BOUNDED RATIONAL ERGODICITY OF THE CYLINDER MAP

ABSTRACT. We consider a skew product over an irrational rotation defined as follows. Let $I = [0, 1)$, $R_\alpha : I \to I$, $R_\alpha(x) = x + \alpha \mod 1$ be an irrational rotation and let

$$f(x) = \begin{cases} 1 & x \in \left[0, \frac{1}{2}\right) \\ -1 & x \in \left[\frac{1}{2}, 1\right) \end{cases}.$$

We call the skew product $T_\alpha : I \times \mathbb{Z} \to I \times \mathbb{Z}$ defined by $T_\alpha(x, y) = (x, y + f(x))$ the cylinder map (aka deterministic random walk). T_α is a measure preserving transformation with respect to the measure $m = m_I \times m_{\mathbb{Z}}$, where m_I is the Lebesgue measure on I and $m_{\mathbb{Z}}$ is the counting measure on $\mathbb{Z}$. Since m is an infinite measure, by a theorem of Aaronson, there can be no sequence of normalizing constants a_n such that the Birkhoff sums $\frac{1}{a_n}\sum_{k=0}^{n-1} G \circ T_\alpha$ converge a.e. to a non-zero, finite limit, for any $G \in L^1(m)$, $\int G dm \neq 0$. Nevertheless, there are properties of an infinite measure preserving transformations that entail weaker modes of convergence for the Birkhoff sums normalized by a sequence of constants. Among these are bounded rational ergodicity and asymptotic distributional stability. Both of these were shown to hold for T_α, when α is a quadratic irrational. In a recent work with H. Nakada and J. Aaronson, we proved that bounded rational ergodicity holds in the case when α is of bounded type. In the talk, I will introduce and explain the mentioned results.