ON THE REAL JACOBIAN CONJECTURE

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The real Jacobian conjecture said: *If $F = (f, g) : \mathbb{R}^2 \to \mathbb{R}^2$ be a polynomial map such that* $\det\big(DF(x)\big)$ *is different from zero for all $x \in \mathbb{R}^2$, then F is injective.*

This conjecture had a negative answer by Pinchuk [4] in 1994. Now several authors look for adding an additional assumption to the fact that $\det\big(DF(x)\big)$ is different from zero for all $x \in \mathbb{R}^2$, in order that the conjecture holds.

Theorem 1. *Let $F = (f, g) : \mathbb{R}^2 \to \mathbb{R}^2$ be a polynomial map such that $\det DF(x)$ is different from zero for all $x \in \mathbb{R}^2$. We assume that the degrees of f and g are equal and that the higher homogeneous terms of the polynomials f and g do not have real linear factors in common, then F is injective.*

Theorem 2. *Let $F = (f, g) : \mathbb{R}^2 \to \mathbb{R}^2$ be a polynomial map such that $\det DF(x)$ is different from zero for all $x \in \mathbb{R}^2$ and $F(0, 0) = (0, 0)$. If the higher homogeneous terms of the polynomials ff_x+gg_x and ff_y+gg_y do not have real linear factors in common, then F is injective.*

Theorem 1 was proved by first time in [3], here we present a more elementary and simple proof provided in the paper [1].

Theorems 1 and 2 are proved using qualitative theory of the ordinary differential equations in the plane.

[1] F. Braun and J. Llibre, *A new qualitative proof of a result on the real jacobian conjecture*, Anais da Academia Brasileira de Ciências **87** (2015), 1519–1524.

[2] F. Braun, J. Giné, J. Llibre, *A sufficient condition in order that the real Jacobian conjecture in $\mathbb{R}^2$ holds*, J. Differential Equations **260** (2016), 5250–5258.

[3] A. Cima, A. Gasull, J. Llibre and F. Mañosas, *Global injectivity of polynomial maps via vector fields*, Automorphisms of Affine Spaces (Curaçao, 1994), 105–123, Kluwer Acad. Publ., Dordrecht, 1995.

[4] S. Pinchuk, *A counterexample to the strong real jacobian conjecture*, Math. Z. **217** (1994), 1–4.

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