# Nucleation phase of condensing zero range process

Johel Beltrán, PUCP - IMCA

Joint work with
C. Landim and M. Jara (IMPA)

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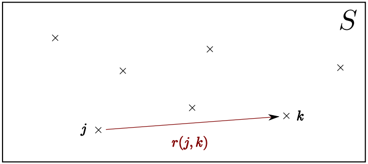

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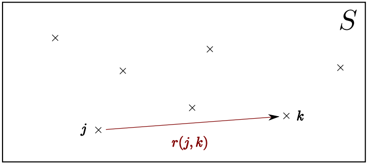


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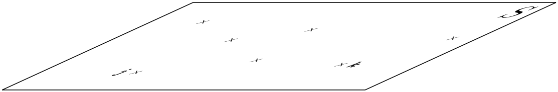

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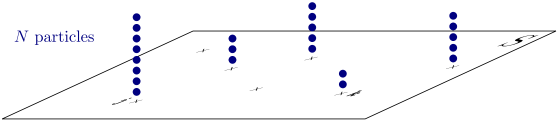


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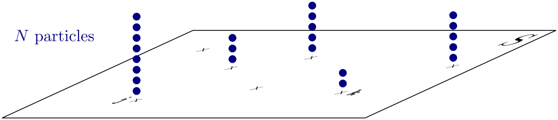


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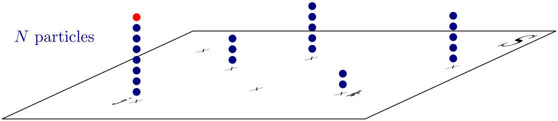


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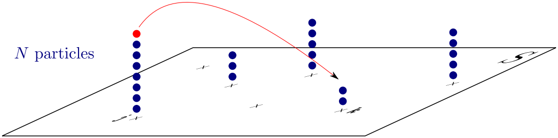


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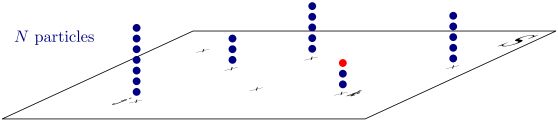


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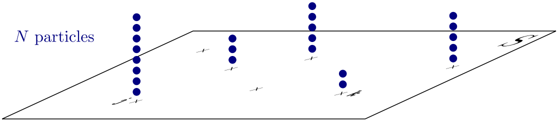


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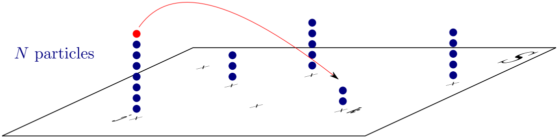


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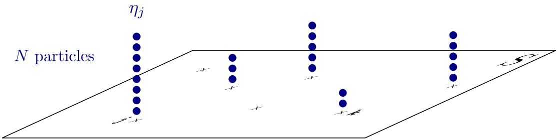


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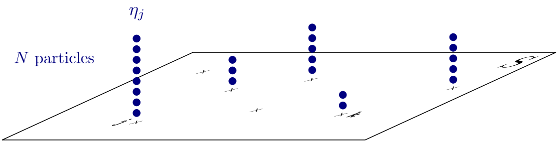


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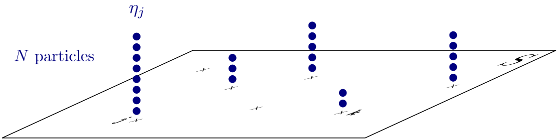


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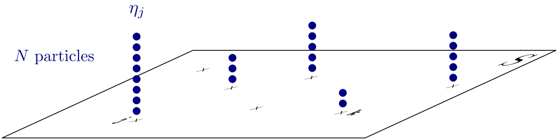


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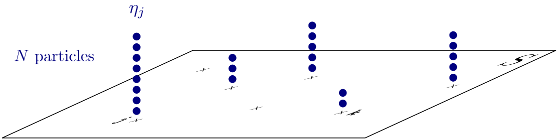


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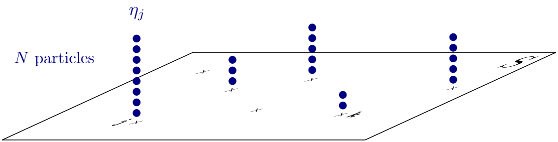


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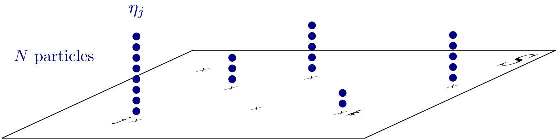


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Does $U^N(t)$ converge?

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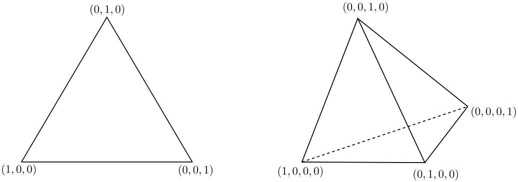

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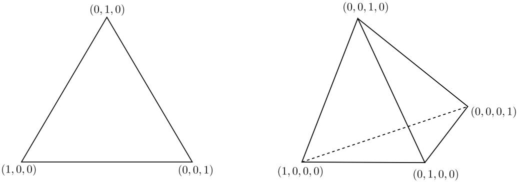


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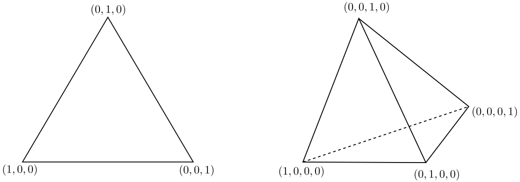


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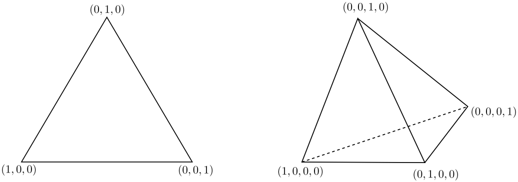


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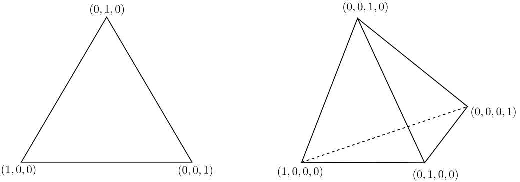


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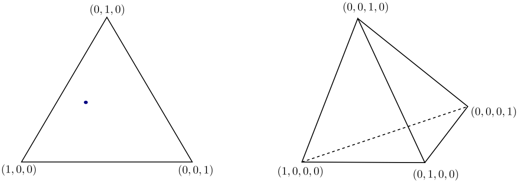


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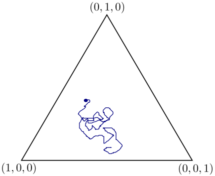


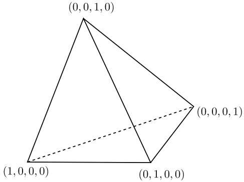


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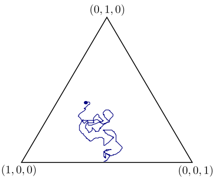


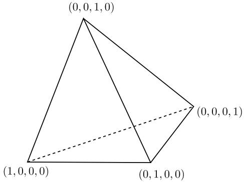


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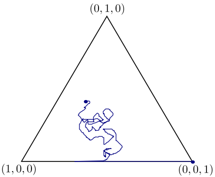


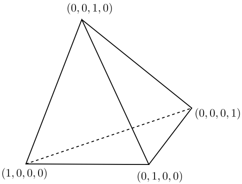


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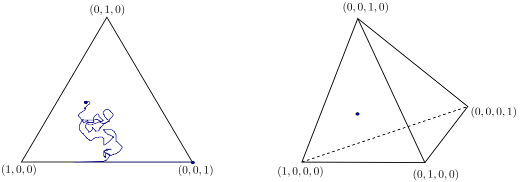


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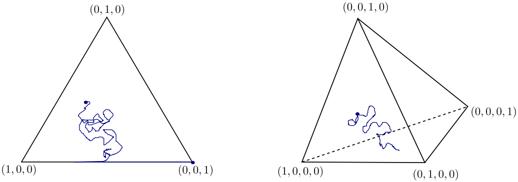


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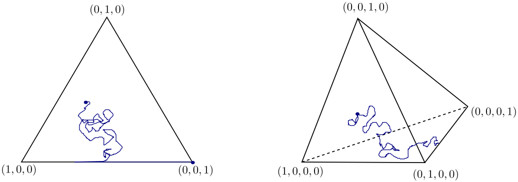


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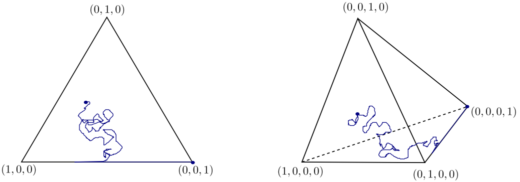


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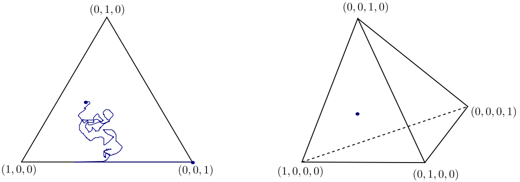


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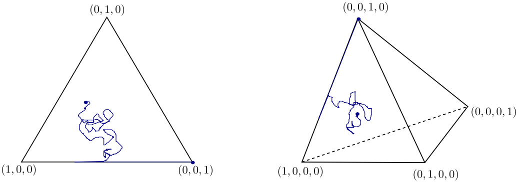


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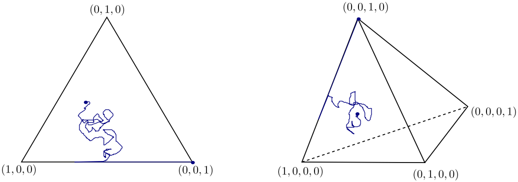


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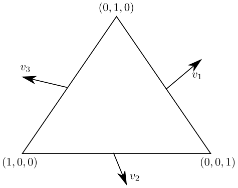

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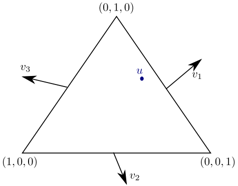

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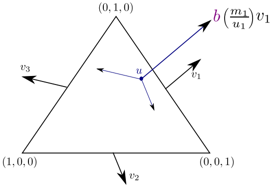

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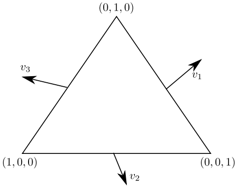


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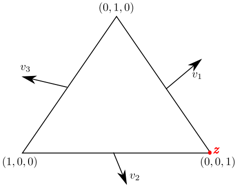


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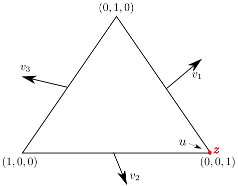


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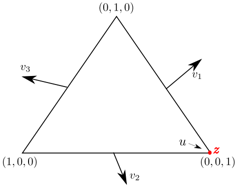


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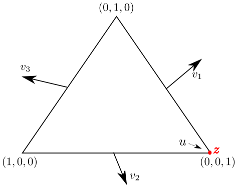


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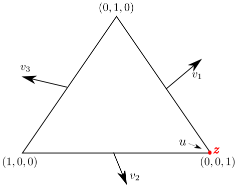


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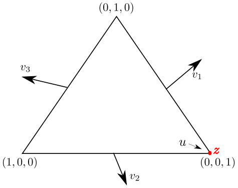


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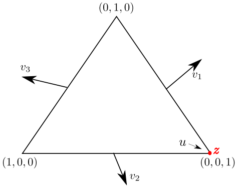


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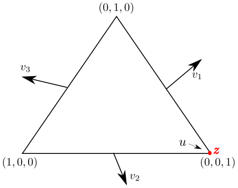


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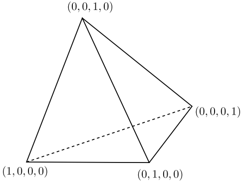

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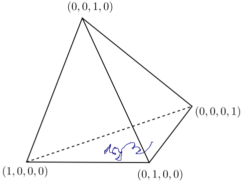

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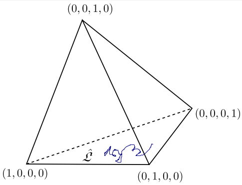

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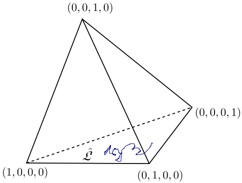


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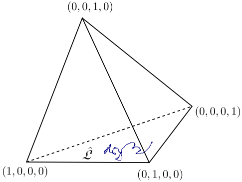


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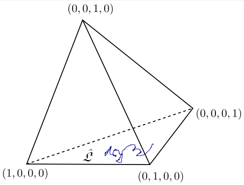


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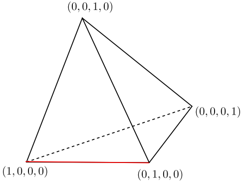


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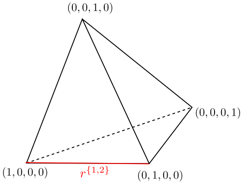


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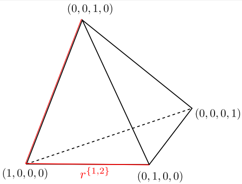


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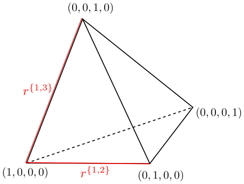


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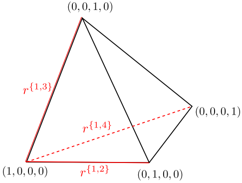


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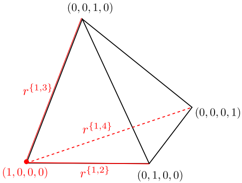


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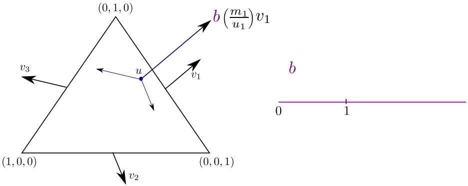

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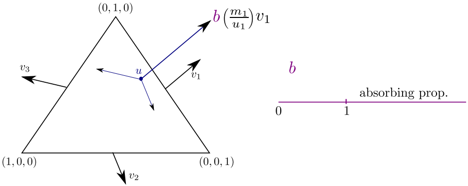

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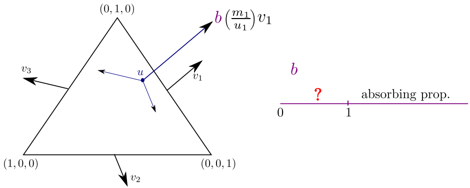


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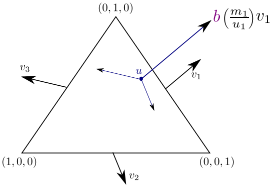


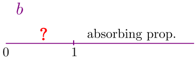


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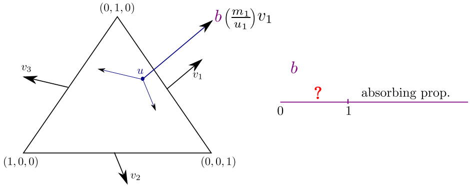


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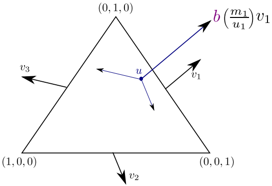


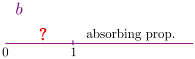


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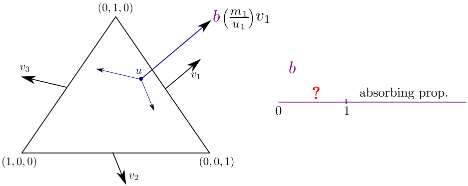


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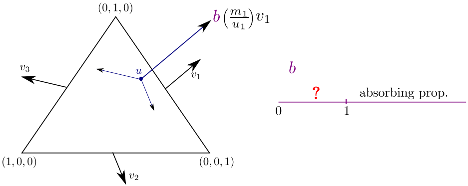


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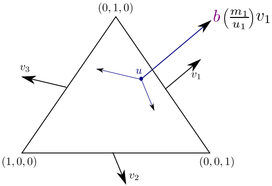


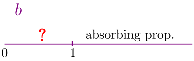


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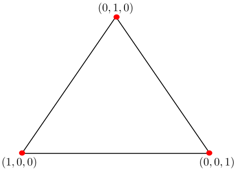


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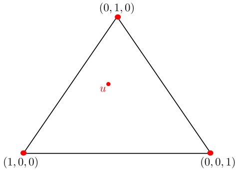


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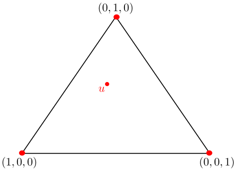


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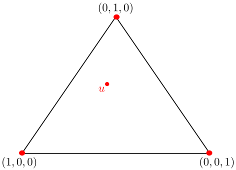


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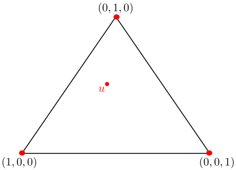


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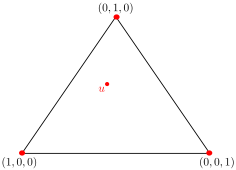


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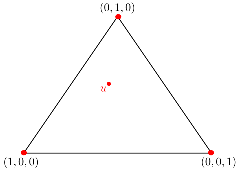


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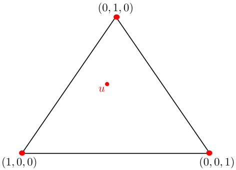


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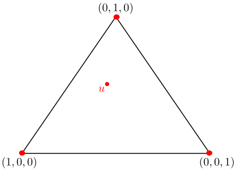


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