y-uniform latinized squares	

Tuesday afternoon contributed talks

Chair: J. Böttcher, `MS.02`

03:30 - 03:50	**K. Markström** Turan densities for 3-graphs
03:55 - 04:15	**P. Hu** Mantel's theorem for random hypergraphs
04:20 - 04:40	**F. Joos** The Erdős-Pósa property for cycles

Chair: J. Haslegrave, `MS.03`

03:30 - 03:50	**I. Karpas** Families avoiding just one pattern
03:55 - 04:15	**C. Bean** Avoiding a pair of vincular and covincular patterns
04:20 - 04:40	**K. Wesek** Grasshopper pattern avoidance

Chair: B. Walczak, `MS.04`

03:30 - 03:50	**C. A. Rodger** Fair 1-factorizations and fair holey 1-factorizations of complete multipartite graphs
03:55 - 04:15	**R. Freij** Warmth and connectivity of edge spaces of graphs
04:20 - 04:40	**T. Johansson** On random k-out subgraphs of large graphs

Chair: A. Liebenau, `B3.02`

03:30 - 03:50	**E. Zamaraeva** On k-threshold functions
03:55 - 04:15	**A. Kisielewicz** A conjecture of Clote and Kranakis
04:20 - 04:40	**A. Burgess** On the Hamilton-Waterloo Problem with odd orders

Wednesday

08:50 - 09:00	Welcome, `MS.02` **Vadim Lozin**
09:00 - 09:55	**Nik Ruškuc** (Chair: Vadim Lozin), `MS.02` *Well quasi-order in combinatorics: embeddings and homomorphisms*
10:00 - 10:30	Coffee break
10:30 - 11:25	**Sergey Norin** (Chair: Oleg Pikhurko), `MS.02` *New tools and results in graph minor structure theory*
11:45 - 12:50	Lunch
12:50	Departure of buses to Warwick Castle
05:10	Departure of buses from Warwick Castle

Thursday

08:50 - 09:00	Welcome, `MS.02` **Vadim Lozin**
09:00 - 09:55	**Chaoping Xing** (Chair: Vadim Lozin), `MS.02` *Optimal rate algebraic list decoding of folded algebraic geometry codes*
10:00 - 10:30	Coffee break
10:30 - 12:05	Contributed talks (4 slots) in 4 parallel sessions Rooms `MS.02`, `MS.03`, `MS.04`, `B3.02`
12:05 - 02:00	Lunch
02:00 - 02:55	**Manuel Bodirsky** (Chair: Agelos Georgakopoulos), `MS.02` *Ramsey Classes: Examples and Constructions*
03:00 - 03:30	Coffee break
03:30 - 05:30	Contributed talks (5 slots) in 4 parallel sessions Rooms `MS.02`, `MS.03`, `MS.04`, `B3.02`
05:35- 06:00	Open problem session 2 (Chair: Peter Cameron), `MS.02`
07:00 - ...	Conference Dinner, Panorama Suite

Thursday morning contributed talks

Chair: J. van den Heuvel, `MS.02`

10:30 - 10:50	**P. Allen** Almost graceful labelling
10:55 - 11:15	**B. Walczak** Asymmetric coloring games on incomparability graphs
11:20 - 11:40	**J. Przybyło** Distant extensions of locally irregular graph colourings
11:45 - 12:05	**S. Legay** The Problem of Tropical Homomorphism in vertex-colored graphs

Chair: K. Tyros, `MS.03`

10:30 - 10:50	**D. Christofides** Diameters of random Cayley graphs
10:55 - 11:15	**M. Conder** Classifying vertex-transitive graphs by their arc-type
11:20 - 11:40	**U. Ahmad** The study of iteration digraphs defined on finite groups
11:45 - 12:05	**S. Pavlíková** New constructions of strongly invertible graphs

Chair: E. Csóka, `MS.04`

10:30 - 10:50	**Y. H. Chen** The incremental network designs of the bottleneck problems
10:55 - 11:15	**J. A. Ellis-Monaghan** Combinatorial strategies for self-assembly
11:20 - 11:40	**R. Xu** Weak safe sets and relaxed safe set
11:45 - 12:05	**P. Ó Catháin** Trades in Hadamard matrices

Chair: F. Joos, `B3.02`

10:30 - 10:50	**K. Gunderson** Friendship hypergraphs
10:55 - 11:15	**I. Moffatt** Ribbon graphs and their minors
11:20 - 11:40	**R. Kwashira** Graph compositions of suspended uniform four Combs
11:45 - 12:05	**S. M. Smith** A new product for permutation groups

Thursday afternoon contributed talks

Chair: R. Mycroft, `MS.02`

03:30 - 03:50	**Ł. Grabowski** Towards a measurable version of Lovász Local Lemma
03:55 - 04:15	**A. Treglown** A random version of Sperner's theorem
04:20 - 04:40	**D. Donovan** Parameter space coverage using Latin Hypercube Sampling
04:45 - 05:05	**F. Skerman** Modularity phase transition in Erdős-Renyi random graphs
05:10 - 05:30	**L. Montero** Tight lower bounds on the number of bicliques in false-twin-free graphs

Chair: S. R. Blackburn, `MS.03`

03:30 - 03:50	**T. S. Griggs** Combinatorics of the sonnet
03:55 - 04:15	**I. Makhlin** A combinatorial formula for affine Hall-Littlewood functions via a weighted Brion Theorem
04:20 - 04:40	**R. Scheidweiler** Ehrhart polynomials and the Erdős multiplication table problem
04:45 - 05:05	**K. Morgan** Certificates for graph polynomials

Chair: D. Christofides, `MS.04`

03:30 - 03:50	**D. Horsley** Alspach's cycle decomposition problem for multigraphs
03:55 - 04:15	**R. Hoyte** Cycle decompositions of the complete graph with a hole
04:20 - 04:40	**E. G. Mphako-Banda** Graph compositions: Some 2-edge connected graphs and some upper bounds
04:45 - 05:05	**M. Merker** Decomposing highly edge-connected graphs into trees of small diameter
05:10 - 05:30	**K. Ehsani** Orientation of graphs with prescribed in-degrees and out-degrees

Chair: J. Przybyło, `B3.02`

03:30 - 03:50	**T. Perrett** A zero-free interval for chromatic polynomials of graphs with 3-leaf spanning trees
03:55 - 04:15	**D. Quiroz** On the chromatic number of exact distance graphs
04:20 - 04:40	**H. Galeana-Sanchez** A characterization of panchromatic patterns
04:45 - 05:05	**C. Merino** The heterochromatic number of hypergraphs coming from matroid structures
05:10 - 05:30	**R. Lang** The list chromatic index of graphs of tree-width 3 and maximum degree at least 7

Friday

08:50 - 09:00	Welcome, `MS.02` **Vadim Lozin**
09:00 - 09:55	**Gary McGuire** (Chair: Vadim Lozin), `MS.02` *Curves over finite fields and linear recurring sequences*
10:00 - 10:30	Coffee break
10:30 - 12:05	Contributed talks (4 slots) in 4 parallel sessions Rooms `MS.02`, `MS.03`, `MS.04`, `B3.02`
12:05 - 02:00	Lunch
02:00	End of Conference

Friday morning contributed talks

Chair: P. J. Cameron, `MS.02`

10:30 - 10:50	**D. B. Penman** Comparable pairs and linear extensions in partially ordered sets
10:55 - 11:15	**J. B. Fawcett** Locally triangular graphs and rectagraphs with symmetry
11:20 - 11:40	**M. Ellingham** Link graphs and an unexpected application of topological graph theory
11:45 - 12:05	**A. Bishnoi** The Alon-Füredi bound

Chair: F. Skerman, `MS.03`

10:30 - 10:50	**J. Haslegrave** Searching for a hidden moving target
10:55 - 11:15	**N. Georgiou** Hat guessing on graphs: solvability and criticality
11:20 - 11:40	**G. E. Farr** The probabilistic method meets Go
11:45 - 12:05	**W. E. Tan** Probabilistic Intuition in Waiter–Client and Client–Waiter games

Chair: D. Bevan, `MS.04`

10:30 - 10:50	**M. Lee** Relative m-covers of generalised quadrangles
10:55 - 11:15	**M. Tannock** Pattern avoidance and non-crossing subgraphs of polygons
11:20 - 11:40	**T. Popiel** Point-primitive generalised polygons
11:45 - 12:05	**D. V. Chopra** On the existence and the maximum number of constraints for some combinatorial arrays

Chair: S. Noble, `B3.02`

10:30 - 10:50	**N. E. Clarke** Skolem labellings of generalized Dutch windmills
10:55 - 11:15	**P. Gordinowicz** Cops, robbers and ordinals
11:20 - 11:40	**J. Foniok** Applications of adjoint functors in graph theory
11:45 - 12:05	**A. Castillo-Ramirez** Memoryless computation and universal simulation

Abstracts of plenary talks

Ramsey Classes: Examples and Constructions

Manuel Bodirsky

This article is concerned with classes of relational structures that are closed under taking substructures and isomorphism, that have the joint embedding property, and that furthermore have the *Ramsey property*, a strong combinatorial property which resembles the statement of Ramsey's classic theorem. Such classes of structures have been called *Ramsey classes*. Nešetřil and Rödl showed that they have the *amalgamation property*, and therefore each such class has a homogeneous Fraïssé limit. Ramsey classes have recently attracted attention due to a surprising link with the notion of extreme amenability from topological dynamics. Other applications of Ramsey classes include reduct classification of homogeneous structures.

We give a survey of the various fundamental Ramsey classes and their (often tricky) combinatorial proofs, and about various methods to derive new Ramsey classes from known Ramsey classes. Finally, we state open problems related to a potential classification of Ramsey classes.

manuel.bodirsky@tu-dresden.de

Institut für Algebra, TU Dresden

Recent developments in graph Ramsey theory

David Conlon

joint work with J. Fox and B. Sudakov

Given a graph H, the Ramsey number $r(H)$ is the smallest natural number N such that any two-colouring of the edges of K_N contains a monochromatic copy of H. The existence of these numbers has been known since 1930 but their quantitative behaviour is still not well understood. Even so, there has been a great deal of recent progress on the study of Ramsey numbers and their variants, spurred on by the many advances across extremal combinatorics. In this survey, we will describe some of this progress.

david.conlon@maths.ox.ac.uk

Mathematical Institute, University of Oxford

Controllability and matchings in random bipartite graphs

Stefanie Gerke

joint work with P. Balister

Motivated by an application in controllability we consider maximum matchings in random bipartite graphs $G = (A, B)$. First we analyse Karp–Sipser's algorithm to determine the asymptotic size of maximum matchings in random bipartite graphs with a fixed degree distribution. We then allow an adversary to delete one edge adjacent to every vertex in A in the more restricted model where each vertex in A chooses d neighbours uniformly at random from B.

stefanie.gerke@rhul.ac.uk

Mathematics Department, Royal Holloway University of London

Some old and new problems in combinatorial geometry I: Around Borsuk's problem

Gil Kalai

Borsuk [1] asked in 1933 if every set of diameter 1 in $\mathbf{R}^d$ can be covered by $d + 1$ sets of smaller diameter. In 1993, a negative solution, based on a theorem by Frankl and Wilson [2], was given by Kahn and Kalai [3]. In this paper I will present questions related to Borsuk's problem.

References.

[1] K. Borsuk and Drei Sätze über die n-dimensionale euklidische Sphäre, *Fund. Math.* **20** (1933), 177–190.
[2] P. Frankl and R. Wilson, Intersection theorems with geometric consequences, *Combinatorica* **1** (1981), 259–286.
[3] J. Kahn and G. Kalai, A counterexample to Borsuk's conjecture, *Bull. Amer. Math. Soc.* **29** (1993), 60–62.

kalai@math.huji.ac.il

Einstein Institute of Mathematics, Hebrew University of Jerusalem, and Department of Mathematics, Yale University

Randomly generated groups

Tomasz Łuczak

We discuss some older and a few recent results related to randomly generated groups. Although most of them are of topological and geometric flavour the main aim of this work is to present them in combinatorial settings.

tomasz@amu.edu.pl

Faculty of Mathematics & Computer Science, Adam Mickiewicz University

Curves over finite fields and linear recurring sequences

Gary McGuire

joint work with O. Ahmadi

We investigate what happens when we apply the theory of linear recurring sequences to certain sequences that arise from curves over finite fields. The sequences we will study are $a_n := \#C(\mathbb{F}_{q^n}) - (q^n + 1)$ where $\#C(\mathbb{F}_{q^n})$ is the number of $\mathbb{F}_{q^n}$-rational points on a curve C defined over $\mathbb{F}_q$.

gary.mcguire@ucd.ie

School of Mathematical Sciences, University College Dublin

New tools and results in graph minor structure theory

Sergey Norin

Graph minor theory of Robertson and Seymour is a far reaching generalization of the classical Kuratowski–Wagner theorem, which characterizes planar graphs in terms of forbidden minors. We survey new structural tools and results in the theory, concentrating on the structure of large t-connected graphs, which do not contain the complete graph K_t as a minor.

snorin@math.mcgill.ca

Department of Mathematics & Statistics, McGill University

Well quasi-order in combinatorics: embeddings and homomorphisms

Nik Ruškuc

The notion of well quasi-order (wqo) from the theory of ordered sets often arises naturally in contexts where one deals with infinite collections of structures which can somehow be compared, and it then represents a useful discriminator between 'tame' and 'wild' such classes. In this article we survey such situations within combinatorics, and attempt to identify promising directions for further research. We argue that these are intimately linked with a more systematic and detailed study of homomorphisms in combinatorics.

nik.ruskuc@st-andrews.ac.uk

School of Mathematics and Statistics, University of St Andrews

Optimal rate algebraic list decoding of folded algebraic geometry codes

Chaoping Xing

We give a construction of folded algebraic geometry codes which are list decodable from a fraction $1 - R - \epsilon$ of adversarial errors, where R is the rate of the code, for any desired positive constant ϵ.

By using explicit towers, we obtain folded algebraic geometry code with the worst-case list size output $O(1/\epsilon)$, matching the existential bound for random codes up to constant factors. Further, the alphabet size of the codes is a constant depending only on ϵ — it can be made $\exp(\tilde{O}(1/\epsilon^2))$ which is not much worse than the lower bound of $\exp(\Omega(1/\epsilon))$. The parameters we achieve are thus quite close to the existential bounds in all three aspects — error-correction radius, alphabet size, and list-size — simultaneously. Our code construction is Monte Carlo and has the claimed list decoding property with high probability. Once the code is (efficiently) sampled, the encoding/decoding algorithms are deterministic with a running time $O_\epsilon(N^c)$ for an absolute constant c, where N is the code's block length.

By using class field towers, we obtain folded algebraic geometry code with the worst-case list size output $O(poly(N))$. The alphabet size of the codes is a constant depending only on ϵ. Our code construction is deterministic. Once the code is efficiently encoded, decoding algorithms are efficiently as well.

xingcp@ntu.edu.sg

Division of Mathematical Sciences, School of Physical & Mathematical Sciences, Nanyang Technological University

Abstracts of contributed talks

The study of iteration digraphs defined on finite groups

U.Ahmad

joint work with M.Moeen

MSC2010 20A05, 20K01, 20K25 05C20

A diagraph is attached with a finite group by utilizing the power map $f : G \to G$ defined by $f(x) = x^k \ \forall\ x \in G$, where k is a fixed natural number and is denoted by $\gamma_G(n, k)$. In this talk, the digraphs associated with generalized quaternions and 2-groups are studied. The basic structure including in-degree, height structure and semi-regularity is discussed for generalized quaternion. Further, the 2-groups are classified with the help of these digraphs.

References.

[1] Min Sha, Digraphs from endomorphisms of finite cyclic groups, *Journal of combinatorial mathematics and combinatorial computing* **83** (2012) 105-120.
[2] L. Somer and M.K*řížek* , On symmetric digraphs of the congruence $x^k \equiv y(modn)$,*Discrete Mathematics* (309) (2009), 1999- 2009.
[3] U. Ahmad , The Power Digraphs of Abelian Groups,*Utilitas Mathematica*(to appear).

uzma.math@pu.edu.pk

Department of Mathematics, University of the Punjab, Lahore, Pakistan

Almost graceful labelling

Peter Allen

joint work with Anna Adamaszek, Michał Adamaszek, Codruţ Grosu and Jan Hladký

05C78, 60G50

Given a graph G, let $\psi : V(G) \to \mathbb{N}$ be an injective vertex labelling. We say $\big|\psi(u) - \psi(v)\big|$ is the *induced label* given to the edge uv of G. Then ψ is a *graceful labelling* of G if the edges of G receive distinct induced labels, and the largest vertex label used is $\max\big(v(G), e(G)+1\big)$. Trivially this many vertex labels must be used, and the question is, given G, whether it is possible to find a graceful labelling, and if not how large the vertex labels must be.

Graceful labellings were introduced by Rosa (though the name was given by Golomb) in the study of *graph packings*, where one wishes to find as many edge-disjoint copies of G in K_t as possible: a graceful labelling of G provides a way to pack copies of G into K_t 'cyclically', and in some cases this can even give a perfect packing of G-copies into K_t, covering all edges.

The most famous conjecture in the area is the Graceful Tree Conjecture, which states that every tree has a graceful labelling and which implies that for any n-vertex tree T we can perfectly pack K_{2n+1} with copies of T. It has been proved only for a few very special classes of trees. We show that for each Δ and $\varepsilon > 0$, if n is sufficiently large then for any n-vertex tree with maximum degree Δ there is a labelling with distinct vertex labels, inducing distinct edge labels, and with the largest label at most $(1+\varepsilon)n$. This implies that we can almost perfectly pack $K_{(2+\varepsilon)n+1}$ with copies of T. I will sketch the proof and discuss extensions to other graph classes.

p.d.allen@lse.ac.uk

Department of Mathematics, London School of Economics

Exponential Domination Number of Cycle Related Graphs

Betül ATAY

joint work with Aysun AYTAÇ

MSC2010, 05C69, 68M10, 68R10

The assessment of the stability and the vulnerability of complex networks is the important concept while studying of them. In the literature, a sort of measures have been given in order to assessment the stability of systems. And also, some graph-theoretic parameters have been formed to provide formulas which calculate the reliability of a network. In this paper, we study the vulnerability of cycles and related graphs to the dominating strategy of a vertex, using a measure called exponential domination number which is a new characteristic for graph vulnerability.

betul.atay@ege.edu.tr

Department of Mathematics, Faculty of Science, Ege University 35100, İzmir/Turkey

Approaching the Moore bound for diameter 3 by Cayley graphs

M. Bachratý

joint work with J. Šiagiová, J. Širáň

05C (Graph Theory)

The largest order $n(d, k)$ of a graph of maximum degree d and diameter k cannot exceed the Moore bound, which has the form $M(d, k) = d^k - O(d^{k-1})$ for $d \to \infty$ and any fixed k. Known results in finite geometries on generalised $(k+1)$-gons imply, for $k = 2, 3, 5$, the existence of an infinite sequence of values of d such that $n(d, k) = d^k - o(d^k)$. This shows that for $k = 2, 3, 5$ the Moore bound can be asymptotically approached in the sense that $n(d, k)/M(d, k) \to 1$ as $d \to \infty$; moreover, no such result is known for any other value of $k \geq 2$. The corresponding graphs are, however, far from vertex-transitive, and there appears to be no obvious way to extend them to vertex-transitive graphs giving the same type of asymptotic result.

The second and the third author (2012) proved by a direct construction that the Moore bound for diameter $k = 2$ can be asymptotically approached by Cayley graphs. Subsequently, the first and the third author (2015) showed that the same construction can be derived from generalised triangles with polarity.

By a detailed analysis of regular orbits of suitable groups of automorphisms of graphs arising from polarity quotients of incidence graphs of generalised quadrangles with polarity, we prove that for an infinite set of values of d there exist Cayley graphs of degree d, diameter 3, and order $d^3 - O(d^{2.5})$. The Moore bound for diameter 3 can thus as well be asymptotically approached by Cayley graphs. We also show that this method does not extend to constructing Cayley graphs of diameter 5 from generalised hexagons with polarity.

mato4247@gmail.com

Faculty of Mathematics, Physics and Informatics, Comenius University

On the enumeration of integer tuples having the same lcm

O. Bagdasar

MSC2010: 05A15, 11Y55, 11B73

Second order recurrences depending on arbitrary complex initial values and recurrence coefficients may be periodic. The function enumerating all self-repeating sequences of given length, involved well-known functions such as Euler's totient function φ and the number of divisors function ω [1].

In general, the periodic linear recurrent sequences of arbitrary order in the complex plane have been characterised in [2]. The enumeration formula for sequences of fixed length involves functions counting the (strictly) increasing tuples of positive integers, having the same least common multiple (lcm).

Here, such functions are expressed as linear combinations of the number of ordered tuples having the same lcm obtained in [3], with coefficients involving the Stirling numbers of second kind [4]. The investigation is linked to novel integer sequences in the OEIS [5], such as A245019, A247517 or A247516.

References.

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`o.bagdasar@derby.ac.uk`

Department of Computing and Mathematics, University of Derby, UK
Faculty of Mathematics and Computer Science, Babeş-Bolyai University, Romania

On the metric dimension of imprimitive distance-regular graphs, II

Robert F. Bailey

05E30, 05C12

The metric dimension of a graph Γ is the least size of a set of vertices $\{v_1, \ldots, v_m\}$ with the property that, for any vertex w, the list of distances from w to each of $v_1, \ldots, v_m$ uniquely identifies w.

In this talk, we continue the study of the metric dimension of imprimitive distance-regular graphs, which I spoke about at the 2013 BCC. Such graphs are either bipartite or antipodal (i.e. being at distance 0 or d is an equivalence relation on the vertices, where d is the diameter). We use a theorem of Alfuraidan and Hall, along with the operations of "halving" and "folding", to reduce the problem to primitive graphs, where known results of Babai apply, except when the diameter is between 3 and 6. In this case, especially when the diameter is 3, we see that unexpected things start to happen.

rbailey@grenfell.mun.ca

Grenfell Campus, Memorial University of Newfoundland

Avoiding a Pair of Vincular and Covincular Patterns

C. Bean

joint work with A. Claesson and H. Ulfarsson

05A05

Babson and Steingrímsson [1] introduced a generalisation of classical permutation patterns, called *vincular patterns* allowing the extra requirement that two letters must be adjacent in the permutation. This was further extended by Bousquet-Mélou *et al.* [2] with *bivincular* patterns. In particular we are interested in what we will call *covincular patterns*, a rotation of the vincular condition, that allows the additional requirement that two letters in our pattern that differ by 1 must also differ by 1 in the permutation.

Simultaneous avoidance of two vincular patterns was looked at by Claesson and Mansour [3]. Allowing one of the patterns to be covincular is a natural follow up question and leads to some interesting results. In particular we look at the number of length n permutations that simultaneously avoid a length 3 vincular and a length 3 covincular pattern.

We see familiar sequences, such as the Catalan and Motzkin numbers, but also some previously unknown sequences which have close links to other combinatorial objects such as ascent sequences, lattice paths and partitions. Where possible we include a generating function for the enumeration which in some cases uses techniques involving q-binomials. We also show an alternative proof that the avoiders of 123 are counted by the Catalan numbers.

References.

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christianbean@ru.is

Reykjavik University

The growth of permutations avoiding 1324

David Bevan

Permutations, Asymptotic enumeration

The enumeration of the class of permutations avoiding the pattern 1324 is notorious for its difficulty. Even the asymptotic growth rate of the 1324-avoiders is currently unknown, although it has been estimated to be close to 11.6 based on numerical analysis of the number of permutations in the class up to length 36. However, rigorous bounds still differ quite markedly from this value. In this talk we outline a proof of a new lower bound of 9.81 for the growth rate, based on an analysis of the structure of certain large subsets of the class.

Key to the proof is the fact that often the distribution of a parameter counting the proportion of a particular substructure in a combinatorial class is concentrated at its limiting mean. In this context, we consider occurrences of patterns in Łukasiewicz paths and establish that in the limit they exhibit a concentrated Gaussian distribution.

David.Bevan@open.ac.uk

Department of Mathematics and Statistics, The Open University

The Alon-Füredi bound

Anurag Bishnoi

joint work with P. L. Clark, A. Potukuchi and J. R. Schmitt

11T55, 11T71, 13B25

In this talk, I will prove an upper bound on the number of zeroes of a multivariate polynomial in a finite grid, assuming that it does not vanish on all points of the grid. A 1993 theorem of Alon and Füredi [1, Theorem 5] gives such a bound in terms of the degree of the polynomial, and in fact the bound is sharp. Our result generalises the Alon-Füredi theorem by taking into account the degree of the polynomial in each variable as well as the total degree. Moreover, we do not assume that the coefficients of the polynomial are from a field, a commutative ring with some additional restrictions on the grid suffice.

I will also show that the theorem of Alon and Füredi can be used to determine the minimum distance of affine cartesian codes, which are a generalisation of the Reed-Muller codes. From this we obtain the well known Schwartz-Zippel lemma as a special case. In fact, more general versions of these results will follow from our bound. Some other applications of the the Alon-Füredi bound and our generalisation will also be discussed, including a classical theorem by Warning which states that *a polynomial* $P \in \mathbb{F}_q[t_1, \ldots, t_n]$ *of degree* $d < n$ *either has no zeroes in* $\mathbb{F}_q^n$ *or has at least* q^{n-d} *zeroes.*

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anurag.2357@gmail.com

Research Group Incidence Geometry, Ghent University

Probabilistic existence results for separable codes

Simon R. Blackburn

94A62, 94A60

Separable codes were defined by Cheng and Miao (IEEE Trans IT, 2011) motivated by applications to the identification of pirates in a multimedia setting. Combinatorially, $\bar{t}$-separable codes lie somewhere between t-frameproof and $(t-1)$-frameproof codes: all t-frameproof codes are $\bar{t}$-separable, and all $\bar{t}$-separable codes are $(t-1)$-frameproof. Results for frameproof codes show that (when q is large) there are q-ary $\bar{t}$-separable codes of length n with approximately $q^{n/t}$ codewords, and that no q-ary $\bar{t}$-separable codes of length n can have more than approximately $q^{\lceil n/(t-1)\rceil}$ codewords.

The talk will outline improved probabilistic existence results for $\bar{t}$-separable codes when $t \geq 3$. More precisely, for all $t \geq 3$ and all $n \geq 3$, there exists a constant κ (depending only on t and n) such that there exists a q-ary $\bar{t}$-separable code of length n with at least $\kappa q^{n/(t-1)}$ codewords for all sufficiently large integers q. This shows, in particular, that the upper bound (derived from the bound on $(t-1)$-frameproof codes) on the number of codewords in a $\bar{t}$-separable code is realistic.

The results above are more surprising after examining the situation when $t = 2$. Results due to Gao and Ge (IEEE Trans IT, 2014) show that an optimal q-ary $\overline{2}$-separable code of length n contains about $q^{2n/3}$ codewords when q is large. Thus optimal $\overline{2}$-separable codes behave neither like 2-frameproof nor 1-frameproof codes.

s.blackburn@rhul.ac.uk

Department of Mathematics, Royal Holloway University of London

Cross-intersecting families

Peter Borg

05D05

Extremal set theory is the study of how small or how large a system of sets can be under certain conditions. A problem in this field that has recently attracted much attention is that of determining the maximum sum or the maximum product of sizes of $k \geq 2$ *cross-t-intersecting* subfamilies of a given family $\mathcal{F}$ of sets; families $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_k$ are said to be cross-t-intersecting if for every i and j in $\{1, 2, \ldots, k\}$ with $i \neq j$, each set in $\mathcal{A}_i$ intersects each set in $\mathcal{A}_j$ in at least t elements. Solutions have been obtained for families $\mathcal{F}$ in general [1] and also for some particular families, such as power sets, levels of power sets, hereditary families, families of permutations, and families of integer sequences. Various observations and results in this area will be presented.

References.

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peter.borg@um.edu.mt

Department of Mathematics, University of Malta

On reliability of the generalized Petersen graphs

Gülnaz Boruzanlı

joint work with John Baptist Gauci

MSC2010, 05C40

The connectivity $\kappa(G)$ and the edge–connectivity $\lambda(G)$ of a connected graph G are important measures for the reliability of graphs, and they give the cardinality of the minimum vertex–cut and the minimum edge–cut, respectively. If every minimum vertex–cut (respectively, edge–cut) isolates a vertex, then the graph G is said to be super connected (respectively, super edge–connected). We show that the generalized Petersen graph $GP[n, k]$ is super connected and super edge–connected for $n \geq 3$, $k \geq 1$ and $n \neq 3k$. The super connectivity (respectively, super edge–connectivity) of a connected graph G is the minimum number of vertices (respectively, edges) that need to be deleted from G in order to disconnect G without creating any isolated vertices. These two measures are denoted by $\kappa'(G)$ and $\lambda'(G)$, respectively. We prove that $\kappa'(GP[n, k]) = \lambda'(GP[n, k]) = 4$ for $n \geq 3$, $k \geq 1$ and $n \neq 3k$. Since the girth of the generalized Petersen graphs is small (at most eight), this class of graphs are good candidates for interconnection networks, and, hence, these results provide more accurate measures for the reliability of such networks.

gulnaz.boruzanli@ege.edu.tr

Department of Mathematics, Ege University

Chromatic thresholds in random graphs

Julia Böttcher

joint work with Peter Allen, Simon Griffiths, Rob Morris, Yoshiharu Kohayakawa, Barnaby Roberts

05C35,05C15,05C80

There has been a recent trend in Combinatorics towards proving 'random analogues' of extremal results in Graph Theory and Additive Number Theory. We are interested in random analogues of the chromatic threshold, which has been completely determined for all graphs only very recently. It turns out that chromatic thresholds and their natural analogues in random graphs behave (non-trivially) differently for certain ranges of the edge probability p. However, the original behaviour can be recovered if we are allowed to delete a few edges from the random graph. In the talk I will present a number of results in this direction. These leave many gaps and open questions, which I will mention as well.

julia.boettcher@lse.ac.uk

Department of Mathematics, London School of Economics

On the Hamilton-Waterloo Problem with odd orders

Andrea Burgess

joint work with Peter Danziger and Tommaso Traetta

MSC2010: 05C70, 05B30, 05C38

The Hamilton-Waterloo problem HWP$(K_v; m, n; \alpha, \beta)$ asks whether there exists a 2-factorization of the complete graph K_v into α m-cycle factors and β n-cycle factors, where $3 \leq m \leq n$. Necessarily, if such a factorization exists, then m, n and v are all odd, m and n are both divisors of v, and $\alpha + \beta = (v-1)/2$. We show that these necessary conditions are sufficient whenever $m \geq 5$, v is a multiple of mn and $v > mn$, except possibly if $\beta \in \{1, 3\}$. For $m \geq 5$ and $v = mn$, we solve the problem whenever $\beta > (n+5)/2$, except possibly if $(v, m, n, \alpha, \beta) = (35, 5, 7, 9, 8)$. Similar results are obtained when $m = 3$, but with a few extra possible exceptions in this case.

andrea.burgess@unb.ca

University of New Brunswick

Remembering Donald Preece

Peter J. Cameron

05-03, 11T99

Donald Preece died on 6 January 2014. He was a familiar presence at BCCs for many years, especially in the role of organiser of the conference concert. But the BCC owes him more than this: in 1999, where the expected venue for the BCC was withdrawn at short notice, Donald stepped in, and with John Lamb's help, ran a successful conference at the University of Kent.

Donald's life contained more than just mathematics, as the extended obituary [1] by Rosemary Bailey makes clear. I will say something about this, and also describe the work he and I did during his last years at Queen Mary, University of London, on primitive lambda-roots [2].

References.

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pjc20@st-andrews.ac.uk

School of Mathematics and Statistics, University of St Andrews

Memoryless Computation and Universal Simulation

A. Castillo-Ramirez

joint work with M. Gadouleau

MSC2010: 20M20, 68Q10, 05E18.

Consider a finite set A and an integer $n \geq 2$. *Memoryless computation* is the study of *instructions* of A^n (i.e. transformations of A^n with at most one nontrivial coordinate function) and the semigroups of transformations that they generate. This model of computation has been recently revitalised by several new results, such as the proof that the full transformation semigroup of A^n may be generated by just $n+1$ instructions (see [1]).

In this talk, which is based in [2], we will introduce the concept of *simulation* as a way of computing a transformation of A^n using $m \geq n$ instructions that may depend on $m-n$ additional coordinates. A transformation of A^m is *n-universal of size m* if the instructions induced by its coordinate functions may simulate any transformation of A^n. We will establish that there is no n-universal transformation of size n, but there is one of size $n+2$. We will also introduce the notions of *sequential*, *parallel* and *quasi-parallel simulation*, and, using combinatorial techniques, we will give bounds for the size and time complexity for each case.

References.

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alonso.castillo-ramirez@durham.ac.uk

School of Engineering and Computing Sciences, Durham University

Which Latin square is the loneliest?

N. Cavenagh

joint work with R. Ramadurai

05B15

Given two Latin squares L_1 and L_2 of the same order, the *hamming distance* between L_1 and L_2 gives the number of corresponding cells containing distinct symbols. If we think of Latin squares as sets of ordered triples, this is given by $|L_1 \setminus L_2|$. Given a specific Latin square L_1, we may wish to know a Latin square L_2 which is closest to it; i.e. for which the Hamming distance is minimized. Equivalently, we may ask for the size of the smallest Latin trade within a given latin square.

It is known that the back circulant Latin square of order n (the operation table for the integers modulo n) has Hamming distance at least $e \log n + 2$ to any other Latin square. We explore whether the back circulant Latin square is the loneliest of all Latin squares; i.e. has greatest minimum Hamming distance to any other Latin square.

nickc@waikato.ac.nz

University of Waikato

The incremental network designs of the bottleneck problems

Yen Hung Chen

68Q25; 68Q17; 68W25; 68M10; 90C27

Given a graph $G = (V, E)$ with nonnegative edge lengths, a set $E_0 \subset E$, a number of time periods $h = |E \setminus E_0|$, and a network optimization problem P, we let $OP(G)$ be the cost of the optimal solution to the network optimization problem P for G. The class of incremental network design problems (INDP) [1] is to find a sequence of edge sets $E_0 \subset E_1 \subset \ldots \subset E_h = E$ with $|E_i \setminus E_{i-1}| = 1$ such that the cumulative cost of the optimal solution of P over all time periods is minimized, i.e., $Min \sum_{i=1}^{h} OP(G_i)$, where $G_i = (V, E_i)$. Motivating applications of the INDP include the infrastructure restorations after a disruption of extreme event and network upgrades and expansions when services increase in demand. Given a graph $G = (V, E)$ with nonnegative edge lengths, a subset $R \subset V$, the bottleneck Steiner tree problem (BSTP) is to find an acyclic subgraph of G interconnecting all vertices in R such that the length of the largest edge is minimized. If $R = \{s, d\}$ contains only two vertices, the minmax path problem (MMPP) is to find a path of G from s to d such that the length of the largest edge is minimized and the shortest path problem (SPP) is to find a path of G from s to d such that the sum of the lengths of all its edges is minimized. The INDP of SPP (i.e., the network optimization problem P for the INDP is the SPP) was shown to be NP-complete and existed a 4-approximation algorithm. In this paper, we focus on the INDP of BSTP and MMPP, called as the incremental bottleneck Steiner tree problem (IBSTP) and the incremental minmax path problem (IMMPP), respectively. We show that there is no polynomial time approximation algorithm achieving a performance ratio of $(1 - \epsilon) * ln|V|$, $0 < \epsilon < 1$, for the IBSTP unless $NP \subseteq DTIME(|V|^{\log\log|V|})$. Then we show that the IMMPP is NP-complete and present the first known approximation algorithm with performance ratio of 4 for the IMMPP.

References.

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yhchen@utaipei.edu.tw

Department of Computer Science, University of Taipei, Taipei, Taiwan

On the Existence and the Maximum Number of Constraints for Some Combinatorial Arrays

D.V. Chopra

joint work with R.M. Low

MSC2010 05B15

The combinatorial arrays we consider are called balanced arrays (B-arrays), which are generalizations of orthogonal arrays. A balanced array T with m rows, N columns, with two symbols and of strength t $(0 \leq t \leq m)$, is an $(m \times N)$-matrix T with two elements (say, 0 and 1) such that in every $(t \times N)$-submatrix T^* of T, we have the following combinatorial condition satisfied: Every $(t \times 1)$-column vector $\underline{\alpha}$ (with i 1s in it) appears with the same frequency (say μ_i, $0 \leq i \leq t$). The vector $\underline{\mu}' = (\mu_0, \mu_1, \ldots, \mu_t)$ is called the index set of the array T, and the number of rows and $\underline{\mu}'$ constitute the parameters of T. Clearly, $N = \sum_{i=0}^{t} \binom{t}{i} \mu_i$. If $\mu_i = \mu$ for each i, then we obtain an orthogonal array (and $N = 2^t \mu$). In this talk, we focus on strength $t = 10$ and derive some existence conditions for these B-arrays. These conditions are then used to obtain $\max(m)$, for a given N. In addition, we present some open problems.

dharam.chopra@wichita.edu

Department of Mathematics and Statistics, Wichita State University, Wichita, KS 67260-0033, USA

Diameters of random Cayley graphs

D. Christofides

joint work with K. Markström

05C80; 05C25; 05C12.

It is well-known that $p(n) = \sqrt{\dfrac{2 \log n}{n}}$ is a threshold for diameter 2 in the case of binomial random graphs.

In this talk we will investigate the threshold for diameter 2 for random Cayley graphs.

References.

[1] D. Christofides and K. Markström, The thresholds for diameter 2 in random Cayley graphs, *Random Structures Algorithms* **45** (2014) 218-235.
[2] D. Christofides and K. Markström, The range of thresholds for diameter 2 in random Cayley graphs, *European Journal Combinatorics* **35** (2014) 141-154.

dchristofides@uclan.ac.uk

UCLan Cyprus

Local connective chromatic number of a graph

Canan Çiftçi

joint work with Pınar Dündar

MSC2010, 05C15, 05C78

A local connective k-coloring of a graph G is a mapping $c : V(G) \longrightarrow \{1, 2, ..., k\}$ such that any non–adjacent two vertices u and v of a color i satisfies $\kappa(u, v) \geqslant i$, where $\kappa(u, v)$ is the maximum number of internally disjoint paths between u and v. The smallest integer k for which there exists a local connective k- coloring of G is called the local connective chromatic number of G, and is denoted $\chi_{lc}(G)$. In this paper, we introduce a new type of graph coloring. We determine the local connective chromatic number of some graphs and give some bounds on the local connective chromatic number.

canan.ciftci@ege.edu.tr

Department of Mathematics, Ege University

Skolem Labellings of Generalized Dutch Windmills

Nancy E. Clarke

joint work with Asiyeh Sanaei

05C78, 05B30

In this talk, we consider Skolem (vertex) labelling and present (hooked) Skolem labellings for generalized Dutch windmills whenever such labellings exist. Specifically, we show that generalized Dutch windmills with more than two cycles cannot be Skolem labelled and that those composed of two cycles of lengths m and n, $n \geq m$, cannot be Skolem labelled if and only if $n - m \equiv 3, 5 \pmod 8$ and m is odd. Showing that a Skolem labelling does not exist is, in general, a complex problem and we present a novel technique for doing so.

nancy.clarke@acadiau.ca

Department of Mathematics and Statistics, Acadia University, Wolfville, NS Canada

Clique-Width, Linear Clique-Width and Well-Quasi-Ordering

A. Collins

joint work with J. Foniok, N. Korpelainen, V. Lozin, V. Zamaraev

05C75

Clique-width is a graph parameter that is most useful for algorithmic graph theory because many difficult algorithmic problems become tractable when restricted to graphs of bounded clique-width. It is a generalisation of the notion of tree-width in the sense that graphs of bounded tree-width have bounded clique-width, but not necessarily vice versa. It was proved by Robertson and Seymour that in the family of minor-closed graph classes the planar graphs constitute a unique minimal class of graphs of unbounded tree-width. It is justified to restrict to minor-closed graph classes when studying tree-width because the tree-width of a graph is never smaller than the tree-width of any of its minors. This is not the case, however, with respect to clique-width, although the clique-width of a graph is never smaller than the clique-width of any of its induced subgraphs. Therefore, when we study clique-width we may restrict ourselves to hereditary graph classes, i.e. graph classes that are closed under taking induced subgraphs.

Not many minimal hereditary classes of unbounded clique-width are known. In this talk I will introduce an infinite family of classes of unbounded clique-width, infinitely many of which are minimal hereditary classes. I will then discuss well-quasi-ordering and describe a canonical antichain that is common to all of these minimal classes.

A.Collins.2@warwick.ac.uk

Mathematics Department, University of Warwick

Classifying vertex-transitive graphs by their arc-type

Marston Conder

(Joint work with with Tomaž Pisanski & Arjana Žitnik)

05E18, 20B25, 05C76

Locally-finite vertex-transitive graphs may be classified according to the action of the automorphism group on the arcs (ordered edges) of the graph. If X is a vertex-transitive graph of finite valency d, with full automorphism group A, then the *arc-type* of X is a graph invariant defined in terms of the lengths of the orbits of the action of the stabiliser A_v of a given vertex v on the set of arcs emanating from v. Specifically, the arc-type is the partition of d as the sum

$$n_1 + n_2 + \cdots + n_t + (m_1 + m_1) + (m_2 + m_2) + \cdots + (m_s + m_s),$$

where $n_1, n_2, \ldots, n_t$ are the lengths of the self-paired orbits, and $m_1, m_1, m_2, m_2, \ldots, m_s, m_s$ are the lengths of the non-self-paired orbits.

For example, if X is arc-transitive then its arc-type is d, while if X is half-arc-transitive then its arc-type is $(d/2 + d/2)$, and if X is zero-symmetric (or equivalently a graphical regular representation of the group A), then $n_i = 1$ and $m_j = 1$ for all i and j.

In this talk I will explain how it can be shown using Cartesian products of 'relatively prime' examples of certain kinds of VT graphs that every partition of the given form occurs as the arc-type of some vertex-transitive graph, with the exception of $1 + 1$ and $(1 + 1)$.

m.conder@auckland.ac.nz

Department of Mathematics, University of Auckland, New Zealand

Generalized solution for the Herman Protocol Conjecture

Endre Csóka

We have a cycle of N nodes and there is a token on an odd number of nodes. At each step, each token independently moves to its clockwise neighbour or stays at its position with probability $\frac{1}{2}$. If two tokens arrive to the same node, then we remove both of them. The process ends when only one token remains. The question is that for a fix N, which is the initial configuration that maximizes the expected number of steps $E(T)$. The Herman Protocol Conjecture says that the 3-token configuration with distances $\lfloor \frac{N}{3} \rfloor$ and $\lceil \frac{N}{3} \rceil$ maximizes $E(T)$. We present an elegant proof of this conjecture not only for $E(T)$ but also for $E\big(f(T)\big)$ for some functions $f : \mathbb{N} \to \mathbb{R}^+$, and we show that this applies for different generalizations of the problem.

csokaendre@gmail.com

Mathematics Institute and DIMAP, University of Warwick

Clique-width of Restricted Graph Classes

K. K. Dabrowski

joint work with A. Brandstädt, S. Huang and D. Paulusma

MSC2010 05C75 "Structural characterization of families of graphs"

The *clique-width* of a graph G, is the minimum number of labels needed to construct G using the following four operations:

- creating a new graph consisting of a single vertex v with label i;
- taking the disjoint union of two labelled graphs G_1 and G_2;
- joining each vertex with label i to each vertex with label j ($i \neq j$);
- renaming label i to j.

Clique-width is of great theoretical interest because many natural algorithmic problems that are NP-complete in general can be solved efficiently on graph classes of bounded clique-width. This includes all problems expressible in monadic second order logic with quantification over vertices, along with other problems such as vertex colouring and Hamiltonian cycle. Clique-width is a tricky parameter to deal with. Indeed, even for low values of c, such as $c = 4$, we do not know if graphs of clique-width c can be detected in polynomial time.

I will describe some of the tools available for dealing with clique-width and summarize our recent work on classifying which classes of graphs have bounded clique-width, in particular for: H-free graphs, H-free bipartite graphs, H-free split graphs, H-free chordal graphs, H-free weakly chordal graphs and (H_1, H_2)-free graphs.

konrad.dabrowski@durham.ac.uk

School of Engineering and Computing Sciences, Durham University

Covers of (pseudo)random graphs by monochromatic subgraphs

Louis DeBiasio

joint work with Deepak Bal

MSC2010 headings: 05C55, 05C38, 05C80

As a footnote in their 1967 paper, Gerencsér and Gyárfás proved that every 2-edge-coloring of K_n has vertex disjoint red and blue *paths* which cover all of the vertices. Later, Lehel conjectured that the same result should hold with *cycles* instead of paths. In 1991, Erdős, Gyárfás, and, Pyber began a systematic study of covers of r-edge-colored complete graphs by monochromatic cycles and trees. Finally, in 2008, after many partial results, Bessy and Thomassé settled Lehel's conjecture.

Recently, Schelp initiated a study of Ramsey-type problems where the host graph is not complete, but has sufficiently large minimum degree. Inspired by this, Balogh, Barat, Gerbner, Gyárfás, and Sárközy conjectured that sufficiently large graphs with minimum degree at least $3n/4$ have the Lehel property. DeBiasio and Nelsen proved that this holds asymptotically and then Letzter proved that it holds exactly.

We will discuss some new results on monochromatic coverings by cycles and trees in which the host graph is (pseudo)random. As an example of such a result, we determine the threshold for which $G_{n,p}$ has the Lehel property.

debiasld@miamioh.edu

Miami University, Oxford, OH

Parameter space coverage using Latin Hypercube Sampling

Diane Donovan

joint work with Kevin Burrage, Pamela Burrage, Thomas McCourt,
Bevan Thompson & Emine Yazici

05B99,05B15,62K99

In this talk I will discuss coverage of t dimensional subspaces of a d dimensional parameter space when performing k trials of Latin Hypercube sampling. The talk will be motivated by applications to Populations of Models and Quantification of Uncertainty. Results of simulations will be presented, with these results leading to the conjecture that the percentage coverage of the parameter space takes the form

$$P(k,n,d,t) = 1 - e^{-k/n^{t-1}},$$

where n represents the number of levels for each parameter. Counting arguments and analytical techniques will be used to validate this conjecture under certain conditions.

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dmd@maths.uq.edu.au

School of Mathematics and Physics,
The University of Queensland,
Brisbane, 4072, Australia

Orientation of Graphs with Prescribed In-Degrees and Out-Degrees

K. Ehsani

joint work with S. Akbari, K. Ozeki, M. Dalirrooyfard, R. Sherkati

05C20 05C21

Suppose that G is an undirected graph and $F(v) \subseteq \mathbb{N}$, for every $v \in V(G)$, then G is called F-avoiding if and only if there exists an orientation O such that $d_O^+(v) \notin F(v)$ for each $v \in V(G)$. Shirazi and Verstrate proved that if $G = (V, E)$ is an undirected graph and $|F(v)| \leq \frac{d_G(v)}{2}$, for every vertex v, then G has a spanning subgraph H such that $d_H(v) \notin F(v)$, for each $v \in V(G)$. In this talk we generalize this result to directed graphs by showing that if G is a bipartite graph and for every vertex v, $F(v)$ is a set of integers such that $|F(v)| \leq \frac{d_G(v)}{2}$, then there exists an orientation O for G such that $d_O^+(v) \notin F(v)$, for any $v \in V(G)$.

Let $f : V(G) \to \mathbb{N}$ be a function. A graph G is f-avoiding if and only if there exists an orientation O such that $d_O^+(v) \neq f(v)$, for each $v \in V(G)$. In this talk it is shown that every connected graph with minimum degree at least 3 is f-avoiding for every function f.

For a graph G, a set of vertices $S \subseteq V(G)$ and a given set of integers N, call G, (S, N)-controllable if and only if there exists an orientation for G such that for every function $f : S \to N$, $d^+(v) = f(v)$, for any $v \in S$. Among other results we also provide a necessary and sufficient condition for (S, N)-controllability of $(2r + 1)$-regular graphs, where $|S| = \frac{n}{2}$ and $N = \{1, 2r\}$ or $N = \{r, r + 1\}$.

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kehsani@ce.sharif.edu

Department of Computer Science and Engineering, Sharif University of Technology

Link graphs and an unexpected application of topological graph theory

Mark Ellingham

joint work with Bin Jia

05C76, 05C60, 05C10

A k-link in a graph is a walk of length k that never uses the same edge twice in succession; a link and its reverse are considered equal. For a given graph G, the k-link-graph $L_k(G)$ has as its vertices the k-links of G, where two k-links are adjacent if they are initial and final subsequences of the same $(k+1)$-link. This generalizes the idea of the line graph, which is the 1-link graph. A natural question is whether $L_k(G)$ uniquely determines the graph G. Whitney proved that $L_1(G)$ determines G for connected G, except in one small case. Xueliang Li showed that $L_2(G)$ determines G for G of minimum degree at least 3. We show that $L_k(G)$ determines G for G of minimum degree at least 3 for all $k \geq 2$. Somewhat surprisingly, part of the proof uses the classification of quadrangular embeddings of 4-regular graphs, which are always on the torus or Klein bottle.

mark.ellingham@vanderbilt.edu

Department of Mathematics, Vanderbilt University, USA

Combinatorial Strategies for Self-Assembly

J. A. Ellis-Monaghan

05C90, 92E10

Newly emerging technologies in self-assembly, both at the nanoscale (for example DNA origami), and the macroscale (for example robotic assembly), are currently generating challenging new design problems for which combinatorics is a natural tool. Often, the self-assembled objects, e.g. lattices or polyhedral skeletons, may be modeled as graphs. Thus, finding optimal design strategies leads to developing algorithms for graphs, addressing computational complexity questions, and finding new graph invariants corresponding to the minimum number of components necessary to build a target structure under various laboratory settings.

We will present recent applications in self-assembly, give results informing the applications, and, since these applications are a rich source of new research directions, describe problems and open questions.

New constructions for large circulant graphs of given degree and diameter

Grahame Erskine

Joint work with David Bevan and Rob Lewis

05C25,05C35

The degree-diameter problem seeks to find the largest possible order of a graph of given diameter and given maximum degree. Many restricted problems are typically studied, and in this talk I will focus on circulant graphs (equivalently, Cayley graphs of cyclic groups).

Most of the best constructions for diameter 2 known in the literature use the structure of finite fields as a basis. I will describe a similar type of construction which removes the need for finite fields and allows us to consider graphs of larger diameter. Using this construction it is possible to obtain asymptotic results for a range of diameters and degrees which are better than any known, both for undirected and directed graphs.

grahame.erskine@open.ac.uk

Department of Mathematics and Statistics, Open University, U.K.

Full subgraphs of a graph

V. Falgas-Ravry

joint work with K. Markström and J. Verstraëte

05C35, 05C70, 05D10

Let G be an n-vertex graph with edge-density p. Following Erdős, Łuczak and Spencer, an m-vertex subgraph H of G is called *full* if it has minimum degree at least $p(m-1)$. Let $f(G)$ denote the order of a largest full subgraph of G, and let $f_p(n)$ denote the minimum of $f(G)$ over all n-vertex graphs G with edge-density p.

We show that for $p:\ n^{-\frac{1}{3}} < p < 1-n^{-\frac{1}{5}}$, the function $f_p(n)$ is of order at least $(1-p)^{\frac{1}{3}}n^{\frac{2}{3}}$, improving on an earlier lower bound of Erdős, Łuczak and Spencer in the case $p = \frac{1}{2}$. Moreover we show that this bound is tight: for infinitely many p near the elements of $\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\}$ we have $f_p(n) = \theta(n^{\frac{2}{3}})$.

As an ingredient of the proof, we show that every graph G on n vertices has a subgraph H on m vertices with $\lfloor \frac{n}{r} \rfloor \leq m \leq \lceil \frac{n}{r} \rceil + 1$ such that for every vertex $v \in V(H)$ the degree of v in H is at least $\frac{1}{r}$ times its degree in G. Finally, we discuss full subgraphs of random and pseudorandom graphs, and introduce several open problems.

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`victor.falgas-ravry@vanderbilt.edu`

Department of Mathematics, Vanderbilt University

The probabilistic method meets Go

G. E. Farr

05C57, 05C80, 05C31

The game of Go — known as *Go* or *Igo* in Japan, *Wéiqí* in China and *Baduk* in Korea — is one of the oldest and deepest board games in the world, and has a huge following in east Asia.

Traditionally, the game involves placing black and white stones on the vertices of a square grid, usually with 19×19 vertices. But it can in fact be played on any graph G. A *position* is an assignment of colours from the set {Black, White} to some subset of $V(G)$. So each vertex is Black, White or Uncoloured. A *legal position* is one in which each monochromatic component is adjacent to some uncoloured vertex. (A *monochromatic component* is a component of the subgraph induced by the vertices of a specific colour.)

Among the most basic questions that can be asked about any game are:

- How many legal positions are there?
- What is the probability that a random arrangement of game elements (stones/pieces/…) on the board gives a legal position?

The author introduced *Go polynomials* to study these questions for Go [1].

In this talk, we apply probabilistic methods to study the asymptotic probability that a random Go position is legal when G is an $n \times n$ square grid, and also when $G \in \mathcal{G}(n;p)$ is an Erdős-Rényi random graph with edge probability p.

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Graham.Farr@monash.edu

Faculty of I.T., Monash University, Clayton, Victoria 3800, Australia

Locally triangular graphs and rectagraphs with symmetry

Joanna B. Fawcett

joint work with John Bamberg, Alice Devillers and Cheryl E. Praeger.

20B25, 05C75, 05E18

Locally triangular graphs are known to be halved graphs of bipartite rectagraphs, which are connected triangle-free graphs in which every 2-arc lies in a unique quadrangle. A graph is locally rank 3 if it has a group of automorphisms G such that for each vertex u, the permutation group induced by the vertex stabiliser of u in G on the neighbourhood of u is transitive of rank 3. One natural place to seek locally rank 3 graphs is among locally triangular graphs. In this talk, we will discuss our classification of the connected locally triangular graphs that are also locally rank 3, which we obtain by classifying the locally 4-homogeneous rectagraphs (with some additional structure).

joanna.fawcett@uwa.edu.au

The University of Western Australia

On the partition graph of a positive integer

M. M. Ferrari

joint work with N. Zagaglia Salvi

MSC2010 05C75

Let n be a positive integer. Denote V_n the set of all ordered partitions of n and W_n the quotient set of V_n with respect to the equivalence relation of rotation.
The partition graph of n, P_n, is the graph having W_n as set of vertices, where two vertices α and β, corresponding to ordered partitions into k and $k+1$, $k>0$, parts respectively, are adjacent when exactly one part a of α is replaced, in β, by a pair of consecutive integers whose sum holds a.
In this paper we investigate some structural properties of this graph. A particular attention is given, in the case of $n = 2t+1$, $t>0$, to the subgraph formed by the vertices having t or $t+1$ parts.

margheritamaria.ferrari@polimi.it

Dipartimento di Matematica, Politecnico di Milano, P.zza Leonardo da Vinci 32, 20133 Milano, Italy

Applications of Adjoint Functors in Graph Theory

J. Foniok

joint work with C. Tardif

05C60, 05C76, 18B35

Knowingly or not, graph theorists have been using adjoint functors for years. In this talk I will present some of the known applications (related to graph colouring, products, the Hedetniemi conjecture as well as complexity of the homomorphism problem). I will mainly introduce several of the many deep open problems in the area, which, in my opinion, deserves more attention than it gets.

For background reading see [1].

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j.foniok@mmu.ac.uk

Manchester Metropolitan University

Warmth and Connectivity of Edge Spaces of Graphs

R. Freij

joint work with A. Dochtermann

MSC2010 headings 05C15, 05E45, 05C81

In this talk, we present and compare two approaches for finding lower bounds on the chromatic number $\chi(G)$ of a graph G. On the one hand basic tools from algebraic topology can be used to show that the connectivity of $\hom(K_2, G)$, the space of homomorphisms from a edge K_2 into G, provides a lower bound on $\chi(G)$. On the other hand, Brightwell and Winkler studied notions of 'long range action' of graph homomorphisms, motivated by constructions in statistical physics, and introduced a graph parameter called the warmth $\zeta(G)$ of a graph G, which is also a lower bound on $\chi(G)$.

We provide evidence for the conjecture that the warmth $\chi(G)$ is always less than three plus the connectivity of $\hom(K_2, G)$. We succeed in establishing the first nontrivial case of the conjecture, by showing that $\zeta(G) \leq 3$ if $\hom(K_2, G)$ has a nontrivial fundamental group (under some additional conditions). We also calculate warmth for a family of 'twisted toroidal' graphs that are important extremal examples in the context of hom complexes. Finally we show that $\zeta(G) \leq n - 1$ if a graph G does not have the complete bipartite graph $K_{a,b}$ for $a + b = n$. This provides an analogue for a similar result in the context of hom complexes.

ragnar.freij@aalto.fi

Department of Communications and Networking, Aalto University

A Characterization of Panchromatic Patterns

Hortensia Galeana-Sanchez

joint work with Ricardo Strausz

05C20

Consider two digraphs D and H. D without loops nor multiple arcs, H possibly with loops. An H- coloration of D is a coloration of the arcs of D with the vertices of H. A directed walk or a directed path C, is an H-walk(respectively an H-path), if $C = (v_0, v_1, \ldots, v_n)$ and (color (v_i, v_{i+1}),color(v_{i+1}, v_{i+2})) $\in A(H)$ for each $0 \leq i \leq n-2$.
A set $S \subseteq V(D)$ is H-independent by walks if for any $x, y \in S, x \neq y$ there is no H- walk between them. And S is H-absorbent by walks if for each $x \in V(D) - S$ there exists an H- walk from x to S. Moreover $N \subseteq V(D)$ is an H-kernel by walks if it is H-independent by walks and H-absorbent by walks. H is a panchromatic pattern whenever for any digraph D, and for every H- coloration of D, D has an H-kernel by walks.
In this talk we characterize panchromatic patterns.

hgaleana@matem.unam.mx

Instituto de Matematicas, Universidad Nacional Autonoma de Mexico

Diameter vulnerability of the generalized Petersen graphs

John Baptist Gauci

joint work with Gülnaz Boruzanlı

MSC2010 05C40

Given a graph G with diameter D and edge–connectivity λ, the diameter vulnerability problem considers the difference between D and the diameter D'_s of the resultant graph G'_s obtained by deleting s $(< \lambda)$ edges from G. The general problem of determining the maximum possible diameter for a graph obtained from a general graph G by deleting s edges has been proved to be NP–complete. A number of particular families of graphs have been studied in literature for their diameter vulnerability. In this work we deal with a subfamily of the generalized Petersen graphs and show that, except for some small cases, the diameter of these graphs remains unchanged on the deletion of one edge and increases by one on the deletion of two edges (or, equivalently, $D'_1 = D$ and $D'_2 = D + 1$). The significance of this result rests in the fact that very few families of graphs exhibit such a behaviour; in fact, for general graphs, it is known that $D'_1 = 2D$ and $D'_2 = 3D - 1$. Also, this result contributes further towards the reliability of interconnection networks modelled by these generalized Petersen graphs since it shows that they are very fault tolerant. In cases when the time delay or the interference in the transmission of signals within a network is directly related to the length of a shortest path between two nodes in the network, link failures between nodes are of a great concern. In these instances, it is desirable that the effect of such failures is minimised by modelling networks by graphs having the difference between D and D'_s as small as possible; hence the suitability of these generalized Petersen graphs.

`john-baptist.gauci@um.edu.mt`

Department of Mathematics, Faculty of Science, University of Malta

Hat guessing on graphs: solvability and criticality

Nicholas Georgiou

joint work with Maximilien Gadouleau

91A12, 05C20

We consider the following version of Winkler's hat guessing game. A team is composed of several players; each player is assigned a hat of a given colour; they do not see their own hat, but can see some other hats, according to a directed graph. The team wins if they have a deterministic strategy such that, for any possible assignment of colours to their hats, at least one player guesses their own hat colour correctly.

We use a directed graph D to specify a hat game $\mathcal{H}_D$, as follows. Each vertex of D represents a player and an arc from u to v means that v can see u. If there is a winning strategy for the game $\mathcal{H}_D$ when played with hats of q different colours, we say that D is *q-solvable.* If D is q-solvable, but no proper subgraph of D is q-solvable, we say that D is *edge-critical for q colours.*

In this talk I will present some new bounds and constructions for solvability and some infinite families of critical graphs for small values of q.

nicholas.georgiou@durham.ac.uk

Department of Mathematical Sciences, Durham University, UK

Cops, robbers and ordinals

P. Gordinowicz

joint work with A. Bonato and G. Hahn

MSC2010 05C57, 05C63, 91A46

The game of cop and robber, introduced by Nowakowski and Winkler in 1983, is played by two players on a graph G, one controlling a cop and the other one a robber, both positioned on vertices of G. The players alternate moving their pieces to distance at most 1 each. The cop win if she capture the robber, the robber wins by escaping indefinitely.

There are a few characterisations of finite cop-win graphs but in the infinite case only one of them still works: for the finite graph G consider the sequence of relation $\{\leq_i\}_{i\in\mathbb{N}}$ on $V(G)$, given by the conditions

- $u \leq_0 v$, when $u = v$
- $u \leq_i v$, if for any $x \in N[u]$ there exist $y \in N[v]$ and $j < i$ such that $x \leq_j y$.

where $N[v]$ denotes (closed) neighbourhood of vertex v. This sequence is monotone increasing, thus there exists an minimum integer k such that $\leq_k = \leq_{k+1}$. Theorem of Nowakowski and Winkler says that the relation $\leq_k$ is total if and only if the graph is cop-win.

In the case of infinite graphs we may consider similar characterisation, but the sequence of the relations may be transfinite and hence it is indexed by the (not necessarily finite) ordinal numbers. Therefore, the characteristic of the sequence is some ordinal — we call it the CR-ordinal. We discuss the following question: which ordinals may be the CR-ordinals of infinite cop-wins graphs?

pgordin@p.lodz.pl

Institute of Mathematics, Lodz University of Technology, Poland

Towards a measurable version of Lovász Local Lemma

Ł. Grabowski

joint work with E. Csóka, A. Máthé, O. Pikhurko, K. Tyros

05C63 Infinite graphs

This is a report on an ongoing work to develop a version of measurable version of Lovász Local Lemma (LLL). As a motivation consider the following result of Erdős-Lovász, which was one of the first applications of LLL. Given a k-coloring $c\colon \mathbb{R} \to \{1, \ldots, k\}$ we say a subset $T \subset \mathbb{R}$ is *multicolored* if it contains all the colors, i.e. $c(T) = \{1, \ldots, k\}$. Erdős and Lovász gave a condition on natural numbers k and m so that for any set $S \subset \mathbb{R}$ such that $|S| = m$ there exists a k-coloring c of $\mathbb{R}$ such that for every $x \in \mathbb{R}$ the set $x + S \subset \mathbb{R}$ is multicolored.

It is natural to ask if we can demand c to be a measurable function. I will state a measurable version of LLL and explain how it allows to deduce the existence of a measurable coloring c.

The right setting for measurable LLL turns out to be *graphings*, which I'll briefly explain. The key idea of the proof of measurable LLL is adapting the constructive proofs of LLL due to Moser and Moser-Tardos to graphings.

graboluk@gmail.com

Mathematics Institue, University of Warwick

Developments in the Khintchine-Meinardus method for asymptotic enumeration

Boris Granovsky

joint work with Dudley Stark,London

Asymptotic enumeration, Multiplicative combinatorial structures and models of statistical physics.

Abstract

A theorem of Meinardus provides asymptotics of the number of weighted partitions under certain assumptions on associated ordinary and Dirichlet generating functions. By applying a method due to Khintchine, we extend Meinardus' theorem to find the asymptotics of the coefficients c_n of multiplicative generating functions of the form $\prod_{k=1}^{\infty} S(a_k z^k)^{b_k} := \sum_{n=1}^{\infty} c_n z^n$ for parameter sequences a_k, b_k and general function $S(z)$. Correspondingly, we reformulate the hypotheses of Meinardus' theorem in terms of multiplicative generating functions and prove a local limit theorem for the model considered. This allows us to prove rigorously the asymptotics of Gentile statistics and to study the asymptotics of combinatorial objects with distinct components. It turns out that for a variety of models covered by the above setting c_n grows exponentially, as $n \to \infty$.

Combinatorics of the sonnet

Terry S. Griggs

MSC2010 05A15 05C38

The Oxford Companion to English Literature defines a sonnet as "a poem consisting of 14 lines ... with rhymes arranged according to one or other of certain definite schemes". It also has a certain internal structure which we use to enumerate all *basic sonnet forms*. We then compare to what extent poets conform to these with particular reference to the sonnets of Shakespeare and a later poet, John Clare (1793 – 1864).

Surprisingly in the course of this analysis we encounter Tutte's (3,8) -cage and recover an elegant construction of this graph due to Coxeter. Finally we look at generalisations of this construction.

Thanks to my colleagues, Jozef Širáň, Rob Lewis and especially Graham Erskine from the Open University for conversations about this work.

t.s.griggs@open.ac.uk

Department of Mathematics and Statistics
The Open University
Walton Hall
Milton Keynes MK7 6AA
UNITED KINGDOM

Friendship hypergraphs

Karen Gunderson

joint work with Natasha Morrison and Jason Semerero

MSC2010 headings: 05C65, 05B05

For $r \geq 2$, an r-uniform hypergraph is called a *friendship r-hypergraph* if every set R of r vertices has a unique 'friend' - a vertex $x \notin R$ with the property that for each subset $A \subseteq R$ of size $r-1$, the set $A \cup \{x\}$ is a hyperedge. In the case $r = 2$, the Friendship Theorem of Erdős, Rényi and Sós states that the only friendship graphs are 'windmills'; a graph consisting of triangles with a single common vertex. For $r \geq 3$, there exist infinite classes of friendship r-hypergraphs, not necessarily uniquely defined. These types of hypergraphs belong to a family that generalises the notion of a Steiner system, since in an r-uniform Steiner system, every set of $r-1$ vertices has a unique friend. In this talk, I shall give some background on these types of hypergraphs and describe new results on both upper and lower bounds on the size of friendship hypergraphs.

karen.gunderson@bristol.ac.uk

University of Bristol

Searching for a hidden moving target

John Haslegrave

05C57, 05C05, 05C85

Pursuit and evasion games on graphs have been widely studied. In the classical Cops and Robbers game, introduced by Parsons [2], the cops and robber take turns to move around a graph, with the cops winning if one of them reaches the robber's position. For a fixed graph and number of cops, is there a strategy which will guarantee success?

We consider a related game where an invisible target moves around a graph and a searcher, who is not constrained by the graph, probes vertices trying to discover its location. Is there a sequence of probes which will guarantee success, even if the target knows the sequence in advance? We give a complete classification of graphs for which such a strategy exists. For each such graph we give a strategy which achieves this in the shortest possible time.

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j.haslegrave@cantab.net

School of Mathematics and Statistics, University of Sheffield

Completing y-uniform latinized squares

A.J.W. Hilton

joint work with John Goldwasser

05B15

Let p, y and n be positive integers with $1 \leq y \leq p$ and $p^2 = yn$. A $p \times p$ y-uniform latinized square is a $p \times p$ matrix on symbols $\sigma_1, \dots, \sigma_n$ arranged so that

(i) each symbol occurs at most once in each row and column,

(ii) each symbol occurs y times altogether.

For $1 \leq r \leq p$, an $r \times p$ y-uniform latinized rectangle R is an $r \times p$ matrix which satisfies (i) above and also (ii)' each symbol occurs at most y times altogether.

An obvious necessary condition for R to be completable to a $p \times p$ y-uniform latinized rectangle is $N(\sigma_i) \geq r + y - p$ $(1 \leq i \leq n)$, where $N(\sigma_i)$ is the number of times that σ_i occurs in R. We show that R can be completed to a y-uniform latinized square if $N(\sigma_i) \geq r + y - p$ $(1 \leq i \leq p)$ and either $\max\{r, y\} = p$ or $\max\{r, y\} < p$ and

$$\min\{r, y\} \leq \frac{p^2 - ry}{p} .$$

We also show that if

$$\min\{r, y\} > \frac{p^2 - ry}{p}$$

and $n \geq \max\{2r, p+r-y\}$, then there is an $r \times p$ y-uniform latinized rectangle satisfying $N(\sigma_i) \geq r + y - p$ $(1 \leq i \leq n)$ which cannot be completed. We suspect that the requirement on n here can be reduced to $n > p$.

Keywords. Latinized squares, Completions

Open problems in finite projective spaces

J. W. P. Hirschfeld

J. A. Thas

51E20

Apart from being an interesting and exciting area in combinatorics with beautiful results, finite projective spaces or Galois geometries have many applications to coding theory, algebraic geometry, design theory, graph theory, cryptology and group theory. As an example, the theory of linear maximum distance separable codes (MDS codes) is equivalent to the theory of arcs in $\mathrm{PG}(n, q)$.

Finite projective geometry is essential for finite algebraic geometry, and finite algebraic curves are used to construct interesting classes of codes, the Goppa codes, now also known as algebraic geometry codes. Many interesting designs and graphs are constructed from finite Hermitian varieties, finite quadrics, finite Grassmannians and finite normal rational curves. Further, most such structures have an interesting group; the classical groups and other finite simple groups appear in this way.

Unsolved problems in some of the following topics are considered:

(1) k-arcs;
(2) k-caps;
(3) Hermitian curves and unitals;
(4) Maximal arcs;
(5) Blocking sets;
(6) Flocks;
(7) Ovoids and spreads;
(8) m-systems and BLT-sets;
(9) Algebraic curves over a finite field.

References.

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[2] J. W. P. Hirschfeld and J. A. Thas, Open problems in finite projective spaces, *Finite Fields Appl.* **32** (2015), 44–81.

`jwph@sussex.ac.uk`

University of Sussex

Alspach's cycle decomposition problem for multigraphs

Daniel Horsley

joint work with Darryn Bryant, Barbara Maenhaut and Ben Smith

05C70, 05B30

In 1981 Brian Alspach posed the problem of showing that the obvious necessary conditions for an edge-decomposition of a complete graph into cycles of specified lengths were also sufficient. In 2014 Darryn Bryant, William Pettersson and I published a proof of this fact. In this talk I will discuss the solution of a generalisation of Alspach's problem to the setting of complete multigraphs. In this case the "obvious" necessary conditions are not so obvious.

danhorsley@gmail.com

School of Mathematical Sciences, Monash University

Cycle decompositions of the complete graph with a hole

Rosalind Hoyte

Daniel Horsley

05C70, 05C38, 05B30

The complete graph of order v with a hole of size u, denoted $K_v - K_u$, is the graph obtained from a complete graph of order v by removing the edges of a complete subgraph of order u. In 2014 Bryant, Horsley and Pettersson proved that the complete graph can be decomposed into cycles of arbitrary lengths, provided that the obvious necessary conditions hold. In this talk we present some results for cycle decompositions of $K_v - K_u$.

Our main result shows that, for odd $m \geq 3$, the graph $K_v - K_u$ can be decomposed into m-cycles for feasible values of u and v when $u \geq m - 2$ and $v \geq u + m + 1$. As a consequence of our result, we find necessary and sufficient conditions for the existence of m-cycle decompositions of $K_v - K_u$ for $m \leq 15$. These results were previously only known for $m \leq 7$. We also discuss a result on decomposing $K_v - K_u$ into short cycles of different lengths.

rosalind.hoyte@monash.edu

Monash University

Mantel's Theorem for Random Hypergraphs

Ping Hu

joint work with József Balogh, Jane Butterfield and John Lenz

05C35 05C80 05C65

A classical result in extremal graph theory is Mantel's Theorem, which states that every maximum triangle-free subgraph of K_n is bipartite. A sparse version of Mantel's Theorem is that, for sufficiently large p, every maximum triangle-free subgraph of $G(n, p)$ is w.h.p. bipartite. Recently, DeMarco and Kahn proved this for $p > K\sqrt{\log n/n}$ for some constant K, and apart from the value of the constant this bound is best possible.

We study an extremal problem of this type in random hypergraphs. Denote by F_5, which sometimes called as the generalized triangle, the 3-uniform hypergraph with vertex set $\{a, b, c, d, e\}$ and edge set $\{abc, ade, bde\}$. One of the first extremal results in extremal hypergraph theory is by Frankl and Füredi, who proved that the maximum 3-uniform hypergraph on n vertices containing no copy of F_5 is tripartite for $n > 3000$. A natural question is for what p is every maximum F_5-free subhypergraph of $G^3(n, p)$ w.h.p. tripartite. We show this holds for $p > K \log n/n$ for some constant K and does not hold for $p = 0.1\sqrt{\log n}/n$.

P.Hu@warwick.ac.uk

Department of Computer Science, University of Warwick

External difference families

Sophie Huczynska

joint work with James A. Davis and Gary L. Mullen

05B10

External difference families generalize the concept of difference families, and were introduced as a method of constructing optimal robust secret sharing schemes. In this work, we introduce *near-complete external difference families*, showing how the existence of such an object implies the existence of a near-resolvable design. We give examples and construction methods, some of which lead to new parameter families of near-resolvable designs.

sophie.huczynska@st-andrews.ac.uk

University of St Andrews

On Computing Graph Pebbling Numbers

Glenn Hurlbert

joint work with Liliana Alcón and Marisa Gutierrez (UNLP, Argentina)

MSC2010 headings 05C85 (68Q17, 05C75)

Graph pebbling is a network model for transporting discrete resources that are consumed in transit. Deciding whether a given configuration on a particular graph can reach a specified target is NP-complete, even for diameter two graphs, and deciding whether the pebbling number has a prescribed upper bound is Π_2^{P}-complete. Recently we proved that the pebbling number of a split graph can be computed in polynomial time. This work continues the program of finding other polynomial classes, moving away from the large tree width, small diameter case (such as split graphs) to small tree width, large diameter case, beginning an investigation on the important subfamily of chordal graphs called k-trees. In particular, we provide a formula for the pebbling number of any 2-path.

ghurlbert@vcu.edu

Department of Mathematics and Applied Mathematics, Virginia Commonwealth University, USA

On random k-out subgraphs of large graphs

Tony Johansson

joint work with Alan Frieze

05C80 Random graphs

We consider random subgraphs of a fixed graph $G = (V, E)$ with large minimum degree. We fix a positive integer k and let G_k be the random subgraph where each $v \in V$ independently chooses k random neighbors, making kn edges in all. When the minimum degree $\delta(G) \geq (\frac{1}{2}+\varepsilon)n$, $n = |V|$ then G_k is k-connected w.h.p. for $k = O(1)$; Hamiltonian for k sufficiently large. When $\delta(G) \geq m$, then G_k has a cycle of length $(1-\varepsilon)m$ for $k \geq k_\varepsilon$. By w.h.p. we mean that the probability of non-occurrence can be bounded by a function $\phi(n)$ (or $\phi(m)$) where $\lim_{n\to\infty} \phi(n) = 0$.

tjohanss@andrew.cmu.edu

Carnegie Mellon University

The Erdős-Pósa property for cycles

Felix Joos

joint work with Henning Bruhn, Tony, Huynh, Oliver Schaudt, Paul Wollan

05C38

A family $\mathcal{F}$ of graphs has the Erdős-Pósa property if there is a function f such that for every integer k, every graph G has k disjoint copies of graphs from $\mathcal{F}$ or contains a set of vertices X meeting every member of $\mathcal{F}$ in G such that $|X| \leq f(k)$.

In a seminal paper Erdős and Pósa proved that the family of cycle has the Erdős-Pósa property. This result has had numerous extensions and generalizations to cycles with further restrictions: long cycles, cycles through a prescribed set of vertices S, and cycles of length $\ell \pmod m$.

We give several extensions of previous results and present a unified approach which implies almost all previous Erdős-Pósa property results for subclasses of cycles up to a better asymptotic behavior of the function f.

References.

[1] H. Bruhn, F. Joos and O. Schaudt, Long cycles through prescribed vertices have the Erdős-Pósa property, *preprint* arXiv:1412.2894.
[2] T. Huynh, F. Joos and P. Wollan, A Unified Erdős-Pósa Theorem for Constrained Cycles via Doubly Group-labeled Graphs.
[3] F. Joos, Parity linkage and the Erdős-Pósa property of odd cycles in highly connected graphs, *preprint* arXiv:1411.6554.

felix.joos@uni-ulm.de

Institut für Optimierung und Operations Research, Universität Ulm, Germany

Families avoiding just one pattern

Ilan Karpas

Eoin Long

MSC2010 headings: 05D05

Let l, n be even natural numbers. A *pattern* p of length l is an element $p = (p_1, \ldots, p_l) \in \{-, +\}^l$. Given such a pattern and two sets $A, B \subset [n]$, we say that the pair (A, B) *forms pattern* p if the following conditions are satisfied:

- $A \triangle B = \{i_1, \ldots, i_l\}$, where $i_1 < i_2 < \ldots < i_l$;
- For $1 \leq j \leq l$, we have $i_j \in A \setminus B$ if $p_j = +$ and $i_j \in B \setminus A$ if $p_j = -$.

We will say that a family $\mathcal{F} \subset 2^{[n]}$ *avoids* pattern p if there are no sets $A, B \in \mathcal{F}$ so that (A, B) forms pattern p. From now on, we assume that $\mathcal{F} \subset \binom{[n]}{n/2}$ and that p has $l/2$ $-$s and $l/2$ $+$s.

How large can a family $\mathcal{F} \subset 2^{[n]}$ be if it avoids pattern p? We say that there is a density theorem for p every such family satisfies $|\mathcal{F}| = o(\binom{n}{n/2})$. Otherwise, we say that there is no density theorem for p.

In this talk I will discuss a number of situations in which such density theorems exist. We show that if p is a pattern consisting of $l/2$ $+$s followed by $l/2$ $-$s (i.e. $p = (+ + \cdots + - - \cdots -))$, then for $l = o(\sqrt{n})$ there is a density theorem for p, while for $l = \Omega(\sqrt{n})$ there is no density theorem for p. For patterns p of length l which alternate between $+$ and $-$ (i.e. $p = (+ - + - \ldots + -))$, we show that there is a density theorem if $l = o(\sqrt{n})$ and that there is no density theorem if $l = \Omega(n)$. Lastly, we show that for a fixed pattern p of constant length, there is a density theorem for p, provided n is large enough.

ilan.karpas@mail.huji.ac.il

The Einstein Institute of Mathematics, The Hebrew University of Jerusalem.

A conjecture on combinatorial structure of permutation groups

Andrzej Kisielewicz

joint work with Mariusz Grech

20B05,20B25,94C10

In 1991, Clote and Kranakis [1] have proved a theorem (formulated below) that turned out to be false. The proof contained a gap with no way to correct it. Subsequently, a few papers have appeared showing that the result is true for various classes of permutation groups. Up to now, only one counterexample to the result is know. We still do not know whether the counterexample is an exception, and the conjecture is in principle true, or there are unknown large classes of permutation groups that falsify the conjecture. In the talk we present some recent results in this direction and argue that the general problem is that the theory of permutation groups lacks results on combinatorial structure of permutation groups.

We consider generalized boolean functions of the form $f : \{0,1\}^n \to \{0,1,\ldots,k-1\}$ referred to as k-valued boolean functions. The set

$$G(f) = \{\sigma \in S_n : f(x_1, x_2, \ldots, x_n) = f(x_{\sigma(1)}, x_{\sigma(2)}, \ldots, x_{\sigma(n)})\}$$

forms a permutation group called the *symmetry group* of f. The conjecture is that *each symmetry group of a k-valued boolean function is a symmetry group of a standard (2-valued) boolean function.*

References.

[1] P. Clote, E. Kranakis Boolean functions, invariance groups, and parallel complexity, *SIAM J. Comput.*, **20** (1991), pp. 553-590.

andrzej.kisielewicz@gmail.com

Institute of Mathematics, University of Wrocław, Poland

On the enumeration of juggling patterns and related combinatorial objects

N. Korpelainen

joint work with O. Bagdasar

05Axx: Enumerative combinatorics, 05A05: Permutations, words, matrices

We consider juggling patterns given by the popular 'siteswap notation'. Famously, the number of juggling patterns of period n, with less than b balls, is given by b^n. [1] However, several questions remain open regarding the enumeration of juggling patterns of fixed length (rather than fixed period). We present a new number sequence enumerating juggling patterns of fixed length under rotational equivalence.

The enumeration of juggling patterns is equivalent to the enumeration of surprisingly many other combinatorial objects: this includes certain subclasses of cyclic permutations, transversals of cyclic latin squares and solutions to toroidal variations of the n-queens problem (in chess). We will give a brief survey of these equivalences and related theory.

References.

[1] J. Buhler, D. Eisenbud, R. Graham & C. Wright, Juggling drops and descents, *The American Mathematical Monthly*, **Vol. 101, No. 6, June - July 1994, p507 - 519** (1994)

N.Korpelainen@derby.ac.uk

Department of Computing & Mathematics, College of Engineering and Technology, University of Derby

Graph compositions of suspended uniform four Combs

R. Kwashira

joint work with E.G. Mphako-Banda , B. Zinsou

05C30, 05A18, 05A05

Graph compositions play an important role in the generalization of both ordinary compositions of positive integers and partitions of finite sets. Graph compositions of certain classes of graphs, like trees, cycle graphs, wheels, etc have been found using generating functions and recurrence relations. In this talk, we use different techniques, in particular the principle of inclusion and exclusion, to count the number of graph compositions of suspended four combs.

References.

[1] A. Knopfmacher and M.E. Mays, Graph Compositions 1: Basic enumeration, *Integers:The Electronic Journal of Combinatorial Number Theory* **1**, (A04), (2001), 1-11.
[2] E.G. Mphako-Banda, Graph compositions and flats of cycle matroids, *Quaestines Mathematicae*, **32**, (2009), 523-527 .

rugare.kwashira@wits.ac.za

University of Witwatersrand, Faculty of science, School of Maths, P.B. X3, Braamfontein, Johannesburg, South Africa

The list chromatic index of graphs of tree-width 3 and maximum degree at least 7

Richard Lang

05C15

In analogy to the chromatic index $\chi'(G)$, the list chromatic index $\mathrm{ch}'(G)$ of a graph G is the smallest integer k so that for each choice of k permisible colours at every edge, there is a proper edge-colouring that picks a permisible colour at every edge. A long standing conjecture states that the list chromatic index always equals the chromatic index. In 1999 Juvan, Mohar and Thomas showed that this is true for graphs of tree-width 2 [1]. We prove the same for 3-trees, and graphs of tree-width 3 and maximum degree at least 7 [2].

References.

[1] M. Juvan, B. Mohar, and R. Thomas, List Edge-Colorings Of Series-Parallel Graphs, *Electronic Journal of Combinatorics* **6** (1999).
[2] R. Lang, On the list chromatic index of graphs of tree-width 3 and maximum degree at least 7, *preprint* `http://xxx.tau.ac.il/abs/1504.02122`.

rlang@dim.uchile.cl

Departamento de Ingeniería Matemática, Universidad de Chile

Relative m-covers of generalised quadrangles

Melissa Lee

joint work with John Bamberg and Michael Giudici

51E12

In 1965, Segre defined the notion of a hemisystem of a Hermitian surface $\mathrm{H}(3, q^2)$, q odd as a subset of the lines that meet each point in half of its lines. In 2011, Penttila and Williford defined relative hemisystems as the analogous concept of hemisystems for q even. They were motivated by the desire to generate rare primitive Q-polynomial 3-class association schemes, and discovered the first infinite family of them as a result of their infinite family of relative hemisystems.

In this talk, I will survey the history of hemisystems and relative hemisystems, why we care about them, and speak about the search for relative hemisystems and and the more general relative m-covers on non-classical generalised quadrangles.

20945795@student.uwa.edu.au

Centre for the Mathematics of Symmetry and Computation at The University of Western Australia

The Problem of Tropical Homomorphism in vertex-colored graphs

Sylvain Legay

joint work with Florent Foucaud, Ararat Harutyunyan, Pavol Hell, Yannis Manoussakis and Reza Naserasr

Coloring of graphs and hypergraphs, Complexity classes

A homomorphism from graph G to graph H is a function f from vertices of G to vertices of H such that for all edge uv in G, $h(u)h(v)$ is an edge in H. Consequently, the problem of existence of a proper colouring is easily generalised to the problem of existence of a homomorphism to a fixed graph H. A theorem of dichotomy for this set of problem is known : If H is bipartite, then H-COLORING is polynomial. If H is not bipartite, then H-COLORING is NP-Complete.
In our work, we study a generalisation of H-COLORING. A tropical homomorphism from vertex-coloured graph (G, c) to vertex-coloured graph (H, c') is a homomorphism h from G to H such that for each vertex v in G, $h(v)$ has the same colour than v. This talk aims to present our results in the search of a dichotomy for the (H, c)-COLORING problems. The main result is that the existence of such a dichotomy is equivalent to the existence of a dichotomy for the Constraint Satisfaction Problems (CSP), a largely studied and still unresolved conjecture. We have also studied the complexity of this problem for some families of graph.

legay@lri.fr

LRI, University Paris South

Cycle Extensions in 0-Block-Intersection Graphs of Balanced Incomplete Block Designs

Jason T. LeGrow

joint work with David A. Pike and Jonathan Poulin

05B05, 05C45, 05C85

A cycle $\mathcal{C}$ in a graph G is said to be *extendable* if there exists another cycle in G which contains all the vertices of $\mathcal{C}$ and exactly one other vertex. A graph G is said to be *cycle extendable* if every non-Hamiltonian cycle in G is extendable.

A *balanced incomplete block design* with parameters $v, k,$ and λ, written BIBD(v, k, λ), is a pair $\mathcal{D} = (V, \mathcal{B})$ of sets, where V is a set of v elements and $\mathcal{B}$ is a set of k-subsets of V—called *blocks*—such that each pair of elements of V occurs in exactly λ blocks in $\mathcal{B}$. The *0-block-intersection* graph of a balanced incomplete block design $\mathcal{D} = (V, \mathcal{B})$ is the graph $\overline{G}_{\mathcal{D}}$ whose vertex set is $\mathcal{B}$ and in which two blocks are adjacent if and only if they do not intersect.

We present a sufficient condition for the 0-block-intersection graph of a BIBD(v, k, λ) to be cycle extendable and briefly discuss a polynomial-time algorithm for finding cycles of any length in such graphs.

References.

[1] Jason T. LeGrow, D. A. Pike, and Jonathan Poulin. Hamiltonicity and Cycle Extensions in 0-Block-Intersection Graphs of Balanced Incomplete Block Designs. Submitted.

jlegrow@uwaterloo.ca

Department of Combinatorics and Optimization, University of Waterloo

The degree-diameter problem for circulant graphs of arbitrary diameter up to degree 10

Robert R. Lewis

05C35

The degree-diameter problem is a search for graphs of given degree and diameter that are extremal in the sense that they have the greatest possible order. In this talk we address the sub-problem of finding families of extremal circulant graphs of arbitrary diameter for a given degree. A circulant graph is the Cayley graph of a cyclic group and is therefore one of the simplest categories of graph to consider.

For circulant graphs up to degree 5, the problem was solved for arbitrary diameter some time ago. For degree 6 to 9, families of largest known graphs for arbitrary diameter have been discovered which are confirmed extremal only for small diameters, [1] [2]. These graphs share some significant attributes across all degrees, 2 to 9. For degree 10, graph families have very recently been discovered that also share the same key attributes. In this talk we will review these families and their common attributes, and propose some conjectures for extremal circulant graphs of higher degree.

References.

[1] R. Dougherty and V. Faber, The degree-diameter problem for several varieties of Cayley graphs, I: The Abelian case, *SIAM Journal on Discrete Mathematics* **17** (2004) 478-519.
[2] R. R. Lewis, The degree-diameter problem for circulant graphs of degree 8 and 9, *Electronic Journal of Combinatorics* **21(4)** (2014) #P4.50.,

robert.lewis@open.ac.uk

Department of Mathematics and Statistics, The Open University, UK

First order convergence of matroids

Anita Liebenau

joint work with Frantisek Kardos, Daniel Kral, Lukas Mach

MSC2010 headings

The model theory based notion of the first order convergence unifies the notions of the left-convergence for dense structures and the Benjamini-Schramm convergence for sparse structures. It is known that every first order convergent sequence of graphs with bounded tree-depth has an analytic limit object called a limit modeling. We establish the matroid counterpart of this result: every first order convergent sequence of matroids with bounded branch-depth representable over a fixed finite field has a limit modeling, i.e., there exists an infinite matroid with the elements forming a probability space that has asymptotically the same first order properties. We show that neither of the bounded branch-depth assumption nor the representability assumption can be removed.

a.liebenau@warwick.ac.uk

University of Warwick

A Combinatorial Formula for Affine Hall-Littlewood Functions via a Weighted Brion Theorem

I. Makhlin

joint work with B. Feigin

MSC2010 headings: 05E10, 52B20, 05E05

I would like to tell about a new combinatorial formula for Hall-Littlewood functions associated with the affine root system of type $\tilde{A}_{n-1}$ (i.e. corresponding to a dominant integral $\widehat{\mathfrak{sl}}_n$-weight).

The formula can be viewed as a weighted sum of exponentials of integer points in a certain infinite-dimensional convex polyhedron. We derive a weighted version of Brion's theorem and then apply it to our polyhedron to prove the formula.

In order to do this we investigate a certain family of polyhedra which naturally generalize the Gelfand-Tsetlin polytopes. This theory is of some interest in its own right.

References.

[1] Boris Feigin and Igor Makhlin, A Combinatorial Formula for Affine Hall-Littlewood Functions via a Weighted Brion Theorem, *preprint* `http://arxiv.org/abs/1505.04269`.

`imakhlin@mail.ru`

National Research University Higher School of Economics, International Laboratory of Representation Theory and Mathematical Physics

Turan densities for 3-graphs

Klas Markström

joint work with Fei Song

05D

Given a 3-uniform hypergraph H the *Turan density* $\alpha(H)$ is the smallest integer such that any 3-uniform hypergraph G, on n vertices, with at least $\alpha(H)\binom{n}{3}$ edges must contain a copy of H.

In this talk I will discuss some results on the possible values of $\alpha(H)$. It is known that there are some subintervals of $[0, 1]$ which contain no Turan densities, and there are values which are known to be accumulation points for the set of Turan densities. I will discuss some examples of each kind.

Klas.markstrom@math.umu.se

Institutionen för matematik och matematisk statistik, Umeåuniversitet, 901 87, Umeå, Sweden

Face 2-coloured triangulations and directed Eulerian digraphs on the sphere

T.A. McCourt

05C10, 05B15, 05C25

Let $\mathcal{G}$ be a properly face 2-coloured (say black and white) triangulation of the sphere with vertex set V. Consider the abelian group $\mathcal{A}_W$ generated by the set V, with relations $r + c + s = 0$ for all white triangles with vertices r, c and s. The group $\mathcal{A}_B$ can be defined similarly, using the black triangles. Then $\mathcal{A}_W$ and $\mathcal{A}_B$ are isomorphic and their finite torsion subgroup is referred to as the canonical group of the triangulation. The study of these canonical groups is motivated by questions on embedding partial latin squares into groups.

From a face 2-coloured triangulation of the sphere three directed Eulerian digraph spherical embeddings can be constructed where the digraphs underlying the embeddings have abelian sand-pile groups isomorphic to the canonical group of the triangulation. Conversely, given a directed Eulerian digraph spherical embedding, D say, it is possible to construct a face 2-coloured triangulation of the sphere whose canonical group is isomorphic to the abelian sand-pile group of the digraph underlying D.

In this talk we shall discuss these correspondences in the case of face 2-coloured triangulations of the sphere which have simple underlying graphs. An application of the discussion will be to bound the order of the canonical group in terms of the number of faces in the triangulation.

thomas.mccourt@plymouth.ac.uk

Department of Mathematics and Statistics, Plymouth University

A new technique for finding small Kirkman Covering and Packing Designs; a KCD(11), a canonical KCD(13), and more examples.

John P. McSorley

MSC2010 headings 05, 51E99, 52C99.

We present a new technique which we call the 'type/unlabelled/labelled' technique for finding, or determining the non-existence of, small Kirkman covering and packing designs. Using this technique and different computational methods, we construct a new KCD(11), and a new canonical KCD(13), and make some progress on the existence question of a KPD(19, 8).

References.

[1] R.Rees, W.Wallis., Kirkman triple systems and their generalisations: a survey, in: Designs 2002, Further Computational and Constructive Design Theory, ed. W.D.Wallis. Kluwer Academic Publishers, (2003).

mcsorley60@hotmail.com

Department of Mathematics,
Southern Illinois University,
Carbondale IL 62901-4408. USA.

Graph modification problems in epidemiology

Kitty Meeks

joint work with Jessica Enright, University of Stirling

05C85, 68R10, 05C90

One of the most natural ways to understand how a disease spreads through a population is to consider the contact between its members: this gives rise to a graph in which vertices represent individuals, and two individuals are joined by an edge if they have contact with each other which could lead to the transmission of the disease.

A key obstacle in using graph theoretic methods to analyse the spread of human diseases in this way is that we do not have good information about the underlying contact graph. When considering instead the spread of disease in livestock, however, there is detailed information about precisely which individuals have contact with one another, thanks to European legislation on the recording of animal movements. Graph theory then becomes very relevant to the design of strategies for introducing controls on this animal contact network so as to restrict the spread of an epidemic.

In this talk I give an overview of how some familiar graph modification problems arise in this application, as well as describing recent joint work with Jessica Enright on algorithms for edge-deletion problems and discussing a situation in which the graph parameter treewidth might be relevant in the real world.

kitty.meeks@glasgow.ac.uk

School of Mathematics and Statistics, University of Glasgow

The heterochromatic number of hypergraphs coming from matroid structures

C. Merino

joint work with J.J. Montellano-Ballesteros

05B35,05C15

The heterochromatic number $h_c(H)$ of a non-empty hypergraph H is the smallest integer k such that for every colouring of the vertices of H with exactly k colours, there is a hyperedge of H all of whose vertices have different colours. This concept was introduced by Arocha et al in [1] and it is closely related to the anti-Ramsey numbers of Erdös et al in [2].

Given a rank-r matroid M, there are several hypergraphs associated to the matroid to consider. One is $C(M)$, the hypergraph where the points are the elements of the matroid and the hyperedges are the circuits of M. The other one is $B(M)$, where here the points are the elements and the hyperedges are the bases of the matroid.

In this talk we will prove that $h_c(C(M))$ equals $r + 1$ when M is not the free matroid $U_{n,n}$, and that $h_c(B(M))$ equals r precisely when M is a paving matroid. We will also list some other partial results, and finish with open problems, some of which are well-known.

References.

[1] J. Arocha, J. Bracho and V. Neumann-Lara, On the Minimum Size of Tight Hypergraphs, *Journal of Graph Theory* **16** (1992) 319-326.
[2] P. Erdös, M. Simonovits and V.T. Sos, Anti-Ramsey theorems, *Colloq. Math. Soc. Janos Bolyai* **10** (1975) 633-643.

merino@matem.unam.mx

IMATE-UNAM

Decomposing highly edge-connected graphs into trees of small diameter

M. Merker

05C40, 05C51, 05C70

The Tree Decomposition Conjecture by Bárat and Thomassen states that for every tree T there exists a natural number $k(T)$ such that the following holds: If G is a $k(T)$-edge-connected graph with size divisible by the size of T, then G can be edge-decomposed into subgraphs isomorphic to T. So far this conjecture has only been verified for paths, stars, and a family of bistars. We prove a weaker version of the Tree Decomposition Conjecture, where we require the subgraphs in the decomposition to be isomorphic to graphs that can be obtained from T by vertex-identifications. This implies the Tree Decomposition Conjecture under the additional constraint that the girth of G is greater than the diameter of T. We also show that the Tree Decomposition Conjecture holds for all trees of diameter at most 4, as well as for some trees of diameter 5.

marmer@dtu.dk

Department of Applied Mathematics and Computer Science, Technical University of Denmark

Ribbon graphs and their minors

Iain Moffatt

05C10, 05C83

In this talk I will introduce a theory of minors of ribbon graphs (or, equivalently, minors of graphs embedded in surfaces). Minors of ribbon graphs differ from those of graphs in that contracting loops is necessary and doing this can create additional vertices and components. Thus the ribbon graph minor relation is incompatible with the graph minor relation.

I will introduce ribbon graphs and their minors, and discuss some of their differences to graph minors, particularly with respect to well-quasi-ordering. I will then describe excluded minor characterisations of some minor closed families of ribbon graphs.

References.

[1] I. Moffatt, Excluded minors and the graphs of knots, *J. Graph Theory*, to appear. `ArXiv:1311.2160`.

[2] I. Moffatt, Ribbon graph minors and low-genus partial duals, *Annals of Combin.*, to appear. `ArXiv:1502:00269`.

iain.moffatt@rhul.ac.uk

Royal Holloway University of London

Tight lower bounds on the number of bicliques in false-twin-free graphs

L. Montero

joint work with M. Groshaus

05C69

A *biclique* is a maximal bipartite complete induced subgraph of G. Bicliques have been studied in the last years motivated by the large number of applications. In particular, enumeration of the maximal bicliques has been of interest in data analysis. Associated with this issue, upper and lower bounds on the maximun number of bicliques were given.

In this work we focus on lower bounds on the minumum number of bicliques of a graph. Despite in general it seems "trivial", since any bipartite complete graph $K_{n,m}$ has just one biclique, it is not considered the fact that all vertices in each partition are false-twins. Since adding false-twin vertices to G does not change the number of bicliques, then a biclique $K_{n,m}$ can be thought as one "equivalent" to $K_{1,1}$. Recall that many of the applications consider the bicliques as a hole group (of one side of bipartition) having a common non-empty subset of characteristics (the other side of the biclique). So, it is natural to think that if there are groups of "twin objects" with the same characteristics, we could only maintain one representative object for each. Following this idea we propose to study the minimum number of bicliques in false-twin-free graphs. In this sense, we prove that any $\{C_4$,diamond,false-twin$\}$-free graph and any $\{K_3$,false-twin$\}$-free graph on $n \geq 3$ and $n \geq 4$ vertices respectively, has at least $\lceil \frac{n}{2} \rceil$ bicliques. We show that these bounds are tight.

lmontero@lri.fr

L.R.I., Université Paris-Sud, Orsay, France

Certificates for Graph Polynomials

Kerri Morgan

joint work with Graham Farr

MSC05C31

The chromatic polynomial gives the number of proper colourings of a graph. This polynomial has been extensively studied in graph theory and statistical mechanics. Recently there has been increasing interest in studying the algebraic properties of chromatic polynomials. Research in this area includes the question of which algebraic numbers can be chromatic roots, chromatic factorisation, and splitting field equivalence, that is, where two chromatic polynomials have the same splitting field.

Morgan and Farr introduced the notion of a *certificate* to prove results on chromatic factorisation. Certificates are a powerful tool in proving properties of the chromatic polynomial and other graph polynomials without the cost of computing the polynomial itself. In fact, certificates provide a general proof theory that can be used for any graph polynomial that has the deletion-contraction property.

In this talk, we will introduce certificates and demonstrate the use of certificates in graph polynomial proofs.

Graph compositions: Some 2-edge connected graphs and some upper bounds

E.G. Mphako-Banda

05C30, 05A18, 05A05, 11B37

Graph compositions of graphs obtained after certain graph operations have been found. Some of the graph operations result in a graph whose graph compositions are related to the graph compositions of the original graphs. One such operation result in 1-edge connected graphs whose graph compositions is given in terms of the original graphs. In this talk we discuss the number of graph compositions of a class of 2-edge connected graphs. In addition, rough upper and lower bounds for graph compositions of any graph G has been set in the literature. We discuss a new upper bound for the number of graph compositions of any graph G.

References.

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eunice.mphako-banda@wits.ac.za

University of Witwatersrand

Hamilton cycles in quasirandom hypergraphs

Richard Mycroft

Joint work with John Lenz and Dhruv Mubayi

05C65

We consider the existence of Hamilton cycles in quasirandom k-graphs of high minimum vertex degree. The notion of quasirandomness we work with is natural; we say that a k-graph H on n vertices is (p, μ)-dense if for any $X_1, \ldots, X_k \subseteq V(H)$ we have

$$e(X_1, \ldots, X_k) \geq p|X_1| \ldots |X_k| - \mu n^k.$$

This generalises a well-studied property for graphs. Also, we write $\delta(H)$ for the minimum vertex degree of H, meaning the largest integer m such that every vertex of H lies in at least m edges of H.

We say that a k-graph C is an ℓ-cycle if its vertices can be ordered cyclically so that every edge of C consists of k consecutive vertices, and successive edges of C intersect in precisely ℓ vertices. A Hamilton ℓ-cycle in a k-graph H is a spanning ℓ-cycle which appears as a subgraph of H.

Our main theorem is that for any fixed $p, \mu, \alpha > 0$, if n is sufficiently large and divisible by $k - \ell$, then any (p, μ)-dense k-graph H on n vertices with $\delta(H) \geq \alpha\binom{n}{k-1}$ contains a Hamilton 1-cycle. Furthermore, we give a construction to show that if $k - \ell$ divides k, then the same conditions do not guarantee the existence of a Hamilton ℓ-cycle in H, even if the minimum vertex degree condition is strengthened to a similar minimum codegree condition. This leaves the validity of the analogous statement for $\ell \geq 2$ with $k - \ell$ not divisible by k as an intriguing open problem.

r.mycroft@bham.ac.uk

School of Mathematics, University of Birmingham

The number of contractible edges in a 4-connected graph having a small number of edges not contained in triangles

Shunsuke Nakamura

joint work with Keiko Kotani

MSC2010 05C40

Let G be a 4-connected graph, and let $\tilde{E}(G)$ denote the set of those edges of G which are not contained in a triangle and let $E_c(G)$ and $E_n(G)$ denote the set of 4-contractible edges and the set of 4-noncontractible edges, respectively. Let $\hat{G}$ denote the induced subgraph of G with edge set $\tilde{E}(G) \cap E_n(G)$. We let Y^* denote the graph of order six defined by $V(Y^*) = \{x_1, x_2, y_1, y_2, z_1, z_2\}$, $E(Y^*) = \{x_1x_2, x_1y_1, x_1y_2, x_2z_1, x_2z_2\}$.
Ando and Egawa proved that if $|\tilde{E}(G)| \geq 15$, then $|E_c(G)| \geq (|\tilde{E}(G)|+8)/4$. In this talk, we prove the following theorem.

Theorem 1.

Let G be a 4-connected graph and suppose that $|\tilde{E}(G)| \geq 1$. Then $|E_c(G)| \geq (|\tilde{E}(G)| + 4)/4$.
Further we have $|E_c(G)| \geq (|\tilde{E}(G)| + 8)/4$ unless one of the following holds:
(1) $|\tilde{E}(G)| = 9$ and $\hat{G} \simeq Y^*$,
(2) $|\tilde{E}(G)| = 6$ and $\hat{G} \simeq 3K_2$,
(3) $|\tilde{E}(G)| = 5$ and $\hat{G} \simeq 2K_2, P_2$,
(4) $|\tilde{E}(G)| = 4$ and $\hat{G} \simeq 2K_2$,
(5) $|\tilde{E}(G)| = 3$ and $\hat{G} \simeq K_2$,
(6) $|\tilde{E}(G)| = 2$ and $\hat{G} \simeq \emptyset$, or
(7) $|\tilde{E}(G)| = 1$ and $\hat{G} \simeq K_2$, where K_n denotes the complete graph of order n and P_n denotes the path of length n.

1414619@ed.tus.ac.jp

Department of Mathematical Information Science, Tokyo University of Science, Shinjuku-ku, Tokyo, 162-8601, Japan

Saturated Graphs of Prescribed Minimum Degree

A. Nicholas Day

05C35: Extremal Problems

A graph G is H-saturated if it contains no copy of H as a subgraph, but the addition of any new edge to G creates a copy of H. Let K_p denote the complete graph on p vertices. Suppose G is a K_p-saturated graph on n vertices with minimum degree at least t. How few edges can G have? In this talk we will consider this question and sketch a proof that shows, for fixed t and p, that G must have at least $tn - O(1)$ edges. This proves and generalises a conjecture of Bollobás [1].

References.

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a.nick.day@gmail.com

School of Mathematical Sciences, Queen Mary University of London

Delta-matroids, ribbon graphs and connectivity

Steve Noble

Carolyn Chun, Deborah Chun, Iain Moffatt, Ralf Rueckriemen

MSC2010 headings 05B35, 05C10

Delta-matroids were introduced and studied by several authors, notably André Bouchet, in the late 1980s. Informally, they are to ribbon (or fat) graphs, what matroids are to graphs. Recent work on ribbon graphs and unconnected work on principal pivot transforms have motivated a resurgence in interest in delta-matroids. We show that various new results or concepts concerning ribbon graphs generalize to delta-matroids and that the splitter theorem for connected matroids may also be extended to delta-matroids.

steven.noble@brunel.ac.uk

Brunel University London

Trades in Hadamard matrices

P. Ó Catháin

joint work with Ian Wanless

05B20, 15B24

Arising originally from the analysis of a family of compressed sensing matrices, Ian Wanless and I recently investigated a number of linear algebra problems involving complex Hadamard matrices. I will discuss our main result, which relates rank-one submatrices of Hadamard matrices, simultaneous representations of a fixed vector with respect to two unbiased bases of a finite dimensional vector space and trades in Hadamard matrices. Only a basic knowledge of linear algebra will be assumed.

p.ocathain@gmail.com

Monash University

Characterisations of Optimal Algebraic Manipulation Detection Codes

M.B. Paterson

joint work with D.R. Stinson

MSC2010 headings 94A62, 94C30, 05B10

Algebraic manipulation detection (AMD) codes are algebraic/combinatorial structures that are closely related to difference sets. They were defined as a generalisation and abstraction of techniques previously used in constructing robust secret sharing schemes, and their use has been proposed for a range of other cryptographic applications. In this talk we consider lower bounds on the success probability of an adversary in attacking an AMD code, as well as combinatorial characterisations of AMD codes meeting these bounds with equality.

m.paterson@bbk.ac.uk

Department of Economics, Mathematics and Statistics, Birkbeck, University of London

New constructions of strongly invertible graphs

S. Pavlíková

05C50 - Graphs and their eigenvalues

Let G be a finite, undirected graph of order n with non-zero eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$. An *inverse* of G is any graph H of order n with spectrum $\lambda_1^{-1}, \lambda_2^{-1}, \ldots, \lambda_n^{-1}$. Given the abundance (and lack) of available bounds on the largest (and the smallest, respectively) positive eigenvalue of a graph, inverses of graphs may provide a useful tool for estimating the smallest positive eigenvalues; this principle has found applications in quantum chemistry.

A way to construct inverses of a graph G is to look for graphs H with the property that the inverse of an adjacency G is similar to an adjacency matrix of H; if such a graph H exists, the graph G is called *strongly invertible*. By a sufficient (but not necessary) condition due to Godsil [2], a graph G with a unique perfect matching M is strongly invertible if contraction of all edges of M produces a bipartite graph out of G. Apart from [2], constructions of strongly invertible graphs have also been studied e.g. by Barik, Neumann and Pati [1], Kirkland and Akbari [3], Kirkland and Tifenbach [4], and Neumann and Pati [5].

We will present new approaches to constructing new strongly invertible graphs from old ones, based on the operation of overlapping a pair of edges. Overlapping turns out to preserve simplicity of strongly invertible graphs. Under certain conditions this operation can be extended to overlapping more than one pair of edges, allowing for further investigation into the structure of strongly invertible graphs.

References

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`sona.pavlikova@stuba.sk`

Faculty of Chemistry and Food Technology, Slovak University of Technology, Bratislava, Slovakia

Comparable pairs and linear extensions in partially ordered sets

D. B. Penman

joint work with Vasileios Iliopoulos and Colin McDiarmid

MSC2010 headings: 06A07

I shall give a progress report on work with Vasileios Iliopoulos and Colin McDiarmid about results relating the number of linear extensions of a partially ordered set to the number of vertices and comparable pairs in it. In particular, the talk will include an upper bound on the number of such extensions and discussion of a result giving the **whp** asymptotic value of the number of linear extensions of a random interval order which shows that the upper bound in question is not too far from the truth in general. There will also be discussion of examples of posets with few linear extensions relative to the number of vertices and comparable pairs.

`dbpenman@essex.ac.uk`

University of Essex

A zero-free interval for chromatic polynomials of graphs with 3-leaf spanning trees

Thomas Perrett

05C31

The *chromatic polynomial* $P(G, t)$ of a graph G is a polynomial with integer coefficients which counts, for each non-negative integer t, the number of proper t-colourings of G. A real number t is called a *chromatic root* of G if $P(G, t) = 0$. An interval $I \subseteq \mathbb{R}$ is called *zero-free* for a class of graphs $\mathcal{G}$ if no $G \in \mathcal{G}$ has a chromatic root in I. It is known [1] that $(-\infty, 0)$, $(0, 1)$, and $(1, 32/27]$ are maximal zero-free intervals for the class of all graphs, but one may find larger zero-free intervals for more restrictive classes. Indeed, Thomassen [2] showed that for graphs with a Hamiltonian path, the interval $(1, t_1]$ is maximally zero-free, where $t_1 \approx 1.295$ is the unique real root of the polynomial $(t-2)^3 + 4(t-1)$. We build on the techniques of Thomassen to prove that, for graphs containing a spanning tree with at most three leaves, the interval $(1, t_2]$ is maximally zero-free, where $t_2 \approx 1.2904$ is the smallest real root of the polynomial $(t-2)^6 + 4(t-1)^2(t-2)^3 - (t-1)^4$.

References.

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[2] C. Thomassen, Chromatic roots and Hamiltonian paths, *J. Combin. Theory Ser. B* **80**, (2000) 218-224.

tper@dtu.dk

Technical University of Denmark

Equitably Coloured BIBDs

David A. Pike

joint work with Robert D. Luther

05B05, 05C15

A balanced incomplete block design (BIBD) with parameters v, k and λ consists of a v-set V of points together with a set $\mathcal{B}$ of k-subsets of V called blocks, such that each 2-subset of V is a subset of exactly λ blocks of $\mathcal{B}$. A colouring of a design $(V, \mathcal{B})$ is a function $f : V \to C$, where $C = \{c_1, \ldots, c_\ell\}$ is a set of elements called colours. A weak colouring of a design is a colouring f such that $|\{f(x) : x \in B\}| > 1$ for each $B \in \mathcal{B}$ (i.e., each block has at least two colours). An equitable colouring is a colouring such that for each block $B \in \mathcal{B}$ the number of points of any colour $c_i \in C$ is within 1 of the number of points of any other colour $c_j \in C$ (i.e., $-1 \leq |B \cap C_i| - |B \cap C_j| \leq 1$, where $C_t = \{x \in V : f(x) = c_t\}$ denotes those points of V having colour c_t). We determine necessary and sufficient conditions for equitably colourable BIBDs.

dapike@mun.ca

Memorial University of Newfoundland

Directed Paths in the Cube

T. Pinto

05C35, 05D05

In 1997, Bollobás and Leader gave tight lower bounds for the number of edge-disjoint paths between disjoint subsets A and B of the hypercube, Q_n, in terms of $|A|$ and $|B|$. They conjectured that this bound holds even when A is a down-set, B is an up-set and the paths are required to be directed (that is, the vertices in the path form a chain). I will sketch a novel compression-type argument that proves a stronger version of this conjecture.

References.

[1] B. Bollobás and I. Leader, Matchings and paths in the cube, *Discrete Applied Mathematics* **75** (1997) 1-8.
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t.a.a.pinto@qmul.ac.uk

School of Mathematical Sciences, Queen Mary University of London

Point-primitive generalised polygons

Tomasz Popiel

joint work with J. Bamberg, S. P. Glasby, C. Schneider, C. E. Praeger

51B12, 20B15

We present some recent results about generalised polygons with collineation groups that act primitively on points. In particular, a group acting point-primitively on a generalised hexagon or octagon must be almost simple of Lie type.

tomasz.popiel@uwa.edu.au

Centre for the Mathematics of Symmetry and Computation, The University of Western Australia

Distant extensions of locally irregular graph colourings

J. Przybyło

MSC2010: 05C15; 05C78

It is well known that there are no *irregular graphs*, understood as simple graphs with pairwise distinct vertex degrees, except the trivial 1-vertex case. A graph invariant called the *irregularity strength* was thus introduced aiming at capturing a level of irregularity of a graph. Suppose that given a graph $G = (V, E)$ we wish to construct a multigraph with pairwise distinct vertex degrees of it by multiplying some of its edges. The least k so that we are able to achieve such goal using at most k copies of every edge is denoted by $s(G)$ and referred to as the irregularity strength of G. Alternately one may consider (not necessarily proper) edge colourings $c : E \to \{1, 2, \ldots, k\}$ with $\sum_{e \ni u} c(e) \neq \sum_{e \ni v} c(e)$ for every pair of distinct vertices $u, v \in V$. Then the least k which permits defining a colouring c with this feature equals $s(G)$. Numerous papers have been devoted to study on this graph invariant since the middle 80's. It also gave rise to many naturally related concepts and the general field of vertex distinguishing graph colourings. The list making up these includes e.g. Zhang's Conjecture, 1–2–3–Conjecture and many others.

Within the talk we shall briefly overview main results concerning these as well as their generalizations aiming at distinguishing vertices at a limited distance in a graph. One of the motivations of introducing these is the well known concept of distant chromatic numbers.

jakubprz@agh.edu.pl

AGH University of Science and Technology

On the chromatic number of exact distance graphs

D. Quiroz

joint work with J. van den Heuvel and H. A. Kierstead

Graph Theory

Given a graph $G = (V, E)$ and a positive integer p, define the *exact distance p graph* $G^{[\natural p]}$ as the graph having vertex set V and edge set $E^{[\natural p]}$ containing an edge xy if and only if the distance in G between vertices x and y (i.e., the number of edges in the shortest path joining x and y) is exactly p.

We consider the chromatic number of exact distance graphs. For p even, this number is unbounded even in the class of trees. However, for odd p the number is known to be bounded for many important classes of graphs. A result of Nešetřil and Ossona de Mendez tells us that for any class with bounded exapansion this number is bounded. This includes all classes with bounded degree and classes closed under taking minors. We present an alternative proof of this result. Our proof relies on the notion of generalized colouring numbers and allows us to obtain better upper bounds for the chromatic number of exact distance graphs for classes such as the class of planar graphs.

d.quiroz@lse.ac.uk

Department of Mathematics. London School of Economics.

Fair 1-factorizations and fair holey 1-factorizations of complete multipartite graphs

C. A. Rodger

Aras Erzurumluoğlu

05C15, 05B15, 05C70

If $V_1, ..., V_p$ are the p parts of the vertex set of $K(n, p)$,the complete multipartite graph with p parts, each of size n, then a *holey k-factor* of deficiency V_i of $K(n, p)$is a k-factor of $K(n, p) - V_i$ for some i satisfying $1 \leq i \leq p$. A *holey k-factorization* is a set of holey k-factors whose edges partition $E(K(n, p))$. Representing each (holey) k-factor as a color class in an edge-coloring, a (holey) k-factorization of $K(n, p)$ is said to be *fair* if between each pair of parts the color classes have size within one of each other (so the edges are shared "evenly" among the permitted (holey) factors). In this talk, the existence of fair (holey) 1-factorizations of $K(n, p)$ will be dicussed. This provides a new construction for symmetric quasigroups of order np with holes of size n. Such quasigroups have the additional property that in the cells in each $n \times n$ "box", the number of times each permitted symbol occurs is within one of the number of times each other permitted symbol occurs; so these quasigroups are as far from those produced by direct products as possible.

`rodgec1@auburn.edu`

Mathematics and Statistics

Ehrhart Polynomials and the Erdős Multiplication Table Problem

Robert Scheidweiler

joint work with Eberhard Triesch

11N25, 11B75

Let

$$P_m(n) := \left\{ \prod_{i=1}^{n} i^{\alpha_i} \,\middle|\, \alpha_i \in \mathbb{N}_0 \text{ and } \sum_{i=1}^{n} \alpha_i = m \right\}$$

be the set of products of m numbers from the set $\{1, \ldots, n\}$. In 1955 Erdős posed the problem of determining the order of magnitude of $|P_2(n)|$. This so-called Erdős Multiplication Table Problem was settled in 2008 by Ford in [2]. Koukoulopoulos determined the order of magnitude of $|P_m(n)|$ in [3]. Recently, Darda and Hujdurović asked in [1] if $|P_m(n)|$ is a polynomial in m of degree $\pi(n)$ - the number of primes not larger than n. Motivated by this question we present and discuss a connection between Ehrhart Theory and the Erdős Multiplication Table Problem.

References.

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scheidweiler@math2.rwth-aachen.de

Lehrstuhl II für Mathematik, RWTH Aachen University

On Covering Dimension of Linear Codes and Matroids

Keisuke Shiromoto

joint work with Thomas Britz

05B35, 94B05, 94B65

The *critical exponent* of a matroid is one of the important parameters in matroid theory and is related to the Rota and Crapo's Critical Problem (cf. [2]). This talk introduces the *covering dimension* of a linear code over a finite field, which is analogous to the critical exponent of a representable matroid. An upper bound on the covering dimension is proven, improving a classical bound for the critical exponent. Finally, a construction is given of linear codes that attain equality in the covering dimension bound.

References.

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[2] J.P.S. Kung, Critical problems, in: Matroid Theory, Seattle, WA, 1995, *Contemporary Mathematics*, **197**, American Mathematical Society, Providence, RI, pp. 1–127, 1996.

keisuke@kumamoto-u.ac.jp

Kumamoto University, Japan

Strong Oriented Graphs with Largest Directed Metric Dimension

Rinovia Simanjuntak

joint work with Yozef G. Tjandra

05C12, 05C20

Let D be a strongly connected oriented graph with vertex-set V and arc-set E. The distance from a vertex u to another vertex v, $d(u, v)$ is the minimum length of oriented paths from u to v. Suppose $B = \{b_1, b_2, b_3, ...b_k\}$ is a nonempty ordered subset of V. The representation of a vertex v with respect to B, $r(v|B)$, is defined as a vector $(d(v, b_1), d(v, b_2), \ldots, d(v, b_k))$. If any two distinct vertices u, v satisfy $r(u|B) \neq r(v|B)$, then B is a resolving set of D. If the cardinality of B is minimum then B is a basis of D and the cardinality of B is the directed metric dimension of D, $dim(D)$.

We proved that if D is a strongly connected oriented graph of order $n \geq 4$, then $dim(D) \leq n - 3$. Furthermore, we characterized strong oriented graphs attaining the upper bound, i.e., strong oriented graphs of order n and metric dimension $n - 3$.

rino@math.itb.ac.id

Combinatorial Mathematics Research Group, Institut Teknologi Bandung

Modularity phase transition in Erdős-Renyi random graphs

F. Skerman

joint work with C. McDiarmid

05C80, 05D40

An important problem in network analysis is to identify highly connected components or 'communities'. The most popular clustering algorithms work by approximately optimising modularity [2]. Given a graph G, the modularity of a partition of the vertex set measures the extent to which edge density is higher within parts than between parts; and the modularity $q(G)$ of G is the maximum modularity of a partition of $V(G)$ and takes a value in the interval $[0, 1)$ where one indicates a highly clustered graph.

To analyse the statistical significance of a partition found in a real network we should consider the maximum modularity of the corresponding random graph [1]. We show that the modularity of an Erdos-Renyi random graph exhibits a phase transition at criticality. In particular, for $\epsilon > 0$ then for edge probability $p \leq 1/n$ with high probability the random graph has modularity at least $1 - \epsilon$. Additionally there is a constant $c = c(\epsilon)$ such that with high probability the random graph with edge probability $p > c/n$ has modularity at most ϵ.

This is joint work with Prof. Colin McDiarmid.

References.

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skerman@stats.ox.ac.uk

Statistics Department, University of Oxford

Flagmatic and Turán densities

J. Sliacan

joint work with Oleg Pikhurko, Konstantinos Tyros

Graph theory, Extremal combinatorics

Subgraph densities are natural quantities to study. One of the systematic ways to do this in dense graphs is by using flag algebras - first coherently introduced by Razborov in 2007. The flag algebras framework is syntax-based and therefore implementable on a PC. The first general purpose flag algebras package was written by Emil Vaughan and is called Flagmatic. We have been extending Flagmatic to accommodate a wider class of density problems. This talk will give a brief introduction to the method of Flag algebras from the point of view of the Flagmatic package, then describe what the package is capable of. A few results will be reproved by Flagmatic to demonstrate its use. A few other will be shown anew to see that the use of Flagmatic goes beyond pedagogical.

j.sliacan@warwick.ac.uk

University of Warwick

A new product for permutation groups

Simon M. Smith

20B07, 20B15, 20B27

There are a number of ways in which one may take the product of two groups. Products which possess some kind of "universal" property (like the free and wreath products), or those which preserve some of the important properties of the input groups, are rare.

Arguably, the most important product in permutation group theory is the wreath product, acting in its product action. The reason for this is that, unlike other products, it preserves a fundamental property called *primitivity* (primitive permutation groups are the basic building blocks from which all finite permutation groups are comprised).

I am going to talk about a new product. It is fundamentally different to the wreath product in product action. Nevertheless, it preserves primitivity under astonishingly similar conditions. Moreover, the product has a "universal" property, and under natural conditions on groups M and N, the product of M and N is simple. The product can be used to easily solve a number of open problems.

sismith@citytech.cuny.edu

City University of New York (City Tech).

On a degree sequence analogue of Pósa's conjecture

Katherine Staden

joint work with Andrew Treglown

05C45, 05C38, 05C35

A famous conjecture of Pósa from 1962 asserts that every graph on n vertices and with minimum degree at least $2n/3$ contains the square of a Hamilton cycle. The conjecture was proven for large graphs in 1996 by Komlós, Sárközy and Szemerédi [1]. I will discuss a degree sequence version of Pósa's conjecture: Given any $\eta > 0$, every graph G of sufficiently large order n contains the square of a Hamilton cycle if its degree sequence $d_1 \leq \cdots \leq d_n$ satisfies $d_i \geq (1/3 + \eta)n + i$ for all $i \leq n/3$. The degree sequence condition here is approximately best possible.

References.

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k.l.staden@warwick.ac.uk

University of Warwick

Counting Partial Latin Rectangles

Rebecca J. Stones

joint work with Raúl Falcón

05B15

We describe four methods of enumerating $r \times s$ partial Latin rectangles on the symbol set $\{1, 2, \ldots, n\}$ with m non-empty cells: an Inclusion-Exclusion method, a chromatic polynomial method, an adaptation of Sade's method, and an algebraic geometry method. These methods have been implemented and give an asymptotic formula for arbitrary fixed m, exact formulae for small fixed m, exact numbers for small r, s, n and the exact number of isomorphism, isotopism, and main classes for small r, s, n.

rebecca.stones82@gmail.com

College of Computer and Control Engineering, Nankai University, Tianjin, China

Probabilistic Intuition in Waiter–Client and Client–Waiter games

Wei En Tan

joint work with Dan Hefetz and Michael Krivelevich

05C57, 91A24

For a finite set X, a family of sets $\mathcal{F} \subseteq 2^X$ and a positive integer q, we consider two types of two player, perfect information games with no chance moves. In each round of the $(1 : q)$ Waiter–Client game $(X, \mathcal{F})$, the first player, called Waiter, offers the second player, called Client, $q + 1$ elements of the board X which have not been offered previously. Client then claims one of these elements and the remaining q elements go back to Waiter. Waiter wins this game if, by the time every element of X has been claimed by some player, Client has claimed all elements of some $A \in \mathcal{F}$; otherwise Client is the winner. Client–Waiter games are defined analogously, with the main difference being that Client wins the game if he manages to claim all elements of some $A \in \mathcal{F}$ and Waiter wins otherwise. In this talk, we will look at the Waiter–Client and Client–Waiter versions of the non–planarity, K_t–minor and non–k–colourability games. For each such game, we give a fairly precise estimate of the unique integer q at which the outcome of the game changes from Client's win to Waiter's win. We also discuss the relationship between our results, random graphs, and the corresponding Maker–Breaker games.

wet916@bham.ac.uk

University of Birmingham

Pattern Avoidance and Non-Crossing Subgraphs of Polygons

M. Tannock

joint work with C. Bean and H. Ulfarsson

05A05, 05C30

We illustrate a connection between independent sets arising from the study of permutations avoiding the pattern 132 and the classical problem of enumerating certain non-crossing subgraphs. The methods developed can be extended to enumerate subclasses of permutations avoiding the pattern 1324, the only principal permutation class of length 4 that remains unenumerated.

More precisely, an independent set of k vertices in a graph we define and a sequence of positive integers of length k determine a unique permutation avoiding 132. Instead of enumerating the independent sets directly we show they are in bijection with the non-crossing subgraphs in a complete graph drawn on a regular polygon. Enumerating these non-crossing subgraphs is a classical problem, see e.g. Comtet [1]. For our purposes we use the generating function found by Flajolet and Noy [2]:

$$F(x, y) = 1 + x \cdot F(x, y) + \frac{xy \cdot F(x, y)^2}{1 - y \cdot (F(x, y) - 1)}.$$

Here x marks the left-to-right-minima of the permutation and y marks vertices in the graph (or edges in the polygon). Setting $y = y/(1-y)$, $x = xy$, and collecting by powers of y gives the Narayana triangle enumerating the permutations avoiding 132 by their number of left-to-right-minima.

Using the same techniques as above we show that there is a generalized polygon whose edges correspond to the vertices in the graph and enumerate certain subclasses of permutations avoiding 1324.

References.

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[2] P. Flajolet and M. Noy, Analytic combinatorics of non-crossing configurations,*Discrete Math.* **204** (1999) 203-229.

murray14@ru.is

Reykjavik University

A random version of Sperner's theorem

Andrew Treglown

joint work with József Balogh and Richard Mycroft

MSC2010 headings 06A07; 05C69

Let $\mathcal{P}(n)$ denote the power set of $[n]$, ordered by inclusion, and let $\mathcal{P}(n,p)$ be obtained from $\mathcal{P}(n)$ by selecting elements from $\mathcal{P}(n)$ independently at random with probability p. A classical result of Sperner [2] asserts that every antichain in $\mathcal{P}(n)$ has size at most that of the middle layer, $\binom{n}{\lfloor n/2 \rfloor}$. In this talk we give an analogous result for $\mathcal{P}(n,p)$: If $pn \to \infty$ then, with high probability, the size of the largest antichain in $\mathcal{P}(n,p)$ is at most $(1+o(1))p\binom{n}{\lfloor n/2 \rfloor}$. This solves a conjecture of Osthus [1] who proved the result in the case when $pn/\log n \to \infty$. Our condition on p is best-possible.

References.

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a.c.treglown@bham.ac.uk

University of Birmingham

Equivalence classes of Dyck paths modulo udu

P. G. Tsikouras[1]

joint work with K. Manes, A. Sapounakis and I. Tasoulas

MSC2010: 05A15

This work deals with lattice paths in the integer plane, which use two kinds of steps, rises (u) and falls (d). A well known class of such paths is the class of *Dyck paths*, which start and end at the same height and lie weakly above it. A wide range of papers dealing with the number of occurrences of a given path τ in a Dyck path, appear frequently in the literature.

In another direction, one can ask for the occurrences of τ to be in fixed positions. More precisely, we consider a particular equivalence relation "$\underset{\tau}{\sim}$" on the set $\mathcal{D}$ of Dyck paths as follows: For $P, Q \in \mathcal{D}$, we have that $P \underset{\tau}{\sim} Q$ iff P, Q have the same length and the positions of the occurrences of τ in P and Q are the same. This equivalence relation was introduced in a recent paper by Baril and Petrossian [1], where the number of equivalence classes has been evaluated for $\tau = \mathrm{uu}, \mathrm{ud}, \mathrm{du}, \mathrm{dd}$.

In this work, we solve the above problem for the more complex case of $\tau = \mathrm{udu}$. For this, we consider the extension of the relation $\underset{udu}{\sim}$ to the set $\mathcal{P}$ of all prefixes of Dyck paths. Next, we define a certain subset $\mathcal{A}$ of $\mathcal{P}$ and we show that each equivalence class of $\underset{udu}{\sim}$ on $\mathcal{P}$ contains exactly one element of $\mathcal{A}$. Then, the elements of $\mathcal{A}$ which are udu-equivalent to some Dyck path are fully characterized. Using this characterization, we evaluate in terms of generating functions the number of these elements of $\mathcal{A}$ and hence the required number of Dyck equivalence classes of $\underset{udu}{\sim}$.

References.

[1] J. L. Baril and A. Petrossian, Equivalence classes of Dyck paths modulo some statistics, *Discrete Math.* **338** (2015) 655-660.

pgtsik@unipi.gr

Department of Informatics, University of Piraeus, Piraeus, Greece

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General stability and exactness theorems

Konstantinos Tyros

joint work with O. Pikhurko and J. Sliacan

MSC2010: 05C35

Classical problems in extremal combinatorics concern the maximal possible density of copies of some graph (or more general of some quantum hypergraph) in the members of some hereditary class of graphs. Such is Turan's problem. Up to now several other special cases have been treated separately. In a joint work with O. Pikhurko and J. Sliacan we investigate necessary conditions in order to be able to calculate the asymptotic densities, as well as, to have stability and exactness.

k.tyros@warwick.ac.uk

Mathematics Institute, University of Warwick, Coventry, CV4 7AL, UK

Generalised Colouring Numbers of Graphs

Jan van den Heuvel

joint work with Patrice Ossona de Mendez, Daniel Quiroz, Roman Rabinovich and Sebastian Siebertz

05C15, 05C75

The colouring number col(G) of a graph G is one plus the minimum integer k such that there exists a linear ordering of the vertices of G in which each vertex v has back-degree at most k, i.e. v has at most k neighbours u that appear before v in the ordering. The colouring number is a structural measure that measures the edge density of subgraphs of G. The colouring number is also a trivial upper bound for the chromatic number of G.

When instead of neighbours u that appear before v in the linear ordering, for some integer $r \geq 1$ we count vertices u that appear before v in the ordering and such that there is an uv-path of length at most r, we get the generalised colouring numbers. In fact, we get different types of generalised colouring numbers, depending on the required position of the internal vertices of the uv-paths in the ordering with respect to u and/or v.

In this talk we give an overview of the relations between those generalised colouring numbers and specific types of colourings of graphs; we give some new bounds on these numbers for specific classes of graphs (such as planar and minor-closed graphs); and we discuss relations between those numbers and structural properties of graphs (such as tree-depth and tree-width).

j.van-den-heuvel@lse.ac.uk

Department of Mathematics, London School of Economics and Political Science, London, UK

Asymmetric Coloring Games on Incomparability Graphs

Bartosz Walczak

joint work with Tomasz Krawczyk

05C15, 05C57

Consider the following game on a graph G: Alice and Bob take turns coloring the vertices of G properly from a fixed set of colors; Alice wins when the entire graph has been colored, while Bob wins when some uncolored vertices have been left. The *game chromatic number* of G is the minimum number of colors that allows Alice to win the game. The *game Grundy number* of G is defined similarly except that the players color the vertices according to the first-fit rule and they only decide on the order in which it is applied. The (a, b)*-game chromatic* and *Grundy numbers* are defined likewise except that Alice colors a vertices and Bob colors b vertices in each round. We study the behavior of these parameters for incomparability graphs of posets with bounded width. We conjecture a complete characterization of the pairs (a, b) for which the (a, b)-game chromatic and Grundy numbers are bounded in terms of the width of the poset; we prove that it gives a necessary condition and provide some evidence for its sufficiency. We also show that the game chromatic number is not bounded in terms of the Grundy number, which answers a question of Havet and Zhu.

walczak@tcs.uj.edu.pl

Theoretical Computer Science Department, Faculty of Mathematics and Computer Science, Jagiellonian University, Kraków, Poland

Symmetries of Latin Squares

Ian M. Wanless

joint work with M. Jayama L. Mendis

05B15;20N05

Paratopism is a well-known action of the wreath product $\mathcal{S}_n \wr \mathcal{S}_3$ on Latin squares of order n. Orbits under paratopism are called *species* (or *main classes*). A paratopism that maps a Latin square to itself is an *autoparatopism* of that Latin square. Let $\mathrm{Par}(n)$ denote the set of paratopisms that are an autoparatopism of at least one Latin square of order n. We prove a number of general properties of autoparatopisms. Between them, they are sufficient to determine $\mathrm{Par}(n)$ for $n \leq 17$. We also make some observations about the asymptotic proportion of paratopisms that lie in $\mathrm{Par}(n)$.

ian.wanless@monash.edu

School of Mathematical Sciences, Monash University, Australia

Subsystems of Netto triple systems

Bridget S. Webb

joint work with Darryn E. Bryant and Barbara M. Maenhaut

05B07

The Netto triple systems are a class of Steiner triple systems having order $q = p^n$ where $n \geq 1$, p is prime, and $q \equiv 7 \pmod{12}$, and there is a unique (up to isomorphism) Netto triple system for each such order. For $q \neq 7$, their full automorphism group acts transitively on unordered pairs of points but not on ordered pairs of points, and they are the only Steiner triple systems with this property.

Netto triple systems are block-transitive, cyclic, uniform, anti-mitre, and are block-regular if and only if $q \equiv 7$ or $31 \pmod{36}$. The elements of a field of order q form the point set of a Netto triple system of order q, and the blocks can be generated from the triple $\{0, 1, \alpha\}$ where α is a primitive sixth root of unity.

In 1975 Robinson conjectured that Netto triple systems of prime order have no non-trivial subsystems. We investigate this conjecture and also study subsystems of Netto triple systems of composite order.

bridget.webb@open.ac.uk

The Open University

Grasshopper pattern avoidance

Krzysztof Węsek

joint work with Michał Dębski and Urszula Pastwa

05A05, 68R15

Pattern avoidance is an important topic in the area of *combinatorics on words*, branch of mathematics partly inspired by the work of Axel Thue at the dawn of 20th century. We say that a pattern p (i.e. finite sequence over a set of variables E) *occurs* in a word w (over an alphabet A) if there exists a substitution f from E to the set of nonempty sequences over A such that $f(p)$ is a block of consecutive elements in w. The classic goal is to construct arbitrarily long words over a small alphabet without occurrence of a given pattern - for example, Thue proved that 3 symbols suffice for the pattern α^2.

In this talk we discuss a new variant of pattern avoidance. For a sequence w, a subsequence $w_{i_1}w_{i_2}...w_{i_n}$ is *almost consecutive* if $(i_{j+1} - i_j) \in \{1, 2\}$ for every j. We say that a pattern p *occurs with jumps* in a word w if p occurs in any almost consecutive subsequence of w. A pattern p is *grasshopper k-avoidable* if there exists an alphabet A of k elements, such that there exist arbitrarily long words over A in which p does not occur with jumps. The minimal such k is the *grasshopper avoidability index* of p. We almost completely determine the grasshopper avoidability index of patterns α^n. We use entropy compression method to obtain results on grasshopper avoidability of patterns in two classes: patterns without variables used exactly once, and patterns which are long in terms of their number of variables. Moreover, we state some open problems concerning this new notion.

k.wesek@mini.pw.edu.pl

Faculty of Mathematics and Information Science, Warsaw University of Technology, Poland

Weak Safe sets and Relaxed Safe set

Renyu Xu

joint work with Sylvain Legay, Yannis Manoussakis, Raquel Agueda Mate, Yasuko Matsui, Leandro Montero, Raquel Diaz Sanchez

05C90

A non-empty subset S of the vertices of a connected graph $G = (V, E)$ is a weak safe set if, for every component D of $G - S$, there exists a component C of $G[S]$ such that $|C| \geq |D|$ whenever there exists an edge of G between D and C. A non-empty subset S is a relaxed safe set if each component A in $G-S$ is at most as large as the sum of the sizes of its adjacent components B_i inside S. The weak safe number $ws(G)$ of G is defined as $ws(G) = \min\{|S| : S$ is a weak safe set of $G\}$, the relaxed safe number $rs(G)$ of G is defined as $rs(G) = \min\{|S| : S$ is a relaxed safe set of $G\}$. By the definition, we have if S is a weak safe set, then S is a relaxed safe set. So $ws(G) \geq rs(G)$. Both the minimum weak safe set and minimum relaxed safe set are NP-complete, we discuss the minimum sizes of weak and relaxed safe sets in connected graphs. For weak safe set, we proved

Theorem 1 For any path P_n of order n, $ws(P_n) = \lceil \frac{n}{3} \rceil$.

Theorem 2 A minimum weak safe set can be found in linear time on trees.

Theorem 3 A minimum weak safe set can be found in polynomial time on interval graphs.

For relaxed safe set, we proved:

Theorem 4 For any path P_n of order n, $rs(P_n) = \lceil \frac{n}{3} \rceil$.

Theorem 5 Let T be a tree of order n with k leaves. Then $rs(T) \geq \frac{n}{k+1}$.

Theorem 6 A minimum relaxed safe set can be found in $O(n^2)$ time on trees.

Theorem 7 A minimum relaxed safe set can be found in polynomial time on interval graphs.

References.

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`renyu.xu@lri.fr`

LRI, University Paris-Sud, France

The Local Stability Method

Liana Yepremyan

joint work with Sergey Norin

MSC2010 headings

In this talk we consider r-uniform hypergraphs, which we shortly call r-graphs. Let $\mathfrak{F}$ be a family of r-graphs, then an r-graph $\mathcal{H}$ is called $\mathfrak{F}$-free if it does not contain any r-graph from $\mathfrak{F}$ as a subgraph. The *Turán Number* $ex(n, \mathfrak{F})$ for a family of r-graphs $\mathfrak{F}$ is defined to be the maximum number of edges in an $\mathfrak{F}$-free r-graph on n vertices. The *Turán density* of the family of r-graphs $\mathfrak{F}$ (or similarly, for an r-graph) is defined to be the following limit

$$\pi(\mathfrak{F}) = \lim_{n \to \infty} \frac{\mathrm{ex}(n, \mathfrak{F})}{\binom{n}{r}}.$$

In this talk we will demonstrate a technique for deriving exact results for the Turán Numbers from so-called *"local stability"* results. We use this method to determine Turán Numbers of generalized triangles for $r = 5, 6$, confirming a conjecture of Frankl and Füredi and Turán numbers of *enlargements* of some r-graphs.

liana.yepremyan@mail.mcgill.ca

Computer Science Department, McGill University

Hamilton saturated hypergraphs of essentially minimum size

A. Żak

joint work with A. Ruciński

MSC2010 05C45, 05C65

For $1 \leq \ell < k$, an *ℓ-overlapping cycle* is a k-uniform hypergraph in which, for some cyclic vertex ordering, every edge consists of k consecutive vertices and every two consecutive edges share exactly ℓ vertices.

A k-uniform hypergraph H is *ℓ-Hamiltonian saturated* if H does not contain an ℓ-overlapping Hamiltonian cycle but every hypergraph obtained from H by adding one more edge does contain such a cycle. Let $\mathrm{sat}(n, k, \ell)$ be the smallest number of edges in an ℓ-Hamiltonian saturated k-uniform hypergraph on n vertices. In the case of graphs Clark and Entringer proved in 1983 that $\mathrm{sat}(n, 2, 1) = \lceil \frac{3n}{2} \rceil$.

For hypergraphs with $k \geq 3$ it seems to be quite hard to obtain such precise results. Therefore, the emphasis is put on the order of magnitude of $\mathrm{sat}(n, k, \ell)$. We proved that for $k \geq 3$ and $\ell = 1$ as well as for all $0.8k \leq \ell \leq k - 1$

$$\mathrm{sat}(n, k, \ell) = \Theta(n^{\ell}).$$

Recently, we have got also an upper bound for all ℓ, k, $1 \leq \ell \leq k - 1$:

$$\mathrm{sat}(n, k, \ell) = O(n^{(k+\ell)/2})$$

which we have improved in the smallest open case to

$$\mathrm{sat}(n, 4, 2) = O(n^{14/5}).$$

The case $\ell = 0$ (perfect matching) is widely open for $k \geq 3$.

zakandrz@agh.edu.pl

AGH University of Science and Technology

Well-quasi-ordering does not imply bounded clique-width

V. Zamaraev

joint work with V. Lozin and I. Razgon

05C99

Well-quasi-ordering is a highly desirable property and frequently discovered concept in mathematics and theoretical computer science. One of the most remarkable recent results in this area is the proof of Wagner's conjecture stating that the set of all finite graphs is well-quasi-ordered by the minor relation [5]. This is, however, not the case for the induced subgraph relation, since the set of cycles forms an infinite antichain with respect to this relation. On the other hand, the induced subgraph relation may become a well-quasi-order when restricted to graphs in particular classes, such as cographs [2] or k-letter graphs [4]. It is interesting to observe that in both examples we deal with graphs of bounded clique-width, which is another property of great importance in mathematics and computer science. Moreover, the same is true for all available examples of graph classes which are well-quasi-ordered by the induced subgraph relation (see e.g. [3]). This raises an interesting question whether the clique-width is always bounded for graphs in well-quasi-ordered classes. This question was formally stated as an open problem by Daligault, Rao and Thomassé in [1]. We answer the question negatively by exhibiting a hereditary class of graphs of unbounded clique-width which is well-quasi-ordered by the induced subgraph relation.

References.

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`V.Zamaraev@warwick.ac.uk`

Mathematics Institute, University of Warwick, UK

On k-threshold functions

E. Zamaraeva

52B05, 52B11, 52B12

The $\{0, 1\}$-valued function f over integer d-dimensional cube $\{0, 1, \ldots, n-1\}^d$ is called threshold if there exists a hyperplane which separates 0-valued points from 1-valued points. Function that can be represented as a conjunction of k threshold functions is called k-threshold function. There is one-to-one correspondence between class of all k-threshold functions for all k and polytopes with integer vertices in E_n^d. I consider essential points and teaching sets of this class and reveal that a polytope with vertices in d-dimensional cube has the unique minimal teaching set which is equal to the set of its essential points. For $d = 2$ I describe structure of minimal teaching set for a polytope and show that cardinality of this set is either $\Theta(n^2)$ or $O(n)$ and depends on perimeter and minimum angle of the polytope.

References.

[1] Zamaraeva E. On teaching sets of k-threshold functions, *preprint* `http://arxiv.org/abs/1502.04340`

`elena.zamaraeva@gmail.com`

Nizhny Novgorod State University, Gagarin ave. 23, Nizhny Novgorod 603600, Russia

Alphabetical index of speakers