

WARWICK DYNAMICAL SYSTEMS WORKSHOP
6–8 JUNE 2016

LOCATION: ROOM MS0.03,
MATHEMATICS INSTITUTE, UNIVERSITY OF WARWICK
COVENTRY, CV4 7AL, UNITED KINGDOM

ABSTRACTS

1) Claude Baesens. *Simplest bifurcation diagrams for monotone families of vector fields on a torus.*

We prove that the bifurcation diagram proposed in [Baesens C, Guckenheimer J, Kim S, MacKay RS, Three coupled oscillators: mode-locking, global bifurcations and toroidal chaos, *Physica D* 49 (1991) 387-475] as the simplest possibility for a monotone two-parameter family of vector fields on a torus is indeed representative of the simplest possible cases. To achieve this we define "simplest" by minimising sequentially the numbers of equilibria, Bogdanov-Takens points, closed curves of centre and of neutral saddle, Reeb components, arcs of rotational homoclinic bifurcation of horizontal homotopy type, necklace points, points of neutral horizontal homoclinic bifurcation, half-plane fan points, points of coexistence of centre and neutral saddle, and degenerate Hopf points. We obtain a variety of simplest cases, including that initially proposed. We provide an example of a family satisfying our definition of simplest. The analysis of this family involves a Hamiltonian approximation and a rescaling of time.

2) Maciej Capiński. *Beyond the Melnikov method: a computer assisted approach.*

We present a Melnikov type approach for establishing transversal intersections of stable/unstable manifolds of perturbed normally hyperbolic invariant manifolds. In our approach, we do not need to know the explicit formulas for the homoclinic orbits prior to the perturbation. We also do not need to compute any integrals along such homoclinics. All needed bounds are established using rigorous computer assisted numerics. Lastly, and most importantly, the method establishes intersections for an explicit range of parameters, and not only for perturbations that are 'small enough', as is the case in the classical Melnikov approach.

3) Luigi Chierchia. *KAM theory for secondary tori.*

In general, in the phase space of nearly-integrable Hamiltonian systems may appear, near simple resonances, "secondary" invariant tori, which are not continuation of integrable Lagrangian tori. We discuss a theory — in the case of analytic mechanical systems — apt to deal with "most" secondary tori, under suitable genericity conditions. This is joint work with Luca Biasco.

4) Jacques Féjoz. *KAM theory and symmetries.*

I describe the influence of Abelian and non-Abelian symmetries on the geometry of KAM tori, with a view to applications to celestial mechanics.

5) Ernest Fontich. *Invariant manifolds of tangent to the identity maps.*

Given a differentiable or analytic map with the origin fixed and the linearization at the origin being the identity we provide conditions for the map to have a stable set. By working with a suitable parameterization of the stable set we study its regularity. We find that under some circumstances there is a loss of regularity (not only at the origin). We study the dependence on parameters. The analogous case for differential equations is also considered. We provide examples. This is joint work with Inmaculada Badomá (UPC) and Pau Martín (UPC).

6) José Pedro Gaivão. *SRB measures for pinball polygonal billiards.*

The billiard map of a polygonal billiard with the standard reflection law is conservative and non-chaotic. Completely different dynamics arises when the reflection law is contracting, i.e. when the reflection angle measured from the normal is a contraction of the incidence angle. In this case, the billiard map is dissipative: the Liouville measure is no longer preserved. In this talk I will discuss recent results obtained in collaboration with Gianluigi Del Magno, João Lopes Dias and Pedro Duarte concerning the existence of SRB measures and their ergodic properties.

7) Marian Gidea. *Arnold Diffusion in a Three- and a Four-Body Problem.*

The Arnold diffusion problem, – asserting that small perturbations exerted on integrable Hamiltonian systems can accumulate to large effects –, is a paradigm in Hamiltonian instability. Despite recent progress on this topic, there are relatively few examples of systems that exhibit the diffusion mechanism. Moreover, some of the available techniques seem difficult to apply to concrete situations. In this talk we will discuss certain models from Celestial Mechanics that exhibit the diffusion phenomenon, and present a geometric/topological methodology to explicitly construct diffusing orbits, under verifiable conditions on the underlying system.

8) Thomas Gilbert. *Conditional return times to rare events in billiard models.*

Motivated by the study of transport processes in some classes of billiard models, we wish to characterize a limiting regime of higher-dimensional billiards such that interaction between some of their degrees of freedom occurs only rarely while others mix fast. Under such conditions, the dynamics of the slow degrees of freedom can be approximated by a Markov jump process. The exponentially-distributed waiting times correspond to the times separating interactions among the slow degrees of freedom and we call them conditional return times or rates. Their definition extends beyond the rare interaction regime and some universal formulas apply.

9) Kei Irie. *A C^∞ -closing lemma for three-dimensional Reeb flows via embedded contact homology.*

We will explain a proof of a C^∞ -closing lemma for Reeb flows on closed contact three-manifolds. As an immediate consequence, it follows that for any closed contact three-manifold with a C^∞ -generic contact form the union of all periodic Reeb orbits is dense.

Our proof is based on recent developments in quantitative aspects of embedded contact homology (ECH). ECH is a kind of Floer homology defined for contact three-manifolds, which has been developed by M.Hutchings and his collaborators. In particular, a key fact is that the asymptotics of ECH spectral invariants recover the volume of a contact manifold, which was proved by D.Cristofaro-Gardiner, M.Hutchings and V.Ramos.

A proof of a C^∞ -closing lemma for Hamiltonian diffeomorphisms of closed surfaces (joint work with Masayuki Asaoka) will be also sketched. The proof uses the above result for Reeb flows and analysis of area-preserving maps near fixed points.

10) Vadim Kaloshin. *Birkhoff Conjecture for convex planar billiards and deformational spectral rigidity of planar domains.*

The classical Birkhoff conjecture states that the only integrable convex planar domains are circles and ellipses. In a joint work with A. Avila and J. De Simoi we show that this conjecture is true for perturbations of ellipses of small eccentricity. It turns out that the method of proof gives an insight into deformational spectral rigidity of planar axis symmetric domains and gives a partial answer to a question of P. Sarnak. The latter is a joint work with J. De Simoi and Q. Wei.

11) Alexey Korepanov. *Homogenization for families of skew products.*

We consider families of fast-slow skew product maps of the form

$$x_{n+1} = x_n + \varepsilon^2 a_\varepsilon(x_n, y_n) + \varepsilon b_\varepsilon(x_n) v_\varepsilon(y_n), \quad y_{n+1} = T_\varepsilon y_n,$$

where T_ε is a family of nonuniformly expanding maps, v_ε is of mean zero and the slow variables x_n lie in \mathbb{R}^d . Under an exactness assumption on b_ε (automatically

satisfied in the cases $d = 1$ and $b_\varepsilon \equiv I_d$), we prove convergence of the slow variables to a limiting stochastic differential equation (SDE) as $\varepsilon \rightarrow 0$.

Our results include cases where the family of fast dynamical systems T_ε consists of intermittent maps, unimodal maps (along the Collet-Eckmann parameters) and Viana maps.

Similar results are obtained also for continuous time systems

$$\dot{x} = \varepsilon^2 a_\varepsilon(x, y, \varepsilon) + \varepsilon b_\varepsilon(x) v_\varepsilon(y), \quad \dot{y} = g_\varepsilon(y).$$

Here, as in classical Wong-Zakai approximation, the limiting SDE is of Stratonovich type $dX = \bar{a}(X) dt + b_0(X) \circ dW$ where \bar{a} is the average of a_0 and W is a d -dimensional Brownian motion.

This is a joint work with Zemer Kosloff and Ian Melbourne.

12) Jean-Pierre Marco. *From the Birkhoff theory of twist maps to Arnold diffusion.*

In this talk we exhibit a large class of a priori unstable nearly integrable C^κ systems on the annulus \mathbb{A}^3 , the dynamics of which can be reduced to that of a polysystem on the annulus \mathbb{A} . This polysystem is a pair (φ, ψ) , where φ is a twist map and ψ is an open correspondence. We then generalize Moeckel's approach to that setting and prove the generic existence of diffusion orbits. This is a joint work with Laurent Lazzarini.

13) Jessica Elisa Massetti. *Normal forms for perturbations of systems possessing a Diophantine invariant torus.*

In 1967 Moser proved the existence of a normal form for real analytic perturbations of vector fields possessing a reducible invariant quasi-periodic Diophantine torus; this showed that the persistence of such an invariant torus is indeed a phenomenon of finite co-dimension. Starting from a geometric interpretation of this result, we will discuss how in some cases the specificity of the system allows the co-dimension to be reduced and, in some situations even eliminated (the "elimination of parameters", set up by Herman, Rüssmann and other authors in the 80-90's).

14) Ian Melbourne. *The Lorenz attractor is exponentially mixing.*

We show that the Lorenz attractor (for the Lorenz equations with the classical parameters) has exponential decay of correlations. Joint work with Vitor Araujo and Paulo Varandas.

15) Narcís Miguel Baños. *The role of stability islands in escape rates through a Cantorus in area preserving maps.*

After the breakdown of an invariant curve of an APM a Cantor set with the same rotation number as the just broken curve persists. Despite the holes in the Cantor

set, created by manifolds of nearby periodic orbits, allow orbits to travel across it, the time to do so can be extremely large.

For concreteness we shall use the Chirikov Standard Map, depending on a parameter k , and look at the behaviour after the destruction of the golden number rotational invariant curve. That is, for $k > k_G = 0.971635\dots$, Greene's threshold. It is well known, thanks to Greene-MacKay renormalisation theory and MacKay-Meiss-Percival transport theory, that the time $T(k)$ to cross (in a suitable sense) the Cantor set, increases and satisfies that $T(k)(k - k_G)^B$ is bounded for a suitable B . In fact this function behaves as a 1-periodic function in a given log scale.

In this talk we will present the results of extensive numerical simulations to obtain $T(k)$ (both average, standard deviation and density) to give evidence of the shape of this periodic function and we will interpret the results by analyzing the role of islands of stability close to the Cantor set.

16) Giannis Moutsinas. *Splitting of separatrices in area-preserving maps close to 1:3 resonance.*

We will look at what happens to a periodic orbit of an autonomous Hamiltonian system with two degrees of freedom, for which the period of oscillation, about the periodic orbit, of neighbouring trajectories is three times the period of the periodic orbit itself, a trajectory at 1:3 resonance. We approach this problem by reducing the study on a 3-dimensional constant-energy surface and then take the first return map, which is 2-dimensional and area-preserving. By the implicit function theorem we know that when we change the chosen energy level we still get a periodic orbit but not exactly at resonance.

The normal form theory for such maps shows us that any map of this kind can be formally conjugated to the flow of an autonomous Hamiltonian of one degree of freedom. This implies that for maps close to resonance there is an invariant set around the origin that is separated by the separatrices of neighbouring saddle points. We will see that in reality these separatrices do not coincide but they meet transversely. However their difference is exponentially small, thus undetected by the normal form. This splitting creates a chaotic region. We will give asymptotics for size of this chaotic region.

17) Anatoly Neishtadt. *On destruction of adiabatic invariance.*

In many problems of classical mechanics and theoretical physics dynamics can be described as a slow evolution of periodic or quasi-periodic processes. Adiabatic invariants are approximate first integrals for such a dynamics. Destruction of adiabatic invariance leads to chaotic dynamics. In the talk it is planned to present a review of some mechanisms of destruction of adiabatic invariance with examples from the charged particle dynamics.

18) Laurent Niederman. *Double exponential stability of quasi-periodic motion in Hamiltonian systems.*

In a joint work with Abed Bounemoura and Bassam Fayad, we prove that generically, both in a topological and measure-theoretical sense, an invariant Lagrangian Diophantine torus of a Hamiltonian system is doubly exponentially stable in the sense that nearby solutions remain close to the torus for an interval of time which is doubly exponentially large with respect to the inverse of the distance to the torus. We also prove that for an arbitrary small perturbation of a generic integrable Hamiltonian system, there is a set of almost full positive Lebesgue measure of KAM tori which are doubly exponentially stable. Our results hold true for real-analytic but more generally for Gevrey smooth systems.

19) Vered Rom-Kedar. *Exponential accelerators.*

A survey of our study of particles acceleration in billiards with moving boundaries will be presented [1-5]. First, a simple mechanism for achieving exponential in time growth of the particles energy will be described [1]. Then, it will be shown that this non-generic example may be extended to a family of robust, generic systems, in arbitrary dimensions, in which exponential acceleration appears, and in which the exponential rate of the energy gain may be predicted by utilizing probabilistic models [2,3]. The extension of these results to billiards with mixed phase space leads to the development of adiabatic theory for non-ergodic systems [4]. Finally, the implication of these results on energy gain for particles that enter and exit such accelerators through a hole in the boundary will be described [5].

Joint work with V. Gelfreich, K. Shah and D. Turaev

- [1] K. Shah, D. Turaev and V. Rom-Kedar, Exponential energy growth in a Fermi accelerator, Phys. Rev. E 81, 056205 (2010).
- [2] V. Gelfreich, V. Rom-Kedar, K. Shah, D. Turaev, Robust exponential accelerators, PRL 106, 074101 (2011).
- [3] V. Gelfreich, V. Rom-Kedar and D. Turaev, "Fermi acceleration and adiabatic invariants for non-autonomous billiards", Chaos 22, 033116 (2012)
- [4] V. Gelfreich, V. Rom-Kedar, D. Turaev, Oscillating mushrooms: adiabatic theory for a non-ergodic system, Journal of Physics A : Mathematical and Theoretical, Volume 47 (Number 39). Article number 395101 (2014).
- [5] K. Shah, V. Gelfreich, V. Rom-Kedar, D. Turaev, Leaky Fermi Accelerators, Phys. Rev. E 91, 062920 (2015).

20) Tere Seara. *Oscillatory orbits in the elliptic restricted planar three body problem.*

The restricted planar three body problem models the motion of a massless body under the Newtonian gravitational force of two bodies evolving in Keplerian ellipses. The circular case is a particular case.

Since Chazy (1922), it is known that the possible states the body $q(t)$ can approach as time tends to infinity are four:

- Hyperbolic: $\|q(t)\| \rightarrow \infty$ and $\|\dot{q}(t)\| \rightarrow c > 0$ as $t \rightarrow \pm\infty$.
- Parabolic: $\|q(t)\| \rightarrow \infty$ and $\|\dot{q}(t)\| \rightarrow 0$ as $t \rightarrow \pm\infty$.
- Bounded: $\limsup_{t \rightarrow \pm\infty} \|q\| < +\infty$.
- Oscillatory: $\limsup_{t \rightarrow \pm\infty} \|q\| = +\infty$ and $\liminf_{t \rightarrow \pm\infty} \|q\| < +\infty$.

Examples of all these types of motion, except the oscillatory ones, were already known by Chazy.

In this talk, we prove the existence of oscillatory motions for any value of the masses of the primaries assuming they move in ellipses whose eccentricity is small enough, as a consequence of the transversal intersection of the stable and unstable manifolds of periodic orbits at “infinity”, and using techniques of Arnold diffusion.

This is a joint work with M. Guardia and P. Martin.

21) Mattia Serra. *Hamiltonian Geometry of Null-Geodesics and the Computation of Coherent Vortex Boundaries.*

Recent results show that boundaries of coherent vortices (elliptic coherent structures) can be computed as closed null-geodesics of appropriate Lorentzian metrics defined on the physical domain of the underlying fluid. Here we show how a Hamiltonian reduction of this geodesic problem simplifies and accelerates the computation of vortex boundaries. We show applications to mesoscale eddy- boundary extraction from satellite-inferred ocean velocity data.

22) Kushal Shah. *Collective Dynamics of Charged Particles in Paul Traps using Tsallis Distribution.*

Paul trap is a device used for confining charged particles using time-periodic electric fields and a Nobel Prize for its discovery was awarded to Wolfgang Paul in 1989. Dynamics of single particles in such traps is well understood but an understanding of the collective dynamics of several particles is still quite elusive. One of the major open problems is regarding the heating observed in such traps. We have studied these collective dynamics using the Vlasov equation under the assumption of Tsallis distribution. In this talk, I will present some of our results and discuss some open problems that need to be addressed. This work was done jointly with V. Saxena.

23) Dmitry Turaev. *Quantum acceleration.*

It is shown that a periodic emergence and destruction of an additional quantum number leads to an exponential growth of energy of a quantum mechanical system subjected to a slow periodic variation of parameters. The main example is given

by systems (e.g., quantum billiards and quantum graphs) with periodically divided configuration space. In special cases, the process can also lead to a long period of cooling that precedes the acceleration, and to the desertion of the states with a particular value of the quantum number.

24) Arturo Vieiro. *Geometry of double resonances of 4D symplectic maps and diffusive properties.*

The crossing of two resonances (a double resonance) in a neighbourhood of an elliptic fixed point of a 4D symplectic map can be described using a suitable Froeschlé-like map. The double resonance structure resembles the product of two penduli, but with a suitable coupling. We will introduce the model and explain the role of the parameters. After describing some geometric and analytic properties of the model, we shall discuss in detail some numerical experiments to investigate the geometry of the crossing and the diffusive properties near a double resonance (passage from one resonance to the other, changes of the local diffusion coefficient along a resonance, etc). Selected parameters corresponding to different configurations within the double resonance structure will be considered. The dynamical differences in all cases will be stressed.

The talk is based on joint works with V. Gelfreich, C. Simó and E. Fontich.

25) Zemer Kosloff. *Symmetric Birkhoff sums in infinite ergodic theory.*

Aaronson showed in 1977 that there is no ergodic theorem in infinite measure space and the failure is very drastic in the sense that normalised Birkhoff sums of positive integrable function either tend to 0 almost surely or to infinity almost surely. Recently it was observed that there are transformations for which the symmetric Birkhoff sums (where the summation is along a symmetric time interval) can behave more regularly. In this talk I will explain this new phenomena and discuss the main result that although the failure is less drastic, there is no ergodic theorem for symmetric Birkhoff sums in infinite measure space and there exists a universal divergence statement. The contents of this talk are a combination of 2 papers, one of them is joint with Benjamin Weiss (Hebrew University) and Jon Aaronson (Tel Aviv).