Warwick Mathematics Institute

Relations between Banach Space Theory and Geometric Measure Theory

08-12 June 2015

Organisers: David Preiss (Warwick), Olga Maleva (Birmingham),
Martin Rmoutil (Warwick), Daniel Seco (Warwick) and Thomas Zürcher (Warwick)

Venue: Room MS.01, Warwick Mathematics Institute, Zeeman Building, University of Warwick, Coventry, United Kingdom

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Program

Times include 5 minutes discussion/break.

	Monday	Tuesday	Wednesday	Thursday	Friday
8:30— 9:30	Registration				
9:30—10:30	Naor	Johnson	Schlumprecht	Godefroy	Azagra
10:30—11:00	*Coffee*	*Coffee*	*Coffee*	*Coffee*	*Coffee*
11:00—12:00	Ostrovskii	Bogachev	Kirchheim	Castillo	Schechtman
12:00—12:30	Pellegrino	Roginskaya	Fonf	Lajara	Sarı
12:30—13:00	Kalenda	Botelho	Zanco	Oja	
13:00—14:30	*Lunch*	*Lunch*	*Lunch*	*Lunch*	*Lunch*
14:30—14:50	Mbombo	R Lechner	Garbulińska	Ain	Khurana
14:50—15:10	Tradacete	Wallis	Reynov	Treialt	Máthé
15:10—15:30	Ortega Castillo	Martínez Cervantes	Albuquerque	Puglisi	
15:30—15:50	Fávaro	Galicer	Talponen	Bargetz	
15:50—16:30	*Coffee*	*Coffee*	*Coffee*	*Coffee*	
16:30—17:00	Baudier	Fabian	Speight	Raja	
17:00—17:30	Avilés	Zajíček	Dymond	Procházka	
17:30—18:00	Ferenczi	Tišer	Wilson	Laustsen	
18:00—18:30	*Evening buffet*	*Evening buffet*	*Evening buffet*		
18:30—				*Conference dinner*	

Null sequences which are defined by ℓ_p spaces

Kati Ain

Let X be a Banach space and let $c_0(X)$ denote the space of null sequences in X. Recently, Delgado and Piñeiro [3] introduced and studied an interesting class of p-null sequences, where $p \geq 1$, which is a linear subspace of $c_0(X)$. On the other hand, there is a strong form of compactness, the p-compactness, that has been studied extensively during last years. The p-null sequences can be characterized via p-compactness as being exactly the null sequences that are relatively p-compact.

Let $1 \leq p \leq \infty$ and $1 \leq r \leq p^*$, where p^* is the conjugate index of p. The concept of p-compactness was generalized in [1] to (p,r)-compactness, so encompassing also other strong forms of compactness. Similarly, we extend the notion of p-null sequences to (p,r)-null sequences. For instance, the (p,p^*)-compactness is exactly the p-compactness and (p,p^*)-null sequences coincide with p-null sequences.

We establish numerous equivalences for a sequence (x_n) in a Banach space X to be a (p,r)-null sequence. One of them is that (x_n) is (p,r)-null if and only if (x_n) is null and relatively (p,r)-compact.

The notion of (p,r)-null sequence is further extended to unconditionally and weakly (p,r)-null sequences, which are also characterized via corresponding forms of (p,r)-compactness.

This is a joint work with Eve Oja and is based on [2].

References

[1] K. Ain, R. Lillemets, E. Oja, Compact operators which are defined by ℓ_p-spaces, Quaest. Math. 35, 145–159 (2012).

[2] K. Ain, E. Oja, On (p,r)-null sequences and their relatives, Math. Nachr. (to appear).

[3] C. Piñeiro, J. M. Delgado, p-Convergent sequences and Banach spaces in which p-compact sets are q-compact, Proc. Amer. Math. Soc. 139, 957–967 (2011).

Nacib Albuquerque

Hölder's inequality on mixed L_p spaces and summability of multilinear operators

Nacib Albuquerque

Recently, Hölder's inequality on mixed L_p spaces was used to obtain several results, new approaches and applications. For instance, the Bohnenblust–Hille and Hardy–Littlewood multilinear inequalities and its recent consequences. We present how this lead us to design a unifying theory on summability of multilinear operators.

Antonio Avilés

Extension operators on balls and on spaces of finite sets

Antonio Avilés

We study extension operators between the compact spaces of subsets of a set X of cardinality at most n. As an application, we show that there are no extension operators between balls of different radii in a nonseparable Hilbert space. This is a joint work with Witold Marciszewski.

Daniel Azagra

On smooth approximation and smooth convex extensions of convex functions

Daniel Azagra

In the first part of this talk we will review some known results on smooth approximation and on smooth extension of functions defined on subsets of Banach spaces, paying special attention to the cases of Lipschitz and of convex functions. In the second part, we will discuss some new results and several open problems about smooth convex extensions of convex functions defined on subsets of $\mathbb{R}^n$ or Banach spaces.

Christian Bargetz

σ-Porosity of the set of strict contractions in a space of non-expansive mappings

Christian Bargetz

Let X be a separable Banach space and $C \subset X$ be a bounded, closed and convex subset. If we equip the set of all non-expansive mappings on C with the metric of uniform convergence, we obtain a complete metric space $\mathcal{M}$. We show that the subset

$$\mathcal{N} = \{f \colon C \to C \colon \operatorname{Lip}(f) < 1\} \subset \mathcal{M}$$

is σ-upper porous. For Hilbert spaces X a similar result has been obtained by F. S. de Blasi and J. Myjak in [1].

This is joint work with Michael Dymond.

References

[1] Francesco S. De Blasi and Józef Myjak. Sur la porosité de l'ensemble des contractions sans point fixe. *C. R. Acad. Sci. Paris Sér. I Math.*, 308(2):51–54, 1989.

Florent Baudier

Introducing almost Lipschitz and nearly isometric embeddability

Florent Baudier

The notions of almost Lipschitz embeddability and nearly isometric embeddability are introduced and discussed. These two relaxations of bi-Lipschitz embeddability appear naturally when trying to faithfully embed proper metric spaces (resp. stable metric spaces) into Banach spaces.

Joint work with G. Lancien.

Vladimir Bogachev

Sobolev and BV classes on infinite-dimensional domains

Vladimir Bogachev

This talk will be a survey of recent results on Sobolev and BV classes of

functions on infinite-dimensional domains with differentiable measures, in particular, with Gaussian measures. Given a reasonable measure on a domain in an infinite-dimensional space, Sobolev classes can be defined by means of completions of suitable classes of smooth functions with respect to natural Sobolev norms or by means of generalized derivatives, similarly to the finite-dimensional case. However, the lack of analogs of Lebesgue measure causes new phenomena. Classes of functions of bounded variation (BV) can be also defined by means of completions and integrations by parts, but in infinite dimensions different definitions arise also due to the fact that one can consider vector measures of bounded variation or of bounded semivariation. For all these classes of Sobolev or BV functions, the problem of extension to the whole space will be discussed and some counter-examples will be considered.

Differential Pietsch measures for dominated polynomials on Banach spaces

Geraldo Botelho

By definition, a continuous n-homogeneous polynomial $P\colon E \longrightarrow F$ between Banach spaces is p-dominated if P sends weakly p-summable sequences in E to absolutely $\frac{p}{n}$-summable sequences in F. Of course the linear case $n = 1$ recovers the ideal of absolutely p-summing linear operators. A Pietsch-type domination theorem asserts that P is p-dominated if and only if there are a constant $C > 0$ and a regular Borel probability measure on $B_{E'}$ endowed with the weak* topology such that

$$\|P(x)\| \leq C \left(\int_{B_{E'}} |\varphi(x)|^p d\mu(\varphi) \right)^{\frac{n}{p}} \text{ for every } x \in E.$$

Any such measure μ is called a *Pietsch measure* for P. It is also well known that if P is p-dominated, then all its derivatives $\widehat{d}^k P(a)$, $a \in E$, $k = 1, \ldots, n$, are p-dominated homogeneous polynomials as well, so all of them admit Pietsch measures. Can we find a common Pietsch measure for P and all its derivatives? We say that a Pietsch measure for P is a *differential Pietsch measure* if it is a Pietsch measure for $\widehat{d}^k P(a)$ for all $a \in E$ and $k = 1, \ldots, n$. In this talk we shall prove that every p-dominated polynomial admits a differential Pietsch measure and give an application of this result to the problem concerning the injectivity of the canonical map $j_p\colon C(B_{E'}) \longrightarrow L_p(\mu)$. This is a joint work

with Daniel Pellegrino and Pilar Rueda. The author is partially supported by CNPq Grant 305958/2014-3 and Fapemig Grant PPM-00326-13.

Kadets = Gurariy

Jesús M. F. Castillo

Gurariy constructed the first separable space $\mathcal{G}$ of almost universal disposition. This space was later shown to be the only separable space of almost universal disposition up to isometries. Moreover, it is isometrically universal for all separable spaces. Kadets constructed a separable Banach space $\mathcal{K}$ with BAP that (isomorphically) contains complemented copies of all separable spaces with BAP; which somehow is optimal since Johnson and Szankowski showed that no separable Banach space can be complementably universal for all separable Banach spaces.

We will show that the Gurariy and Kadets space are "the same" object, although in different categories. In particular:

- The Gurariy space:
  1. Is a unique object in the category of separable Banach spaces and into isometries (single arrows).
  2. It can be obtained as the Fraïssé limit of finite dimensional rational Banach spaces and single arrows.
  3. It can be constructed via an ω-times iterated push-out from a countable dense set of single arrows between finite-dimensional Banach spaces.
  4. It contains isometric copies of all separable Banach spaces.
- The Kadets space:
  1. Is a unique object in the category of separable Banach spaces with 1-FDD and into isometries admitting norm one projections (double arrows).
  2. It can be obtained as the Fraïssé limit of finite dimensional rational Banach spaces and double arrows.

3. It can be constructed via an ω-times iterated push-out from a countable dense set of double arrows between finite-dimensional Banach spaces.
4. It contains isometric 1-complemented copies of all separable Banach spaces with 1-FDD. As a by-product, it contains isometric copies of all separable Banach spaces.

We will also discuss how abstract (in particular, nonseparable) Gurariy and Kadets spaces look like, touching the questions: How many non-isomorphic Gurariy (or Kadets) spaces having a given density character are there? Does a Kadets space contain complemented copies of spaces with BAP?

This talk is based on a joint work with Yolanda Moreno.

Michael Dymond

On the structure of universal differentiability sets

Michael Dymond

Subsets of $\mathbb{R}^n$ containing a point of differentiability of every Lipschitz function are a subject of significant modern research. In this talk we discuss the nature of such sets, focussing in particular on various strong structural properties that they exhibit. We also indicate possible directions of future research in this area.

The research presented in this talk formed part of my PhD thesis, supervised by Olga Maleva.

Marián Fabian

Rich families in Asplund spaces and separable reduction of Fréchet (sub)differentiability

Marián Fabian

Let $(\mathcal{P}, \prec)$ be an up-directed poset. A family $\mathcal{R} \subset \mathcal{P}$ is called rich if it is (i) cofinal/dominating/saturating, and (ii) it is σ-complete, that is, every increasing sequence in it has supremum belonging again to $\mathcal{R}$. Let $(X, \|\cdot\|)$ be an Asplund, rather non-separable, Banach space, i.e., every separable subspace of it has separable dual. Let $\mathcal{S}_{\square}(X \times X^*)$ denote the family of all

"rectangles" $V \times Y$ where $V,\ Y$ are any closed separable subspaces of X and X^*, respectively, and endow it with the partial order "$\subset$". Then there exists a rich family $\mathcal{A} \subset \mathcal{S}_{\square}(X \times X^*)$ such that for every $V \times Y \in \mathcal{A}$ the assignment $Y \ni x^* \longmapsto x^*|_V \in V^*$ is an *isometry onto* V^*. This structural result is then used to find a rich subfamily $\mathcal{R} \subset \mathcal{A}$ such that for every $V \times Y \in \mathcal{R}$, every $v \in V$, and every $v^* \in \partial(f|_V)(v)$ there is a unique $x^* \in \partial f(v) \cap Y$ such that $x^*|_V = v^*$ and $\|x^*\| = \|v^*\|$; here ∂f means the Fréchet subdifferential of a function $f\colon X \longrightarrow (-\infty, +\infty]$. This way, we get (exact) separable reductions of several statements from variational analysis: $\partial f(x) \neq \emptyset$, fuzzy sum rule for $\partial(f_1 + f_2)(x)$, etc. The talk is based on a forthcoming paper by Marek Cúth and the lecturer.

Vinícius Vieira Fávaro

New topologies for some spaces of n-homogeneous polynomials and applications on hypercyclicity of convolution operators

Vinícius Vieira Fávaro

Let $n \in \mathbb{N}$ and suppose that $(\mathcal{P}_{\Delta}({}^nE), \|\cdot\|_{\Delta})$ is a quasi-normed space of n-homogeneous polynomials defined on E such that the inclusion $\mathcal{P}_{\Delta}({}^nE) \hookrightarrow \mathcal{P}({}^nE)$ is continuous and the space $\mathcal{P}_f({}^nE)$ of all finite type n-homogeneous polynomials is contained in $\mathcal{P}_{\Delta}({}^nE)$. Suppose that a normed space

$$(\mathcal{P}_{\Delta'}({}^nE'), \|\cdot\|_{\Delta'}) \subset \mathcal{P}({}^nE')$$

is such that the Borel transform

$$\mathcal{B}\colon\ (\mathcal{P}_{\Delta}({}^nE)', \|\cdot\|) \to (\mathcal{P}_{\Delta'}({}^nE'), \|\cdot\|_{\Delta'})$$

given by $\mathcal{B}(T)(\varphi) = T(\varphi^n)$, for all $\varphi \in E'$ and $T \in \mathcal{P}_{\Delta}({}^nE)'$, is a topological isomorphism.

In this work we develop general approach to create a slightly different space, instead of $\mathcal{P}_{\Delta}({}^nE)$, which is a normed space and preserves the duality result above. More precisely, we create a new topology for $\mathcal{P}_{\Delta}({}^nE)$ such that the completion of this space, denoted by $\left(\mathcal{P}_{\widetilde{\Delta}}({}^nE), \|\cdot\|_{\widetilde{\Delta}}\right)$, becomes a Banach space and preserves the duality result above, that is,

$$\widetilde{\mathcal{B}}\colon\ \left(\mathcal{P}_{\widetilde{\Delta}}({}^nE)', \|.\|\right) \longrightarrow \left(\mathcal{P}_{\Delta'}({}^nE'), \|.\|_{\Delta'}\right)$$
$$\widetilde{\mathcal{B}}(T)(\varphi) = T(\varphi^n)$$

is a topological isomorphism.

The technique to create this topology uses results of Banach and topological vector spaces theory. Using [1] and this new technique we generate new examples of hypercyclic convolution operators in situations where the known techniques are not effective.

This is a joint work with D. Pellegrino.

The author is supported by FAPESP Grant 2014/50536-7.

References

[1] F. J. Bertoloto, G. Botelho, V. V. Fávaro, A. M. Jatobá, *Hypercyclicity of convolution operators on spaces of entire functions*. Ann. Inst. Fourier (Grenoble) **63** (2013), 1263–1283.

Valentin Ferenczi

Approximate Ramsey properties of finite dimensional ℓ_p spaces

Valentin Ferenczi

We present new Ramsey properties of finite dimensional ℓ_p spaces, $0 < p < +\infty$. This extends results of Gromov–Milman and Odell–Rosenthal–Schlumprecht, and gives a new proof of the extreme amenability of the isometry group of the space L_p. The proof uses concentration of measure for approximate equi-partitions. Joint work with Jordi Lopez-Abad, Brice Mbombo, and Stevo Todorcevic.

Vladimir Fonf

Applications of almost over total sequences

Vladimir Fonf

A sequence of elements in the dual Banach space is called almost over total (AOT) if annihilator of any its subsequence has finite co-dimension (in the initial space). We prove that any normalized AOT sequence is relative norm-compact and give several applications of this fact.

Daniel Galicer

Energy Integrals, Metric Embedding Theory and Summing Operators

Daniel Galicer

Let $K \subset \mathbb{R}^n$ be a compact set endowed with the metric $d_\alpha(x,y) = |x-y|^\alpha$, where $0 < \alpha < 1$. A classical result of Schoenberg and von Neumann asserts that there exist a minimum r for which the metric space (K, d_α) may be isometrically embedded on the surface of a Hilbert sphere of radius r. We provide estimates of these radii for several centrally symmetric convex bodies K. To this end, we study the energy integral

$$\sup \int_K \int_K |x-y|^{2\alpha} d\mu(x) d\mu(y),$$

where the supremum runs over all finite signed Borel measures μ on K of total mass one. We bound this value by the mean width of K or the 2α-summing norm of certain operator. In the case where K is an ellipsoid or $K = B_q^n$, the unit ball of ℓ_q^n (for $1 \leq q \leq 2$), we obtain the correct asymptotical behavior of the least possible radius.

Joint work with Daniel Carando and Damián Pinasco.

Joanna Garbulińska-Wgrzyn

Universal structures in Banach spaces

Joanna Garbulińska-Wgrzyn

We shall present several results concerning isometrically universal structures in separable Banach spaces.

Gilles Godefroy

Lipschitz-free Banach spaces and the approximation properties

Gilles Godefroy

Let M be a metric space, equipped for convenience with a distinguished point 0. The space of real-valued functions on M which vanish at 0 is a Banach space, which happens to be isometric to the dual of the closed linear span of the Dirac measures. This predual $\mathcal{F}(\mathcal{M})$ is called the free space over M and it provides in particular a linearization diagram for Lipschitz maps between

metric spaces. Although free spaces are easy to define, their structure is quite difficult to analyse. In this talk, we will investigate the relations between the validity of Grothendieck's bounded approximation property for the free space over a compact metric space M, the existence of linear extension operators for Lipschitz functions and the existence of extensions for Banach-space valued Lipschitz functions.

William B. Johnson

Some recent results in non separable Banach space theory

William B. Johnson

I'll discuss three recent papers, one with Gideon Schechtman, the second with Amir Bahman Nasseri, Schechtman, and Tomasz Tkocz, the third with Tomasz Kania and Schechtman. The papers are unrelated but connected in the sense that they are all concerned with the structure of classical non separable Banach spaces; namely, L_p, ℓ_∞, and ℓ_∞^c (the space of bounded functions that have countable support), respectively.

Ondřej Kalenda

Vector-valued Mokobodzki theorem

Ondřej Kalenda

A classical theorem of Mokobodzki says that whenever X is a compact convex set and $f\colon X \to \mathbb{R}$ is an affine function of the first Baire class, the function f is the limit of a sequence of affine continuous functions. We investigate possible extensions of this result to the vector-valued case. It turns out that the validity of such an extension is related to the approximation properties of the range space. In particular, if E is a Banach space enjoying the λ-bounded approximation property, and $f\colon X \to B_E$ is an affine mapping of the first Baire class, then there are affine continuous mappings $f_n\colon X \to \lambda B_E$ pointwise converging to f. This is a part of a joint work with Jiří Spurný.

Divya Khurana

Subspaces of L_p defined on trees

Divya Khurana

In this talk we will consider a class of subspaces of L_p first introduced by

Bourgain, Rosenthal and Schechtman. These spaces may be defined in two different ways; however for this talk we will consider the realization through well founded trees. We will show some glimpses of L_p index theory of this class.

Currents in infinite dimensions

Bernd Kirchheim

Integral currents serve as a geometrically meaningful class of generalized surfaces that possesses compactness properties allowing to handle variational problems. This extended the theory of Caccioppoli sets into the situation of general, but finite, dimension and codimension.

Later, metric (integral) currents of finite dimension were introduced in a more intrinsic way, and some progress towards the solution of Plateau's problem in infinite dimensions was made. We will discuss this concept and the relation to ambient Banach space.

Differentiability on L^p of a vector measure

Sebastián Lajara

We study the properties of Gâteaux, Fréchet, uniformly Gâteaux and uniformly Fréchet smoothness of the space $L^p(m)$ of scalar p-integrable functions with respect to a positive vector measure m. Our main results establish that if $p > 1$, then the space $L^p(m)$ has one of these properties whenever the Banach lattice X where the measure m takes its values has the corresponding property.

Joint work with L. Agud, J. M. Calabuig and E. A. Sanchez-Perez.

Niels Laustsen

A weak*-topological dichotomy in the dual unit ball of the Banach space of continuous functions on the first uncountable ordinal, with applications in operator theory

Niels Laustsen

Denote by $[0, \omega_1)$ the locally compact Hausdorff space consisting of all countable ordinals, equipped with the order topology, and let $C_0[0, \omega_1)$ be the Banach space of scalar-valued, continuous functions which are defined on $[0, \omega_1)$ and vanish eventually. In joint work with Tomasz Kania (Lancaster) and Piotr Koszmider (Polish Academy of Sciences, Warsaw), we show that a weak*-compact subset of the dual space of $C_0[0, \omega_1)$ is either uniformly Eberlein compact, or it contains a homeomorphic copy of a particular form of the ordinal interval $[0, \omega_1]$. This dichotomy yields a unifying approach to most of the existing studies of the Banach space $C_0[0, \omega_1)$ and the Banach algebra $\mathscr{B}(C_0[0, \omega_1))$ of bounded, linear operators acting on it, and it leads to several new results, as well as to stronger versions of known ones.

Richard Lechner

Localization and projections on bi-parameter BMO

Richard Lechner

We prove that for any operator T on bi-parameter BMO the identity factors through T or $I - T$. Bourgain's localization method provides the conceptual framework of our proof. It consists in replacing the factorization problem on the non-separable bi-parameter BMO by its localized, finite dimensional counterpart. We solve the resulting finite dimensional factorization problems by exploiting the geometry and combinatorics of colored dyadic rectangles.

http://arxiv.org/abs/1410.8786

Gonzalo Martínez Cervantes

Riemann integrability versus weak-continuity

Gonzalo Martínez Cervantes

We study the relation between weak-continuity and Riemann integration in

Banach spaces. A Banach space is said to have the weak Lebesgue property if every Riemann integrable function from the unit interval into it is weakly continuous almost everywhere. We present several results concerning the weak Lebesgue property. The main result of this section is that the weak Lebesgue property is stable under ℓ_1-sums. The study of some non-separable Banach spaces with the weak Lebesgue property has led us to an interesting problem in Set Theory and Geometric Measure Theory that will be discussed in this talk.

Measuring sets with translation invariant Borel measures

András Máthé

We study Borel sets B in $\mathbb{R}$ and in Polish groups for which there is a translation invariant Borel measure assigning positive and σ-finite measure to B. Let us call these sets measured. Davies and later Elekes and Keleti constructed Borel and compact subsets of $\mathbb{R}$ which are not measured.

Is it true that the union of two measured sets is measured? Or perhaps, on the contrary, is it true that every non-empty Borel set in $\mathbb{R}$ is a union of (countably many) measured sets?

To answer the second question in the negative, first we consider non-locally compact Polish groups and prove a statement reminiscent of the theorem that Haar null sets form a σ-ideal. Then, to certain Banach spaces with suitable Schauder bases, we associate compact subsets of $\mathbb{R}$ which are not a union of countably many measured sets.

To answer the first question in the negative the following 'paradox' is presented. Let A and B be Borel subsets of $\mathbb{R}$ of zero Lebesgue measure, and assume that B is of the second category. Then B is larger than A in the sense that there are Borel partitions $A = A_1 \cup A_2$, $B = B_1 \cup B_2$ and gauge functions g_1, g_2 such that the Hausdorff measures satisfy $H^{g_i}(B_i) = 1$, $H^{g_i}(A_i) = 0$ $(i = 1, 2)$.

Brice Rodrigue Mbombo

Ramsey properties for finite dimensional normed spaces

Brice Rodrigue Mbombo

Given $d \leq m$, let $\mathbb{E}_{d,m}$ be the set of all $m \times d$ matrices $(a_{i,j})$ such that:

(i) $\sum_{j=1}^{d} |a_{i,j}| \leq 1$ for every $1 \leq i \leq m$.

(ii) $\max_{i=1}^{m} |a_{i,j}| = 1$ for every $1 \leq j \leq d$.

These matrices correspond to the linear isometric embeddings from the normed space ℓ_∞^dinto ℓ_∞^m, in their unit bases.

Using the Graham–Rothschild Theorem on partitions of finite sets, we prove the following: for every integers d, m and r and every $\varepsilon > 0$ there exists $n \geqslant m$ such that for every coloring of $\mathbb{E}_{d,n}$ into r-many colors there is $A \in \mathbb{E}_{m,n}$ and a color $\widetilde{r} < r$ such that $A \cdot \mathbb{E}_{d,m} \subseteq (c^{-1}\{\widetilde{r}\})_\varepsilon$. We extend this result, first for embeddings between *polyhedral* normed spaces, and finally for arbitrary finite dimensional normed spaces. As a consequence, we obtain that the group of linear isometries of the *Gurarij* space is extremely amenable. A similar result for positive isometric embeddings gives that the universal minimal flow of the group of affine homeomorphisms of the Poulsen simplex is the Poulsen simplex itself.
This is a joint work (in progress) with Dana Bartošová (University of São Paulo) and Jordi López-Abad (ICMAT Madrid).

Assaf Naor

On the bi-Lipschitz structure of transportation cost metrics

Assaf Naor

The geometry of spaces of probability measures on metric spaces, equipped with transportation-cost metrics, has been studied extensively for many years, but mainly from the isometric perspective. The structure of such spaces from the bi-Lipschitz perspective remains largely a mystery, with many longstanding questions remaining open. In this talk we will survey such fundamental questions and explain some partial progress. We shall also discuss applications of such questions to areas such as metric expanders and Lipschitz extension.

Principles of local reflexivity revisited

Eve Oja

The principle of local reflexivity (PLR) is a powerful tool in the theory of Banach spaces and its applications. The PLR shows that the bidual X^{**} of a Banach space X is "locally" almost the same as the space X itself.

The PLR was discovered by Lindenstrauss and Rosenthal [LR] in 1969. It was improved by Johnson, Rosenthal, and Zippin [JRZ] in 1971. Since then, many new proofs, refinements, and generalizations of the PLR have been given in the literature. For instance, there is a version of the PLR for Banach lattices due to Conroy and Moore, and Bernau, revisited in [LO]. Very recently, a PLR respecting subspaces was discovered in [O].

In the present talk, we discuss old and new principles of local reflexivity from a general viewpoint (together with applications if there is time).

References

[JRZ] W. B. Johnson, H. P. Rosenthal, M. Zippin, On bases, finite dimensional decompositions and weaker structures in Banach spaces, Israel J. Math. 9 (1971) 488–506.

[LO] A. Lissitsin and E. Oja, The convex approximation property of Banach spaces, J. Math. Anal. Appl. 379 (2011), 616–626.

[LR] J. Lindenstrauss, H. P. Rosenthal, The $\mathcal{L}_p$ spaces, Israel J. Math. 7 (1969) 325–349.

[O] E. Oja, Principle of local reflexivity respecting subspaces, Adv. Math. 258 (2014) 1–12.

Strong pseudoconvexity in Banach spaces

Sofía Ortega Castillo

Pseudoconvexity is a property of open sets that generalizes geometric convexity. The topic of pseudoconvexity has been widely studied in the theory of functions of several complex variables due to its close relationship to domains of existence, domains of holomorphy and holomorphically convex domains.

We became interested in strong pseudoconvexity due to two results in complex analysis of several variables related to the cluster value problem for Banach spaces. In this talk we provide some notions of strong pseudoconvexity in the Banach space setting such as local uniform pseudoconvexity, uniform pseudoconvexity and strict pseudoconvexity. We also prove that 2-uniformly PL-convex spaces, such as L_p for $1 \leq p \leq 2$, have uniformly pseudoconvex unit ball. In contrast we prove that the unit balls of L_p, for $p \geq 2$, are not even strictly pseudoconvex.

Mikhail Ostrovskii

Metric characterization of the Radon-Nikodým property in Banach spaces

Mikhail Ostrovskii

The Radon-Nikodým property (RNP) can be characterized in many different analytic, geometric, and probabilistic ways. The RNP plays an important role in the theory of metric embeddings (works of Cheeger, Kleiner, Lee, and Naor (2006–2009)). In this connection Johnson (2009) suggested the problem of metric characterization of the RNP. The main goal of the talk is to explain the solution of this problem in terms of thick families of geodesics.

Daniel Pellegrino

The Bohnenblust–Hille and Hardy–Littlewood inequalities

Daniel Pellegrino

We discuss the recent advances related to the investigation of the Bohnenblust–Hille and Hardy–Littlewood inequalities, and its applications. Although many new results have been proved in the last years, several challenging problems remain open. Special attention will be dedicated to the state of the art of the search of the optimal constants of these inequalities.

Antonín Procházka

Low distortion embeddings of uniformly discrete spaces

Antonín Procházka

We will see an example of a bounded uniformly discrete metric space M with the following property: If M embeds into a Banach space X bi-Lipschitz with distortion strictly less than 2 then X linearly contains ℓ_1. A refinement of the construction of M allows for a proof of the following theorem: $C([0,\omega^\alpha])$ does not embed bi-Lipschitz with distortion strictly less than 2 into $C([0,\omega^\beta])$ if $\beta < \alpha$. Finally, for natural finite subspaces M_n of M we will see that embedding M_n into a Banach space X bi-Lipschitz with distortion $D < 2$ implies that ℓ_1^m is $(1+\varepsilon)$-isomorphic to a subspace of X. We will discuss the trade-off between D, ε, n and m. The talk contains joint results with Luis Sánchez González.

Daniele Puglisi

Banach spaces with C(K) Calkin algebras

Daniele Puglisi

We deal with the following question: Let A be a Banach algebra, does there exist a Banach space X so that the Calkin algebra of X is isomorphic, as a Banach algebra, to A?

Matías Raja

Renorming superreflexive spaces

Matías Raja

Among the different equivalent norms on a given super-reflexive space, it is quite natural to look for the one that makes the modulus of convexity $\delta_{\|\cdot\|}(t)$ as large as possible. In a recent work, we have investigated this question from an asymptotic point of view: to find a equivalent norm on X such that the limit $\lim_{t\to 0}\delta_{\|\cdot\|}(t) = 0$ converges in the feasible slowest manner. Our results provide a quite satisfactory answer to this problem. For instance, from our arguments it is easy to deduce the classical Pisier's renorming with power type modulus of convexity. However, if we only care about the power type modulus we are loosing information about the geometry of the space. Different moduli of convexity can be compared using a finer partial order.

We will give some applications to generalized notions of type and cotype in Banach spaces.

Oleg Reynov

On products of nuclear operators

Oleg Reynov

The following question was asked to me (privately) by Boris Mitjagin at the Aleksander Pelczynski Memorial Conference (Bedlewo, 2015): Is it true that a product of two nuclear operators in Banach spaces can be factored through a trace class operator in a Hilbert space?

Considering convolution operators in function spaces, we show that the answer is negative. We discuss some close problems in connection with the question. E.g., for which $s, r \in (0, 1]$ and $p > 0$ the products of s-nuclear and r-nuclear operators factor through S_p-operators in Hilbert spaces?

Moreover, we consider also some properties of the products of tree and more nuclear operators.

Maria Roginskaya

Partial power-boundedness and the Hausdorff dimension of the unitary point spectrum

Maria Roginskaya

If an operator is power bounded then its unitary point spectrum is countable (a result by Jamison). If an operator has only some subsequence of its powers bounded then its unitary point spectrum can be rather large and can be measured by its Hausdorff dimension. I'll discuss this result and some later development.

Bünyamin Sari

Separable elastic Banach spaces are universal

Bünyamin Sari

We prove a conjecture of Johnson and Odell asserting that separable elastic Banach spaces contain a copy of $C[0, 1]$. This is a joint work with Dale Alspach.

Gideon Schechtman

Metric X_p inequalities

Gideon Schechtman

A new nonlinear inequality of the flavour of the nonlinear version of the inequalities for type and cotype will be presented. This is a nonlinear extension of a linear inequality that was proved by Johnson, Maurey, Schechtman and Tzafriri in 1979 (and resembles the X_p inequality of Rosenthal). The formulation (and proof) of the new inequality completes the search for bi-Lipschitz invariants that serve as an obstruction to the embeddability of L_p spaces into each other, the previously understood cases of which were metric notions of type and cotype, which however fail to certify the nonembeddability of L_q into L_p when $2 < q < p$. Among the consequences of the new inequality are new quantitative restrictions on the bi-Lipschitz embeddability into L_p of snowflakes of L_q and integer grids in ℓ_q^n, for $2 < q < p$.

Joint work with Assaf Naor.

Thomas Schlumprecht

Closed Ideals of $L(\ell_p \oplus \ell_q)$

Thomas Schlumprecht

We solve a question asked by A. Pietsch, and prove that the space of linear bounded operators on $\ell_p \oplus \ell_q$ has infinitely many closed sub ideals.

Gareth Speight

A Measure Zero Universal Differentiability Set in the Heisenberg Group

Gareth Speight

The Heisenberg group $\mathbb{H}^n$ is a metric measure space equipped with translations and dilations. Lipschitz functions on $\mathbb{H}^n$ are Pansu differentiable almost everywhere, but $\mathbb{H}^n$ admits no bilipschitz embedding into a Euclidean space. We show there exists a measure zero 'universal differentiability set' in $\mathbb{H}^n$ containing a point of Pansu differentiability for every real-valued Lipschitz function. The proof adapts known techniques from Banach space theory, showing that existence of an 'almost maximal horizontal directional derivative' implies Pansu differentiability.

Joint work with Andrea Pinamonti.

Jarno Talponen

On natural density, orthomodular lattices, measure algebras and non-distributive L^p spaces

Jarno Talponen

In this talk different sorts of generalized measure algebras are discussed.

It turns out that if $\mathcal{B}$ is a Boolean algebra included in the natural way in the collection $\mathcal{D}/_\sim$ of all equivalence classes of natural density sets of the natural numbers, modulo null density, then $\mathcal{B}$ extends to a σ-algebra $\Sigma \subset \mathcal{D}/_\sim$ and the natural density is σ-additive on Σ.

The main tool employed in the argument is given in a more general setting, involving a kind of quantum state function, more precisely, a group-valued submeasure on an orthomodular lattice.

At the end we discuss the construction of 'non-distributive L^p spaces' by means of submeasures on lattices.

Jaroslav Tišer

Differentiability of Convex and Quasiconvex functions

Jaroslav Tišer

New criterion of Γ nullness gives a shorter proof of the result that a convex continuous function on a separable Asplund space is Fréchet differentiable Γ a.e. and also the new result of Gâteaux differentiability of quasiconvex functions.

Pedro Tradacete

Random unconditionally convergent bases in Banach spaces

Pedro Tradacete

A series $\sum_n x_n$ in a Banach space is *randomly unconditionally convergent* when $\sum_n \epsilon_n x_n$ converges almost surely on signs $(\epsilon_n)_n$ (with respect to the Haar probability measure on $\{-1,1\}^{\mathbb{N}}$). P. Billard, S. Kwapień, A. Pelczyński and Ch. Samuel introduced the notion of random unconditionally convergent (RUC) coordinate system $(e_i)_i$ in a Banach space. We will present new results on RUC basis, exploring the connections between this notion and classical unconditionality.

Silja Treialt

Duality results on some bounded approximation properties

Silja Treialt

The bounded approximation property for pairs (X, Y) consisting of a Banach space X and a fixed subspace Y was recently introduced by Figiel, Johnson, and Pełczyński [FJP].

Johnson [J] proved that if the dual space X^* of a Banach space X has the bounded approximation property, then X has the bounded duality approximation property. We extend Johnson's result to the pairs of Banach spaces as follows. The pair $(X^*, Y^\perp)$ has the bounded approximation property if and only if the pair (X, Y) has the bounded duality approximation property.

We examine lifting possibilities in the more general case of bounded convex approximation properties with applications to their other special cases. These include the metric versions and the bounded positive approximation property for Banach lattices.

This is a joint work with Eve Oja.

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Ben Wallis

Constructing Banach ideals using upper ℓ_p-estimates

Ben Wallis

Using upper ℓ_p-estimates for normalized weakly null sequence images, we describe a new family of operator ideals $\mathcal{WD}_{\ell_p}^{(\infty,\xi)}$ with parameters $1 \leq p \leq \infty$

and $1 \leq \xi \leq \omega_1$. These classes contain the completely continuous operators, and are distinct for all choices $1 \leq p \leq \infty$ and, when $p \neq 1$, for infinitely many $1 \leq \xi \leq \omega_1$. For the case $\xi = 1$, there exists an ideal norm $\|\cdot\|_{(p,1)}$ on the class $\mathcal{WD}_{\ell_p}^{(\infty,1)}$ under which it forms a Banach ideal. We also prove that each space $\mathcal{WD}_{\ell_p}^{(\infty,\omega_1)}(X,Y)$ is the intersection of the spaces $\mathcal{WD}_{\ell_p}^{(\infty,\xi)}(X,Y)$ over all $1 \leq \xi < \omega_1$.

Bobby Wilson

Tangents of σ-finite curves and scaled oscillation

Bobby Wilson

In this talk we will discuss two problems. The first problem is which properties of a set E allow for the existence of a set of positive measure on which a function $f\colon \Omega \subset \mathbb{R}^n \to \mathbb{R}$ is differentiable assuming the lower scaled oscillation of f finite for all $x \in \Omega \setminus E$. The lower scaled oscillation is defined as $l_f(x) := \liminf_{r\to 0} \sup_{\|x-y\|<r} \|f(x) - f(y)\|/r$. We show that if E is countable, then f is always differentiable on a set of positive measure. On the other hand, we will show that for any nonzero gauge function g there is an almost nowhere differentiable function with $\mathcal{H}^g(E) = 0$.

The second problem we will discuss was initiated by the observation that the graph of a function with finite lower scaled oscillation has σ-finite $\mathcal{H}^1$ measure. This naturally leads to the question whether every curve of σ-finite length has a tangent at every point of a set of positive $\mathcal{H}^1$ measure. We answer this question affirmatively.

We do not yet know whether or not these results can be extended to infinite dimensional Banach spaces with Radon-Nikodym property. This is joint work with Marianna Csörnyei.

Luděk Zajíček

Smoothness via directional smoothness and Marchaud's theorem in Banach spaces

Luděk Zajíček

(Based on a recent joint article with M. Johanis.)

Classical Marchaud's theorem (1927) asserts that if f is a bounded function on $[a,b]$, $k \in \mathbb{N}$, and the $(k+1)$th modulus of smoothness $\omega_{k+1}(f;t)$ is so small that $\eta(t) = \int_0^t \frac{\omega_{k+1}(f;s)}{s^{k+1}}\,\mathrm{d}s < +\infty$ for $t > 0$, then $f \in C^k\big((a,b)\big)$ and $f^{(k)}$ is

uniformly continuous with modulus $c\eta$ for some $c > 0$ (i.e. in our terminology f is $C^{k,c\eta}$-smooth). A generalization of Marchaud's theorem for mappings f between Banach spaces easily follows from the classical one-dimensional version and the following theorem: If f is locally bounded and $C^{k,\omega}$-smooth on every line, then it is $C^{k,c\omega}$-smooth for some $c > 0$. This theorem is proved using a converse of Taylor theorem (dealing with $C^{k,\omega}$-smooth functions) proved by M. Johanis (2014).

On the structure of the Almost Overcomplete and Almost Overtotal sequences in Banach spaces

Clemente Zanco

A sequence in a separable Banach space X $\langle$resp. in the dual space $X^*\rangle$ is said almost overcomplete AOC $\langle$resp. almost overtotal AOT $\rangle$ whenever the closed linear span of any of its subsequences has finite codimension in X $\langle$resp. the annihilator (in X) of each of its subsequences has finite dimension$\rangle$. We provide information about the structure of such sequences. In particular it can happen that, an AOC $\langle$resp. AOT $\rangle$ given sequence admits countably many not nested subsequences such that the only subspace contained in the closed linear span of each of such subsequences is the trivial one $\langle$resp. the closure of the linear span of the union of their annihilators in X of such subsequences is the full $X\rangle$.

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