Davis-Warwick Probability Workshop

Organisers: Erik Slivken and Oleg Zaboronski

14-16 of December, 2015.
Mathematical Sciences Building, University of California at Davis

1 Programme

1.1 Monday

Morning session chair: Saul Jacka

Afternoon session chair: Erik Slivken

Talk	Time	Speaker	Title
	08:50-09:00	Saul Jacka (Warwick)	Introduction, greetings
1	**09:00-09:45**	Janko Gravner (UC Davis)	Jigsaw percolation: a nucleation analysis
2	**09:45-10:30**	Elisabetta Candellero (Warwick)	Percolation and isoperimetric inequalities.
	10:30-11:00	**COFFEE**	**BREAK**
3	**11:00-11:45**	Tobias Johnson (USC)	Size bias coupling and the spectral gap for random regular graphs
4	**11:45-12:30**	Nick Simm (Warwick)	Random matrix theory and log-correlated Gaussian fields
	12:30-14:00	**LUNCH**	**BREAK**
5	**14:00-14:45**	Shishi Luo (UC Berkeley)	Scaling limits of a model for selection at two scales
6	**14:45-15:30**	Yacine Barhoumi (Warwick)	On the splitting phenomenon in the Sathe-Selberg theorem: universality of the Gamma factor
	15:30-16:00	**TEA**	**BREAK**
7	**16:00-16:45**	Stefan Adams (Warwick)	Random field of gradients and elasticity
8	**16:45-17:30**	Allan Sly	TBA

1.2 Tuesday

Morning session chair: Janko Gravner

Talk	Time	Speaker	Title
9	**09:00-09:45**	Saul Jacka (Warwick)	Nonlinear PDEs, optimal control and the policy improvement algorithm.
10	**09:45-10:30**	Axel Saenz (UC Davis)	On the completeness of the Bethe ansatz for the periodic ASEP model.
	10:30-11:00	**COFFEE**	**BREAK**
11	**11:00-11:45**	Paul Chleboun (Warwick)	Relaxation and mixing of kinetically constrained models.
12	**11:45-12:30**	Bruno Nachtergaele (UC Davis)	Entanglement Dynamics of Disordered Quantum *XY* Chains
	12:30	**LUNCH**	**BREAK**
	15:30	**TOUR OF THE FUTURE WARWICK IN CA**	**SITE AND DINNER IN ROSEVILLE**

Transporation to Roseville: a chartered bus will depart from Unitrans Bus Barn on Garrod Drive at 15:30 sharp.

1.3 Wednesday

Morning session chair: Elisabetta Candellero

Afternoon session chair: Bruno Nachtergaele

Talk	Time	Speaker	Title
13	**09:00-09:45**	Jonathan Warren (Warwick)	A stochastic solution to the two dimensional Toda equations.
14	**09:45-10:30**	David Aldous (UC Berkeley)	Waves in a Spatial Queue: Stop-and-Go at Airport Security
	10:30-11:00	**COFFEE**	**BREAK**
15	**11:00-11:45**	Mihail Poplavskyi (Warwick)	On the asymptotic behaviour of the pure complex spectrum probability for the real Ginibre ensemble
16	**11:45-12:30**	Amanda Young (UC Davis)	Applications of a Modified Martingale Method for Estimating Spectral Gaps
	12:30-14:00	**LUNCH**	**BREAK**
17	**14:00-14:45**	Max Fathi (UC Berkeley)	A transport-information inequality for Markov chains on discrete spaces.
18	**14:45-15:30**	Michael Bishop (UC Davis)	Gap Dependency on Half Spaces in the Product Vacua and Boundary State models
	15:30-16:00	**TEA**	**BREAK**
19	**16:00-16:45**	Yumeng Zhang (UC Berkeley)	Counting solutions of random regular NAESAT beyond condensation threshold
20	**16:45-17:30**	Fraydoun Rezakhanlou (UC Berkeley)	Generalized Smoluchowski Equations and Scalar Conservation Laws

2 Abstracts

1. **Janko Gravner**

   Title: Jigsaw percolation: a nucleation analysis.

   Abstract: Jigsaw percolation is a model for collaborative problem solving: a nonlocal process that iteratively merges connected clusters in a deterministic puzzle graph by using connectivity properties of arandom people graph on the same set of vertices. We presume the people graph is random while the puzzle graph is a fixed deterministic graph. The main question is to estimate the probability that the puzzle is solved, that is, that the process eventually produces a single cluster. Particularly sharp answers can be obtained for the one dimensional ring and two dimensional torus puzzles where bootstrap percolation techniques can be used. The talk is on joint work with David Sivakoff.

2. **Elisabetta Candellero**

   Title: Percolation and isoperimetric inequalities.

   Abstract: In this talk we will discuss some relations between percolation on a given graph G and its geometry. There are several interesting questions relating various properties of a graph, such as growth or dimension, and the process of percolation on it. In particular we will look for conditions under which its critical percolation threshold is non-trivial, that is: $p_c(G)$ is strictly between zero and one. In a very influential paper on this subject, Benjamini and Schramm asked whether it was true that for every graph satisfying $dim(G) > 1$, one has $p_c(G) < 1$. We will explain this question in detail and present some recent results that have been obtained in this direction. This talk is based on a joint work with Augusto Teixeira (IMPA, Rio de Janeiro, Brazil).

3. **Tobias Johnson**

   Title: Size bias coupling and the spectral gap for random regular graphs.

   Abstract: The smaller the second eigenvalue of a regular graph, the stronger the expansion properties of the graph. Since this connection was discovered in the 1980s, researchers have tried to pinpoint the second eigenvalue of random regular graphs. The most prominent work in this direction was Joel Friedman's proof of Noga Alon's conjecture from 1985 that for a random d-regular graph on n vertices, the second eigenvalue is almost as small as possible, with high probability as n tends to infinity with d held fixed.

   We consider the case of denser graphs, where d and n are both growing. Here, the best result (Broder, Frieze, Suen, Upfal 1999) holds only if $d = o(n^{1/2})$. We extend this to $d = O(n^{2/3})$. Our result relies on new concentration inequalities for statistics of random regular graphs based on the theory of size biased couplings, an offshoot of Stein's method. The theory we develop should be useful for proving concentration inequalities in other settings involving dependence beyond random regular graphs.

   This is joint work with Nicholas Cook and Larry Goldstein.

4. **Nick Simm**

   Title: Random matrix theory and log-correlated Gaussian fields.

   Abstract: I will describe recent results on relations between random matrix theory and log-correlated Gaussian fields. A rather famous example of the latter is the Gaussian Free Field, which appears in numerous areas of mathematical physics and probability. We will show that certain objects from random matrix theory converge to what look like one-dimensional "cuts" of this field, depending on the way the field is regularized. The main example will concern the log characteristic polynomial of a Hermitian random matrix, for which I will present Theorems describing the convergence in law on both mesoscopic and macroscopic scales. Given time, in the case of the GUE, I will present a (non-heuristic and conjectural) discussion on the extreme value statistics of the characteristic polynomial.

5. **Shishi Luo**

   Title: Scaling limits of a model for selection at two scales.

   Abstract: Natural selection can act in opposing directions at different biological scales. For example, a fast-replicating virus strain is at an advantage within a host because it out-competes slower-replicating strains. It is at a disadvantage at the between-host scale, however, if it causes host morbidity and is less frequently transmitted. We model this phenomenon as a particle process and prove the weak convergence of this stochastic process under two natural scalings. The first scaling leads to a deterministic nonlinear integropartial differential equation with dependence on a single parameter directly related to the pre-limiting model parameters. We show that the fixed points of this differential equation are Beta distributions and that their stability depends on the parameter value and the behavior of the initial data around 1. The second scaling leads to a measure-valued Fleming-Viot process, a stochastic process that is frequently associated with a population genetics. This is joint work with Jonathan Mattingly.

6. **Yacine Barhoumi**

   Title: On the splitting phenomenon in the Sathe-Selberg theorem: universality of the Gamma factor.

   Abstract: We consider several classes of sequences of random variables whose Laplace transform presents the same type of splitting phenomenon when suitably rescaled. Answering a question of Kowalski-Nikeghbali, we explain the apparition of a universal term, the Gamma factor, by a common feature of each model, the existence of an auxiliary randomisation that reveals an independence structure. The class of examples that belong to this framework includes random uniform permutations, random polynomials or random matrices with values in a finite field and the classical Sathe-Selberg theorems in probabilistic number theory. We moreover speculate on potential similarities in the Gaussian setting of the celebrated Keating and Snaith's moments conjecture. (Joint work with R. Chhaibi)

7. **Stefan Adams**

   Title: Random field of gradients and elasticity.

   Abstract: Random fields of gradients are a class of model systems arising in the studies of random interfaces, random geometry, field theory, and elasticity theory. These random objects pose challenging problems for probabilists as even an a priori distribution involves strong correlations, and are likely to be an universal class of models combining probability, analysis and physics in the study of critical phenomena. They emerge in the following three areas, effective models for random interfaces, Gaussian Free Fields (scaling limits), and mathematical models for the Cauchy-Born rule of materials, i.e., a microscopic approach to nonlinear elasticity. The latter class of models requires that interaction energies are non-convex functions of the gradients. Open problems over the last decades include unicity of Gibbs measures, the scaling to GFF and strict convexity of the free energy. We present in the talk first results for the free energy and the scaling limit at low temperatures using Gaussian measures and rigorous renormalisation group techniques yielding an analysis in terms of dynamical systems. The key ingredient is a finite range decomposition for parameter dependent families of Gaussian measures. (partly joint work with S. Mueller and R. Kotecky)

8. **Alan Sly TBA**

9. **Saul Jacka**

   Title: Nonlinear PDEs, optimal control and the policy improvement algorithm.

   Abstract: The talk will show how to embed fully non-linear elliptic and parabolic pdes as Hamilton Jacobi Bellman equations, discuss what this tells us about the existence of solutions and explore the possibility of numerical solutions via a (family of) Policy Improvement Algorithm(s).

10. **Axel Saenz**

    Title: On the completeness of the Bethe ansatz for the periodic ASEP model.

    Abstract: In recent work with collaborators Norman Do (Monash Uni.) and Eric Brattain (UC Davis), we have shown that the Bethe ansatz is complete for the periodic ASEP model. The ASEP model is a continuous Markov process which describes a system of N particles on a ring lattice of L sites and each particle has probability p (resp. $1-p$) to jump right (resp. left). In our work, we use ideas from topology and complex geometry to obtain our results. I will expose the ideas behind this methods and discuss the applications of this methods for future work in scaling limits and the KPZ universality class.

11. **Paul Chleboun**

    Title: Relaxation and mixing of kinetically constrained models.

    Abstract: We study the relaxation and out-of-equilibrium dynamics of a family of kinetically constrained models (KCMs) called the d-dimensional East-like processes. KCMs are spin systems on integer lattices, where each vertex is labelled either 0 or 1, which evolve according to a very simple rule: i) with rate one and independently for each vertex, a new value 1/0 is proposed with probability $1-q$ and q respectively; ii) the proposed value is accepted if and only if the neighbouring spins satisfy a certain constraint. Despite of their apparent simplicity, KCMs pose very challenging and interesting problems due to the hardness of the constraints and lack of monotonicity. The out-of-equilibrium dynamics are extremely rich and display many of the key features of the dynamics of real glasses, such as; an ergodicity breaking transition at some critical value, huge relaxation times close to the critical point, and dynamic heterogeneity (non-trivial spatio-temporal fluctuations of the local relaxation to equilibrium). We discuss recent advances on the out-of-equilibrium dynamics of the East-like processes, including the dependence of the relaxation and mixing time on the system size, density, and dimension. We also look at simulations which motivate some interesting limit shape conjectures. This is joint work with Alessandra Faggionato and Fabio Martinelli.

12. **Bruno Nachtergaele**

    Title: Entanglement Dynamics of Disordered Quantum XY Chains.

    Abstract: One of the expected signatures of Many-Body Localization is logarithmic (as opposed to ballistic) growth of bipartite entanglement starting from a product initial condition. For a class of disordered quantum XY chains, and a large class of product initial states, we prove that the entanglement satisfies a constant bound, independent of time and system size. Therefore, although disordered XY chains display many of the expected generic features of Many-Body Localization, the dynamics of entanglement appears to be even more strongly localized than is generically expected and observed numerically in other model systems. (Work in collaboration with Houssam Abdul-Rahman (U Alabama at Birmingham), Robert Sims (U Arizona), and Gunter Stolz (U Alabama at Birmingham). This research was supported by the National Science Foundation under Grants DMS-1069320 (G.S.) and DMS-1515850 (B.N.), and by a grant from the Simons Foundation (No. 301127 to R.S.).)

13. **Jon Warren**

    Title: A stochastic solution to the two dimensional Toda equations.

    Abstract: I will explain how to construct a random (and non-smooth) solution to the Toda equations starting from the stochastic heat equation, and ask whether this tells us anything new about the heat equation. This is based on work with Chin Lun, and earlier work with Neil O'Connell.

14. **David Aldous**

    Title: Waves in a Spatial Queue: Stop-and-Go at Airport Security.

    Abstract: Imagine you are the 170th person in line at an airport security checkpoint. As people reach the front of the line they are being processed steadily, at rate 1 per unit time. But you move less frequently, and when you do move, you typically move several units of distance, where 1 unit distance is the average distance between successive people standing in the line.

This phenomenon is easy to understand qualitatively. When a person leaves the checkpoint, the next person moves up to the checkpoint, the next person moves up and stops behind the now-first person, and so on, but this "wave" of motion often does not extend through the entire long line; instead, some person will move only a short distance, and the person behind will decide not to move at all. Around the k'th position in line, there must be some number $a(k)$ representing both the average time between your moves and the average distance you do move. This talk describes a stochastic model in which in can be proved that $a(k)$ grows as order $k^{1/2}$. A more refined analysis (work in progress) suggests sharp asymptotics in terms of the coalescing Brownian motion model.

15. **Mikhail Poplavskyi**

    Title: On the asymptotic behaviour of the pure complex spectrum probability for the real Ginibre ensemble.

    Abstract: We discuss a new result for the probability that a large real Ginibre matrix has no or very few real eigenvalues. The asymptotic behaviour of the probability can be guessed by using a deep connection between the real eigenvalues process and Annihilating Brownian Motions (ABM's), first discovered in papers of R. Tribe and O. Zaboronski. We give a rigorous proof of the guess by using the determinant representation of the probability. We also show how to exploit the connection between ABM's to and the real Ginibre ensemble to predict the asymptotic behaviour of the biggest real eigenvalue. The talk is based on a joint paper with E.Kanzieper, C.Timm, R.Tribe, O.Zaboronski.

16. **Amanda Young**

    Title: Applications of a Modified Martingale Method for Estimating Spectral Gaps.

    Abstract: We introduce a form of the martingale method for estimating the spectral gap of a quantum spin system that relies on an increasing sequence of frustration free Hamiltonians rather than an increasing sequence of finite volumes. In particular, this new form is more easily applied to systems with periodic boundary conditions and we use it to prove a non-vanishing spectral gap for finitely correlated state models with periodic boundary conditions and a unique ground state. Further applications to AKLT type models in one and two dimensions with both open and periodic boundary conditions are in progress. This research is supported in part by the National Science Foundation under Grant DMS-1515850.

17. **Max Fathi**

    Title: A transport-information inequality for Markov chains on discrete spaces.

    Abstract: In this talk, I will explain how to prove an inequality relating the transport distance and the Fisher information for Markov chains on discrete spaces with positive Ollivier-Ricci curvature, as well as its applications to the study of concentration of measure. Joint work with Yan Shu.

18. **Michael Bishop**

    Title: Gap Dependency on Half Spaces in the Product Vacua and Boundary State models

    Abstract: We consider for a family of quantum spin 1/2 systems called the Product Vacua and Boundary State (PVBS) models defined on subsets of the d dimensional lattice $\mathbf{Z}_d$ with Hamiltonians composed of sums of non-commuting local projections. For any set of parameters for the PVBS model, we prove a simple geometric condition on the half-space which determines the existence or non-existence of a spectral gap. As a corollary, we prove the existence or non-existence of a spectral gap for the model defined on the entire lattice $\mathbf{Z}_d$. This research was supported in part by the National Science Foundation under Grant DMS-1515850.

19. **Yumeng Zhang**

    Title: Counting solutions of random regular NAESAT beyond condensation threshold.

    Abstract: We consider the k-NAESAT problem on d random regular graphs and determine its log partition function in the so called condensation regime, where standard first moment estimation is known to inflate by a constant fraction. Our result partially verifies the "one step replica symmetry breaking" picture from statistical physics that in this regime, the solution space is dominated by a few large clusters of constant size. Our method is based on analyzing a weighted Belief Propagation using a novel encoding of local neighborhood. This is joint work with Allan Sly and Nike Sun.

20. **Fraydoun Rezakhanlou**

    Title: Generalized Smoluchowski Equations and Scalar Conservation Laws.

    Abstract: By a classical result of Bertoin, if initially a solution to Burgers' equation is a Levy process without positive jumps, then this property persists at later times. According to a theorem of Groeneboom, a white noise initial data also leads to a Levy process at positive times. Menon and Srinivasan observed that in both aforementioned results the evolving Levy measure satisfies a Smoluchowski–type equation. They also conjectured that a similar phenomenon would occur if instead of Burgers' equation, we solve a general scalar conservation law with a convex flux function. Though a Levy process may evolve to a Markov process that in most cases is not Levy. The corresponding jump kernel would satisfy a generalized Smoluchowski equation. Along with Dave Kaspar, we show that a variant of this conjecture is true for monotone solutions to scalar conservation laws. I also formulate some open question concerning the analogous questions for Hamilton-Jacobi PDEs in higher dimensions.