Ideas About Quantum Metastability

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£: EPSRC

PLAN

Problem: relaxation & non-ergodicity in quantum systems
(both closed = unitary & open = dissipative)
vs. slow relaxation in classical glasses

1. Slow relaxation in closed quantum systems due to dynamical constraints

2. Signatures of many-body localisation in the absence of disorder

3. Towards a theory of metastability in (open) quantum systems

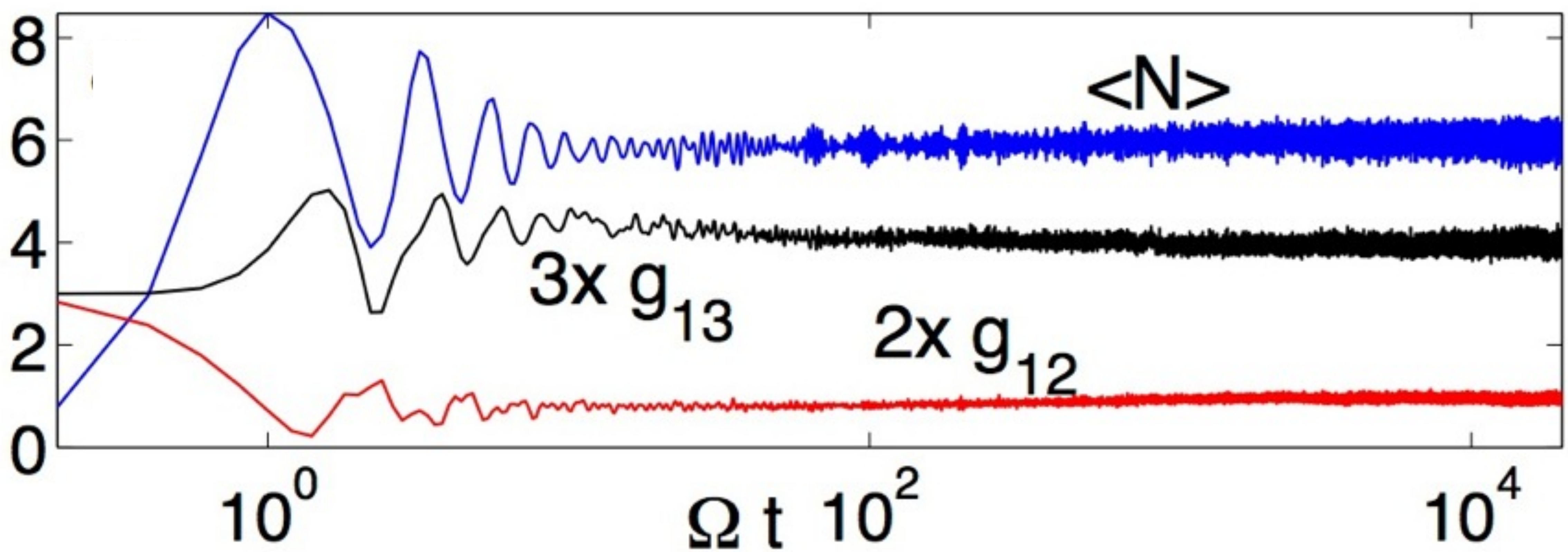
Generic Q. many-body systems "**equilibrate**" (become stationary):

no degenerate E gaps → dephasing in E basis → diagonal ensemble

{Short, Popescu, Linden, Reimann, Eisert, Goldstein, many others}

$$\rho(t) \longrightarrow \omega = \lim_{t\to\infty} \frac{1}{t}\int_0^t U_{t'}\rho_0 U_{t'}^\dagger dt' \qquad |\langle A(t)\rangle - \mathrm{Tr}(A\omega)| \text{ small}$$

E.g. $H = \Omega\sum_{k=1}^{L}\sigma_k^x + \Delta\sum_{k=1}^{L} n_k + V\sum_{k\neq m}^{L}\frac{n_m n_k}{|m-k|^6}$ (Rydbergs)



{Lesanovsky-Olmos-JPG, **PRL 2010**}

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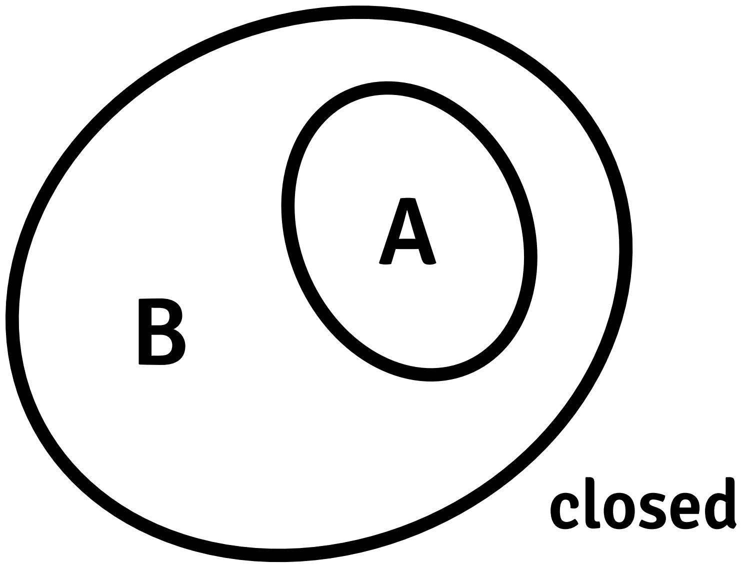
Most Q. many-body systems also "**thermalise**":

{Deutsch, Srednicki, Goldstein, Eisert, Rigol, Gogolin, many others}



$$\rho_A = \mathrm{Tr}_B\, |\psi\rangle\langle\psi| \longrightarrow \rho_A = \mathrm{Tr}_B\, \rho_{\mathrm{th}}$$

$$\rho_{\mathrm{th}} = e^{-\beta H}$$

β set by $\langle\psi_0|H|\psi_0\rangle$

thermalisation ↔ ergodicity (independence of initial conditions)

eigenstate thermalisation hypothesis (ETH):

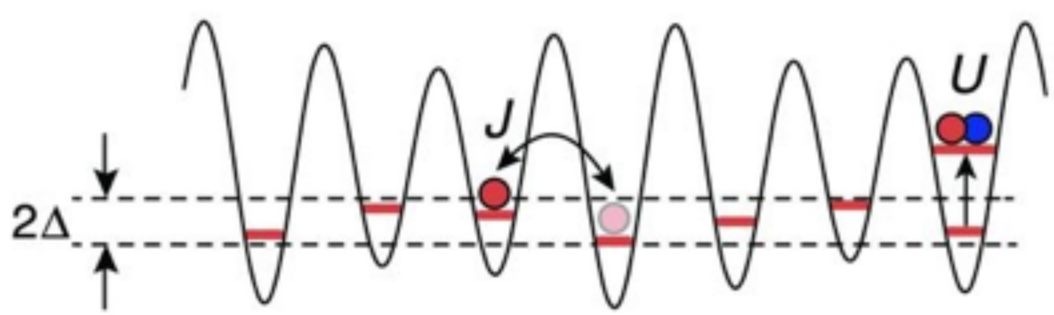
$$\langle E|\mathcal{O}_{\mathrm{local}}|E\rangle = \text{smooth } F(E)$$

Ergodic & thermal → non-ergodic & **many-body localised (MBL)**

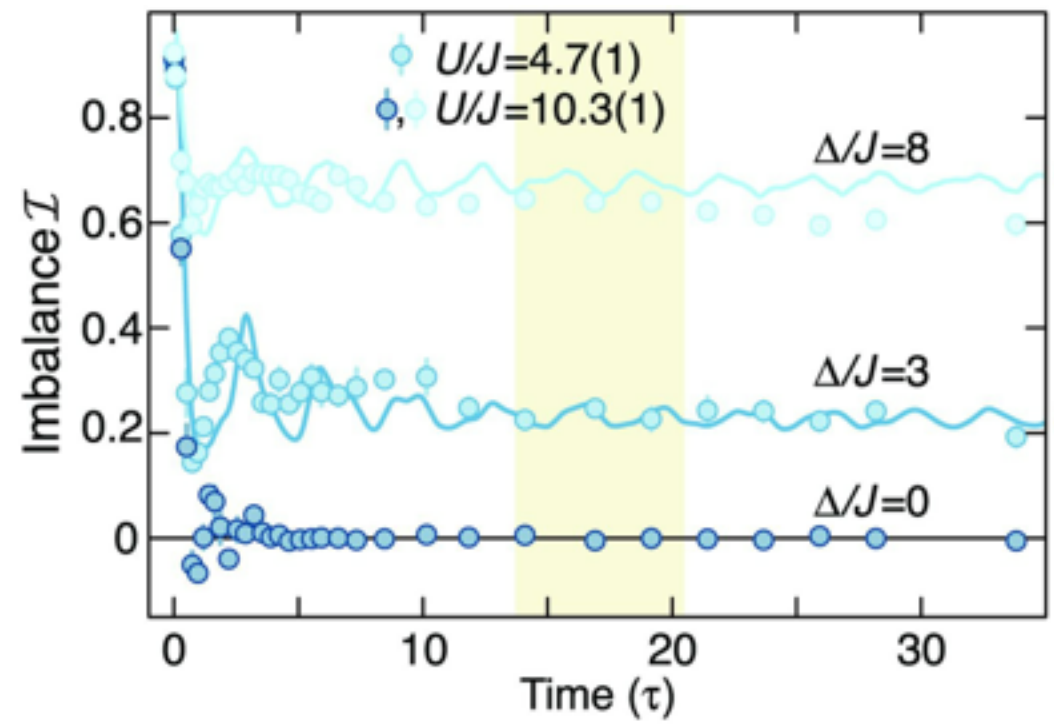
{Basko-Aleiner-Altshuler, Huse+, Prosen+, Abanin+, Moore+, Altman+, many others}

Cf. Anderson localisation, **but with interactions**

MBL transition **driven by quenched disorder** (rnd h small = thermal, rnd h large = MBL)



{Schreiber et al 2015}



MBL = **breakdown of ETH** (excited states = Area law)

MBL often thought of as a **"glass transition"**

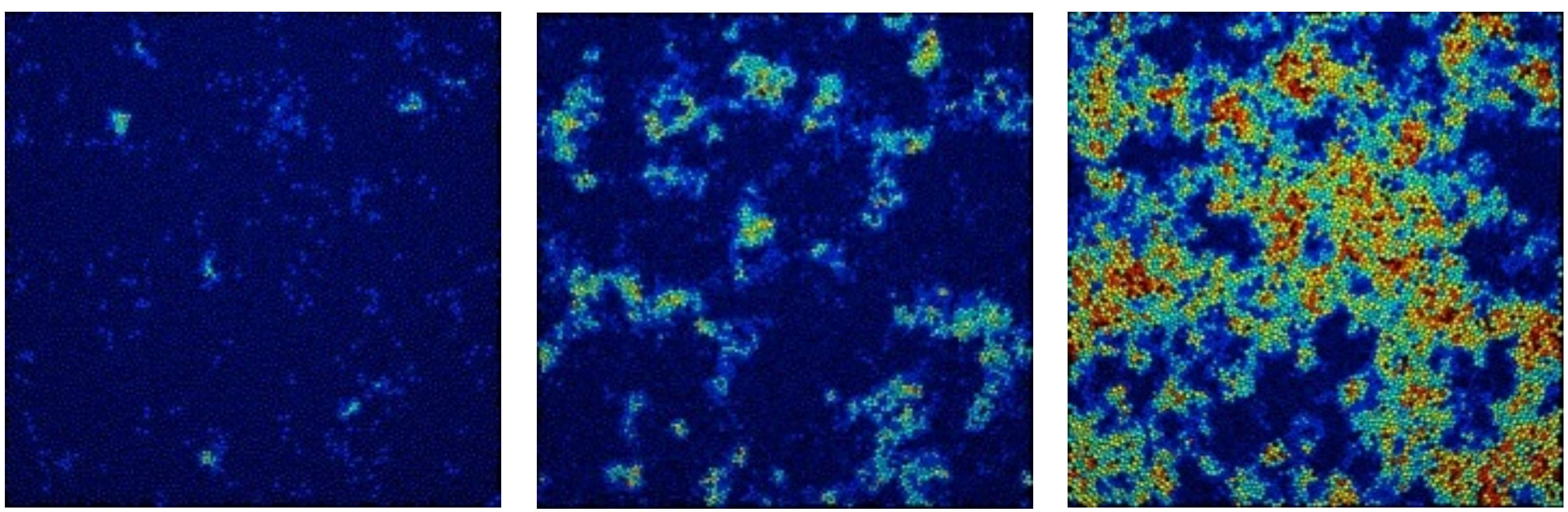
What can we say about slow relaxation in Q.S. and MBL from what we know of classical glasses?

Classical glasses

- Slowdown & eventual arrest
- Timescales not divergent
- No disorder

Relaxation is intermittent and spatially heterogeneous

$t \ll \tau_\alpha$ $\quad$ $t \approx \tau_\alpha$ $\quad$ $t \gg \tau_\alpha$



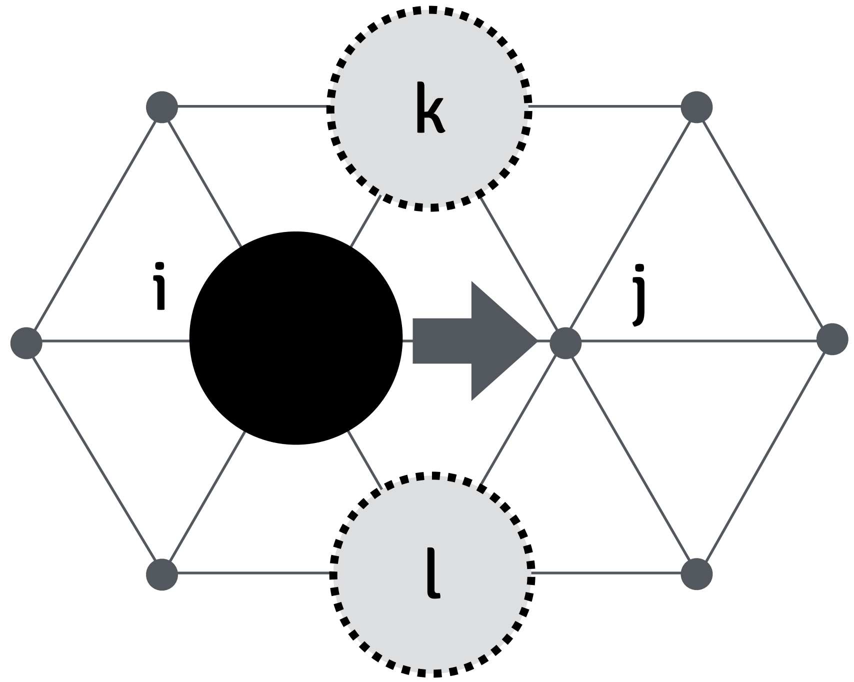
e.g.
50:50 L-J mixture
{Hedges 2009}

Due to (effective) local constraints on dynamics {JPG-Chandler, **PRL 2002**}

1. Quantum slow relaxation due to dynamical constraints

{van Horssen-JPG, 2016}

Steric (excluded volume) constraint on hopping:



classical = **constrained lattice gas**

{Kob-Andersen, Jackle+}

$$H = -\sum_{\langle ij \rangle} \underbrace{(1 - n_k n_l)}_{\text{constraint}} \left\{ \lambda (\sigma_i^+ \sigma_j^- + \sigma_j^+ \sigma_i^-) - (1-\lambda) \left[n_i (1 - n_j) - n_j (1 - n_i) \right] \right\}$$

constraint ("1-vacancy assisted" or 1-TLG)

Rokhsar-Kivelson pt: $\lambda = \frac{1}{2} \longrightarrow H = -\text{ class. Master op. } \mathbb{W} \longrightarrow \partial_t |P\rangle = \mathbb{W} |P\rangle$

Away from RK: $\lambda \neq \frac{1}{2} \longrightarrow$ Large-deviations: **active** $\left(\lambda > \frac{1}{2}\right)$ **inactive** $\left(\lambda < \frac{1}{2}\right)$

1. Quantum slow relaxation due to dynamical constraints

{van Horssen-JPG, 2016}

$$H = -\sum_{\langle ij \rangle} (1 - n_k n_l) \left\{ \lambda(\sigma_i^+ \sigma_j^- + \sigma_j^+ \sigma_i^-) - (1-\lambda) \left[n_i(1-n_j) - n_j(1-n_i) \right] \right\}$$

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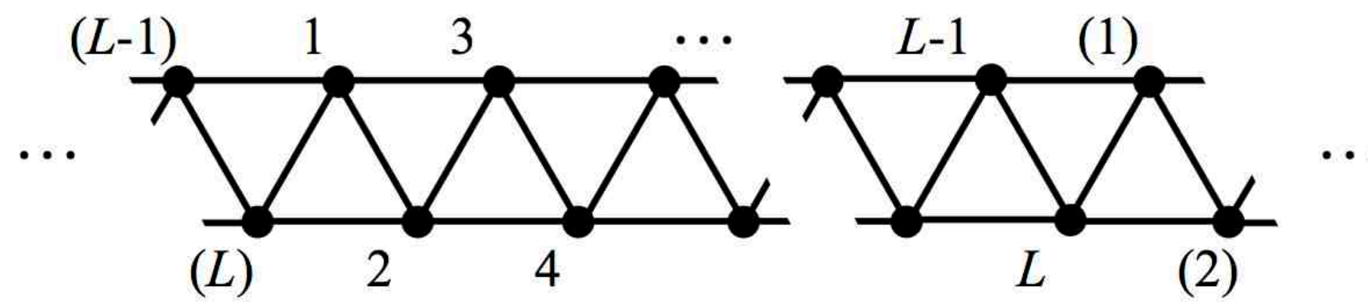
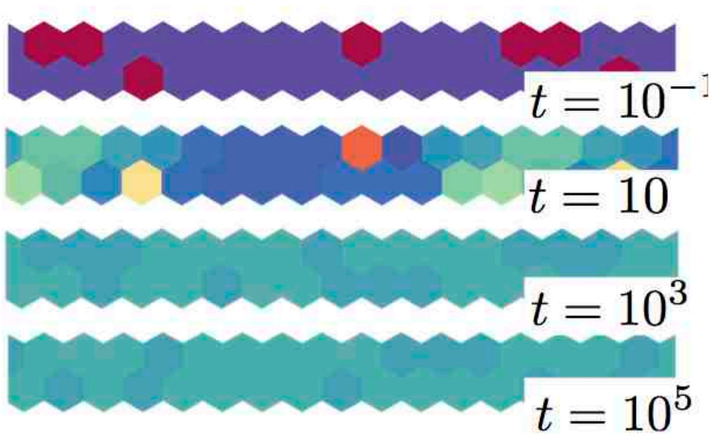
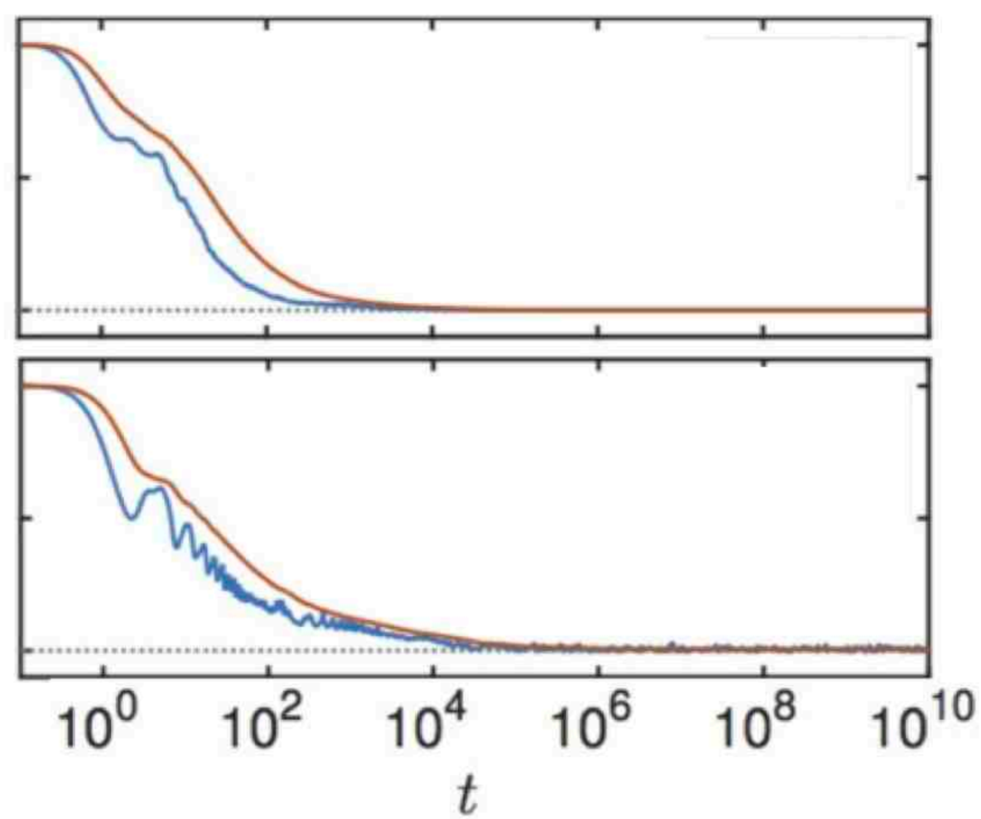


$\lambda = 0.8$



active = fast



$$\Phi(t) = \sum_{\langle ij \rangle} |\langle n_i(t) \rangle - \langle n_j(t) \rangle|^2$$

$$c(t) = \sum_i \langle n_i(0) n_i(t) \rangle$$

$N = 24$

$M = 20$

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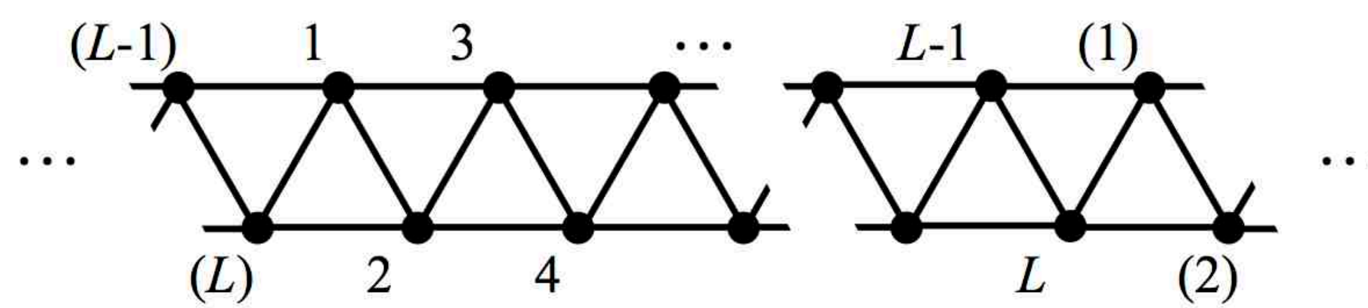
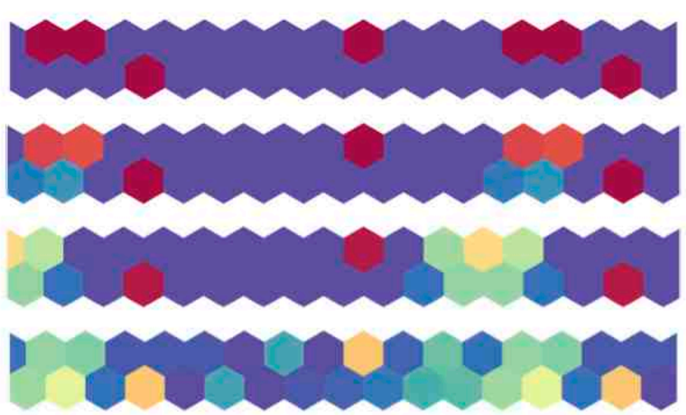
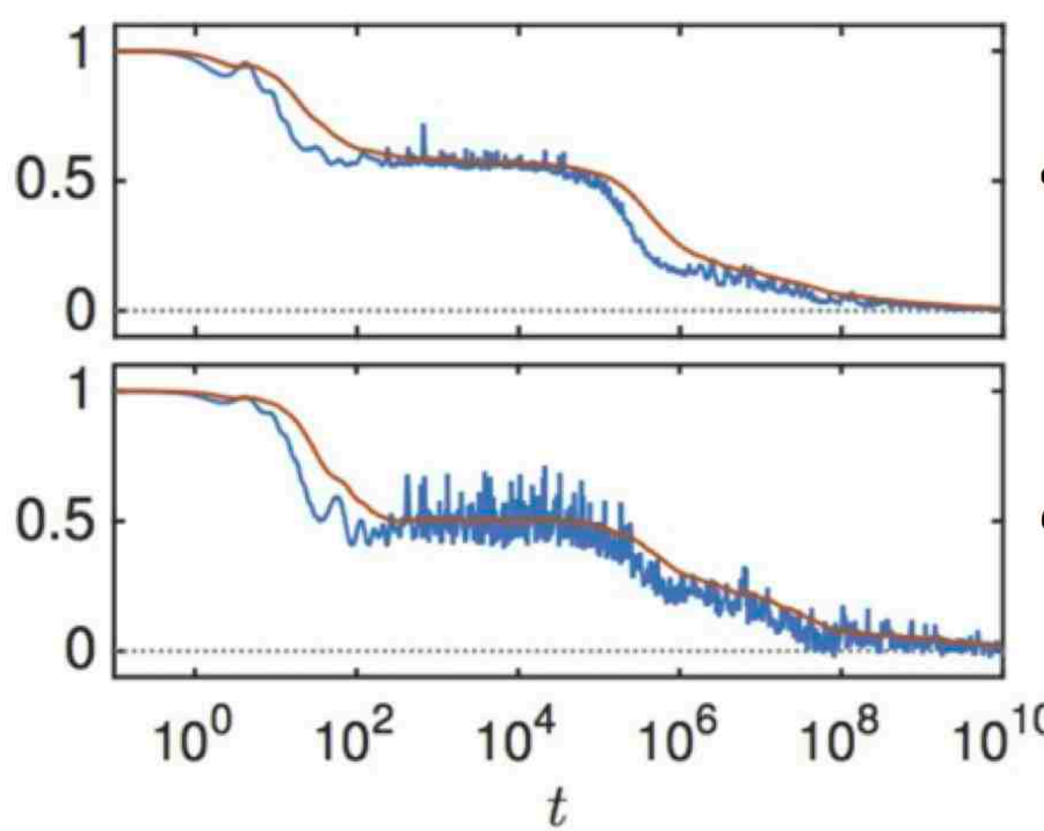


$\lambda = 0.2$



inactive = slow



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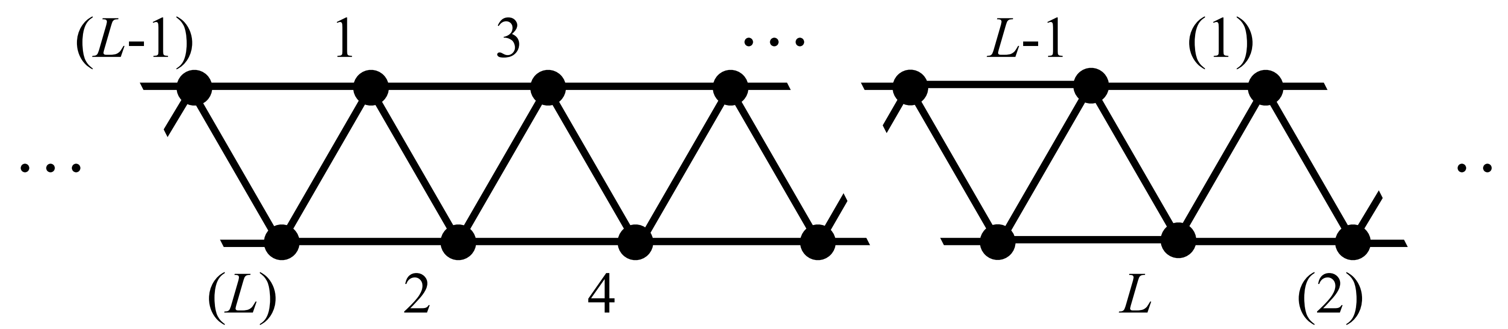
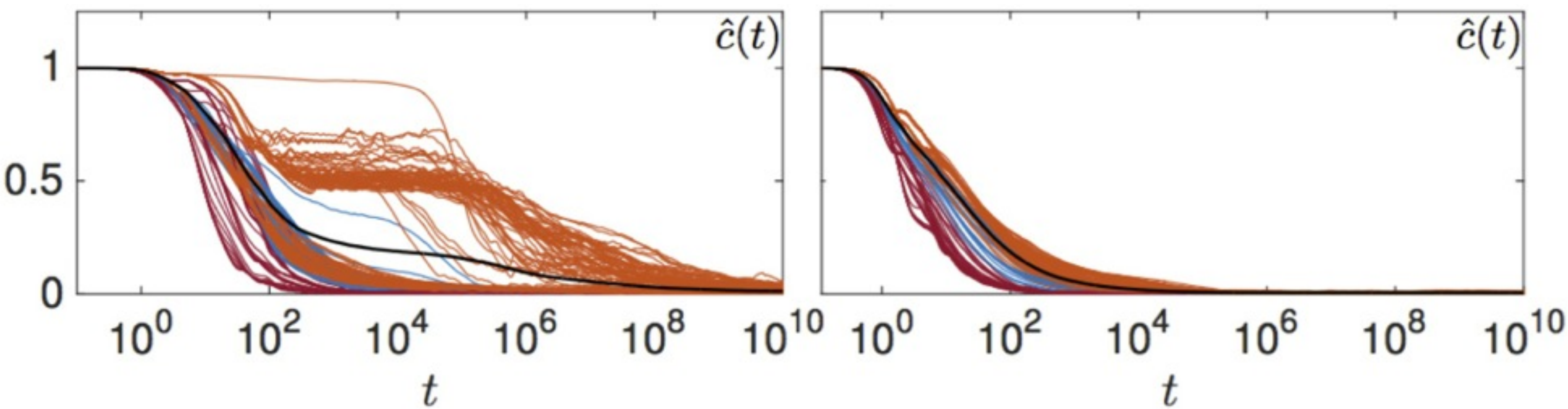


$\lambda = 0.2$ $\lambda = 0.8$



metastability = memory of initial conditions

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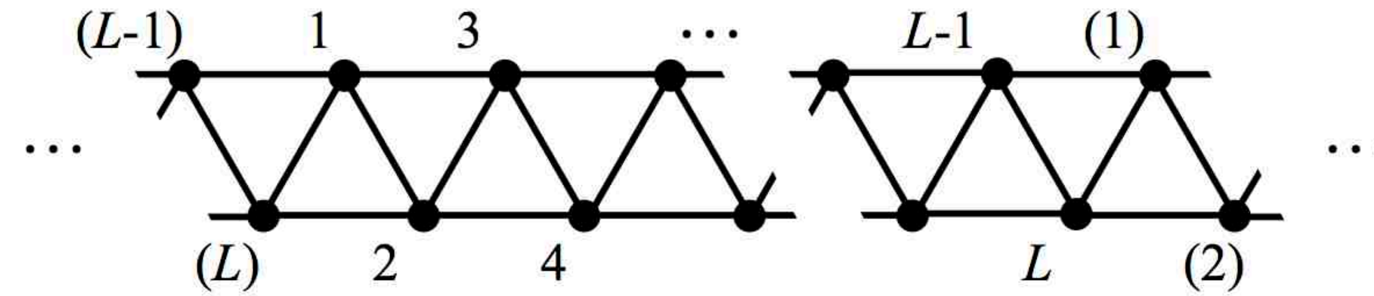

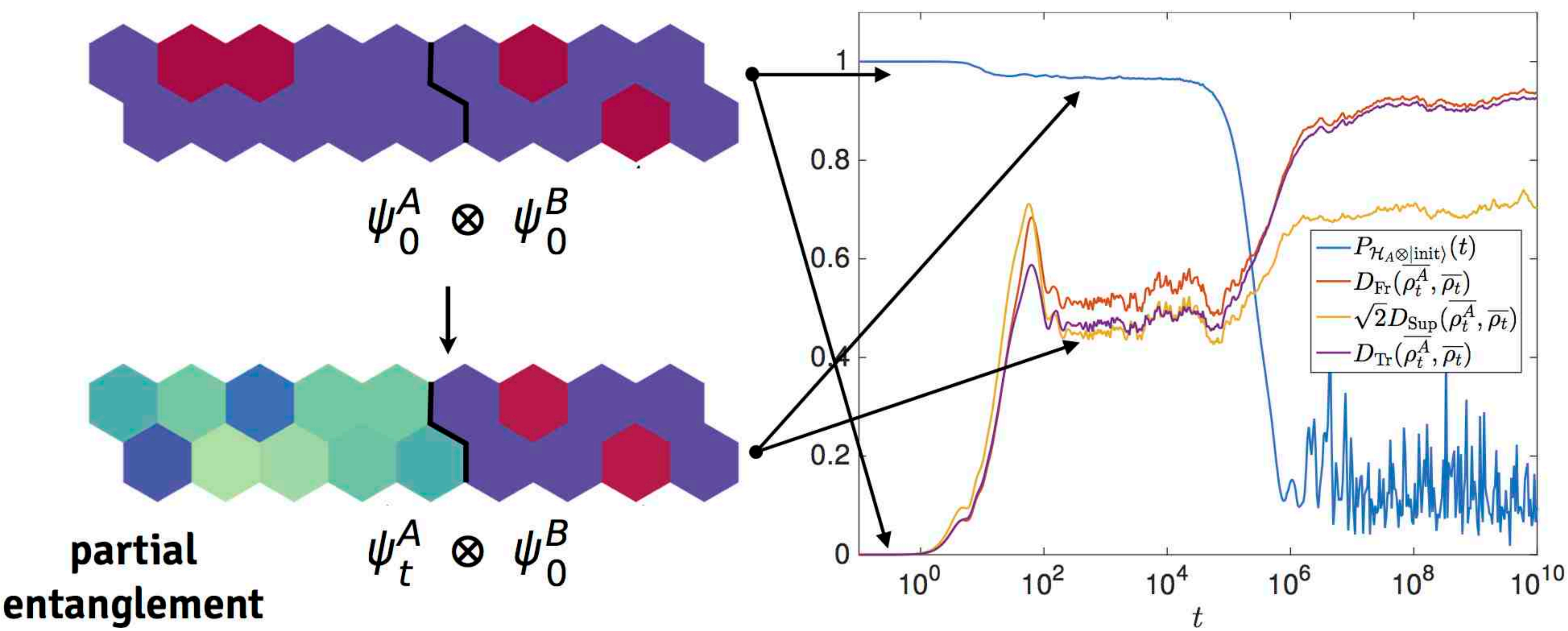

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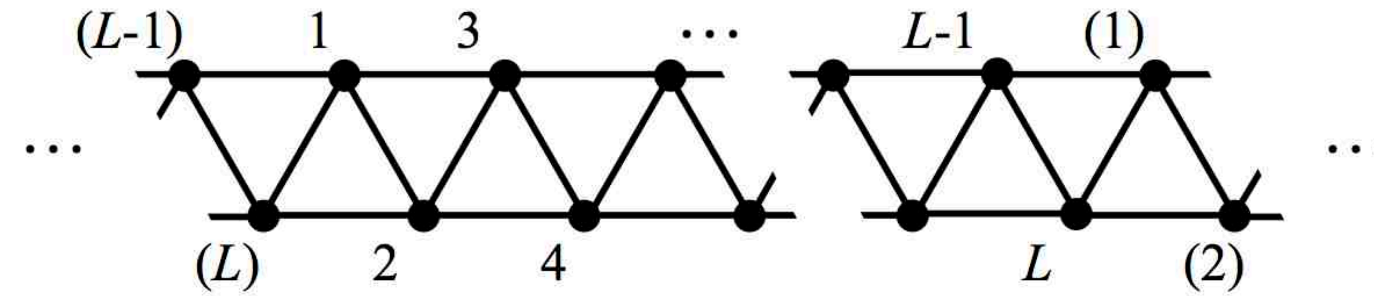

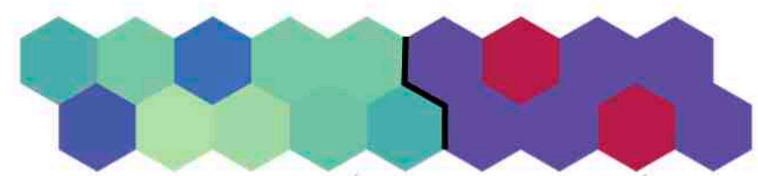

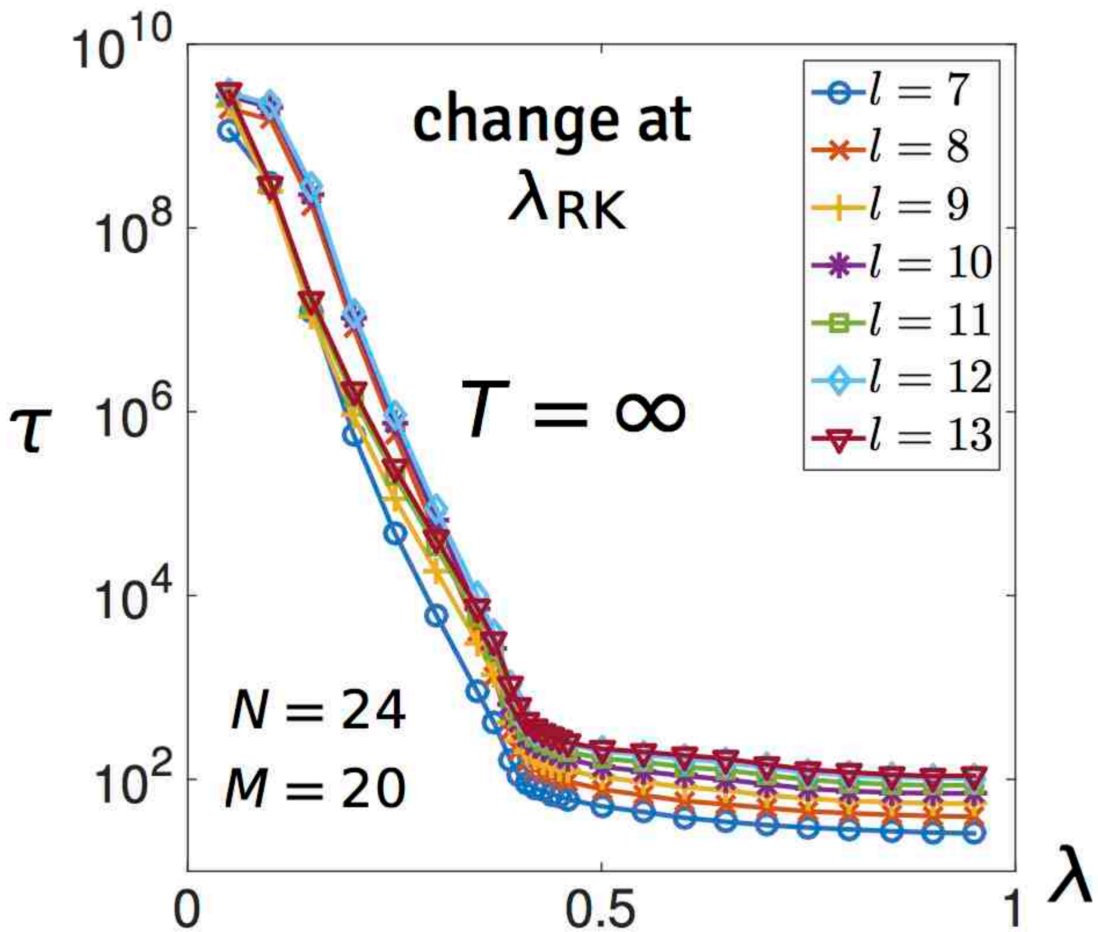

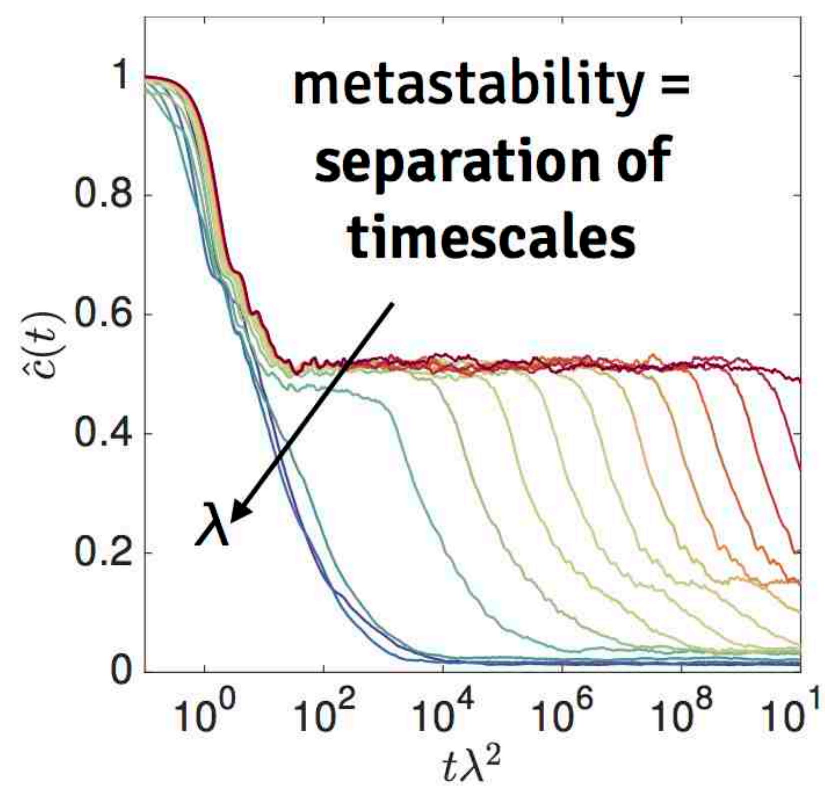

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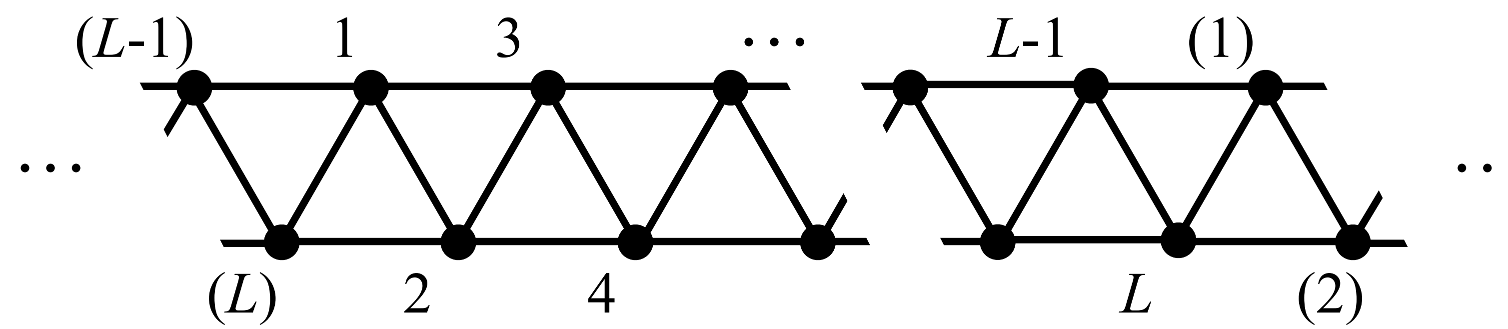
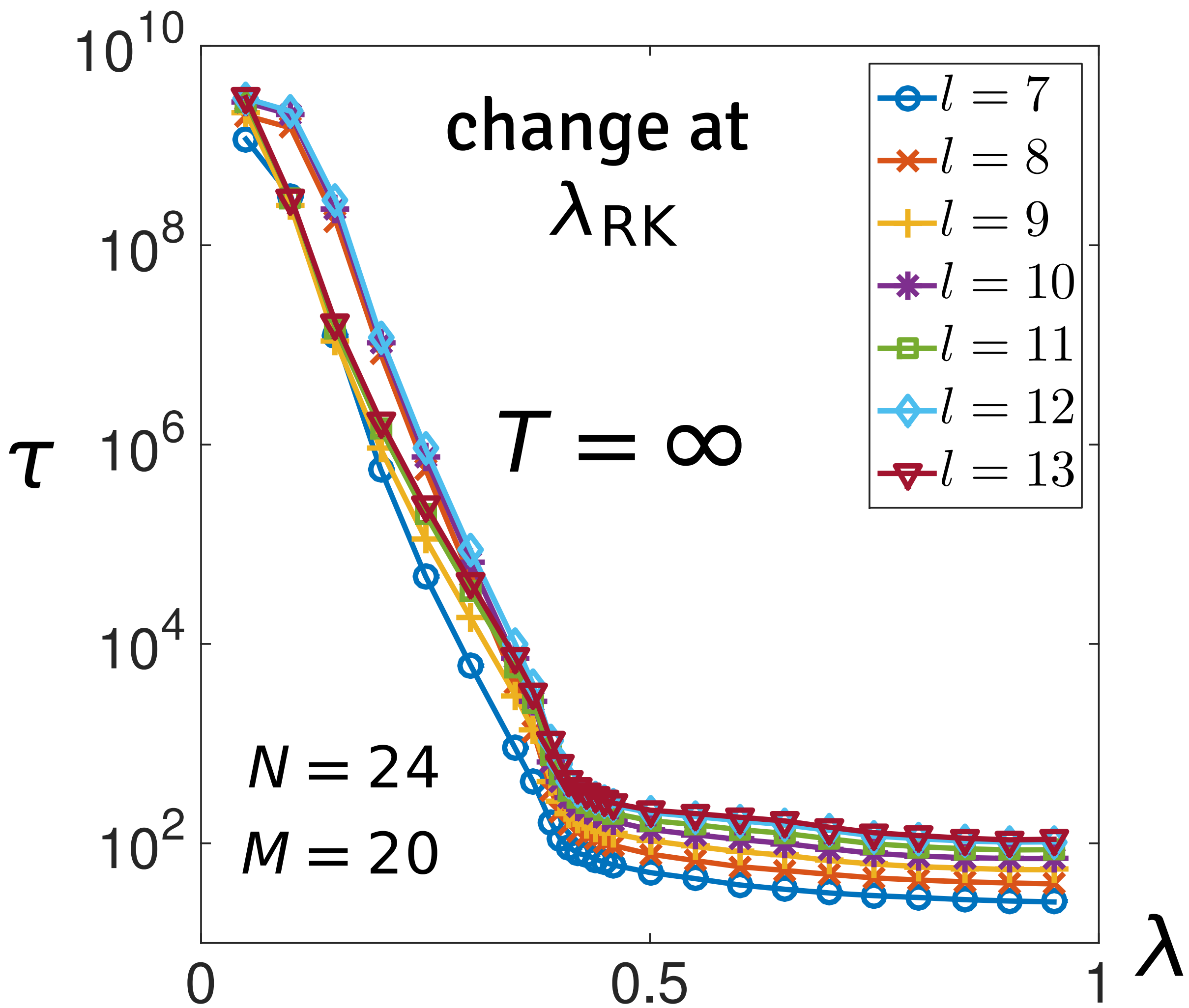
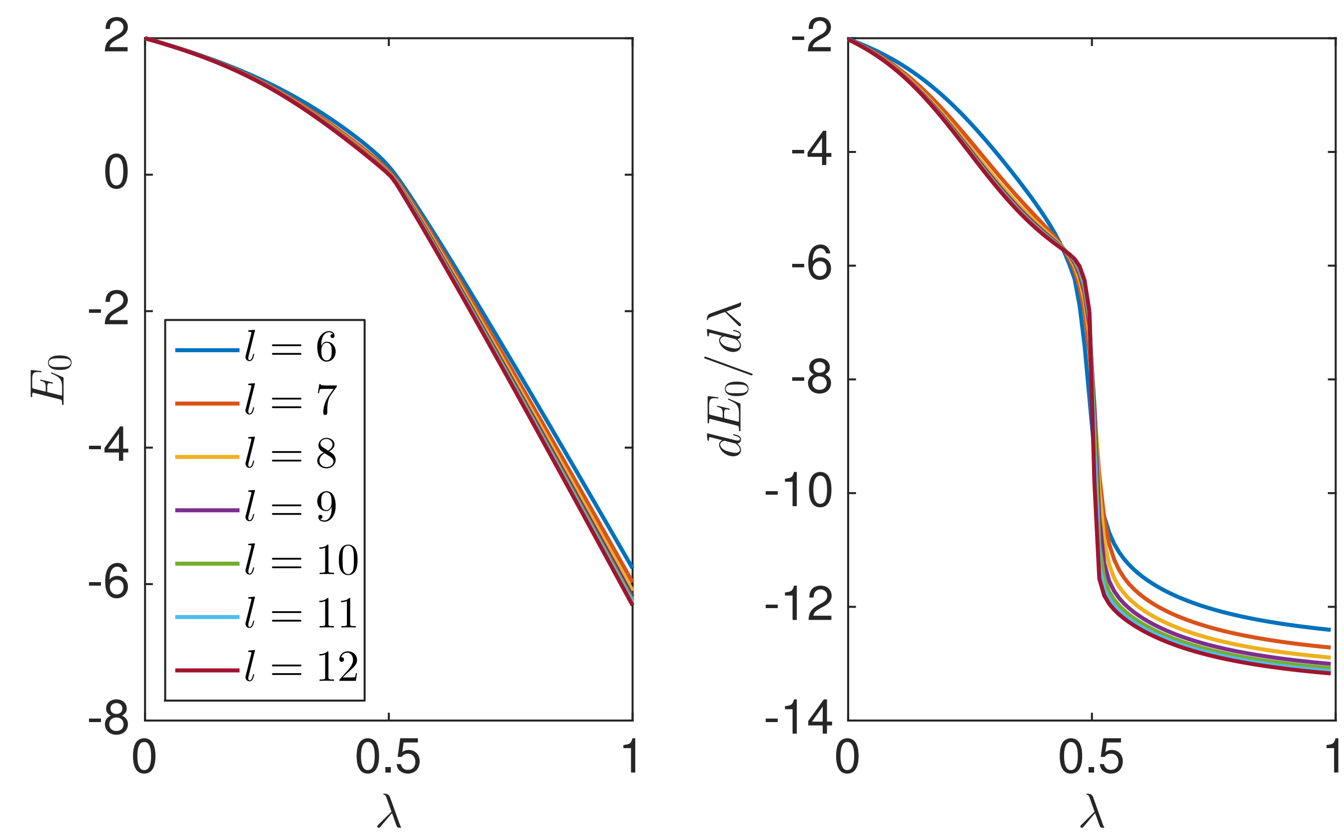






1st order singularity in g.s.

cf. classical **active-inactive** transitions

2. Dynamics of MBL in a quantum glass (w/o disorder)

{van Horssen-Levi-JPG, **PRB 92, 100305(R) 2015**}

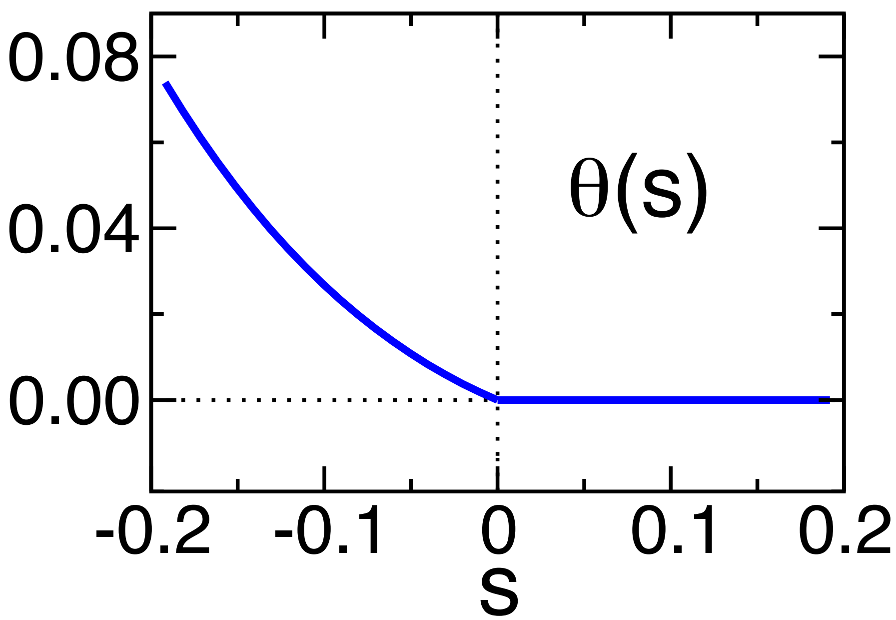
$$H = -\sum_i n_{i+1}\left(e^{-s}\sigma_i^x - 1\right)$$

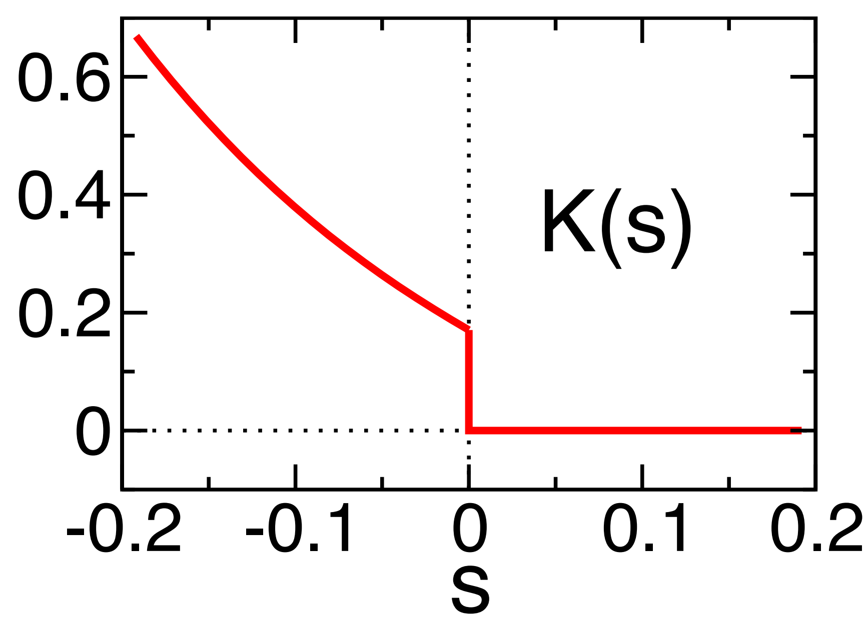
$$e^{-s} = \lambda/(1-\lambda) \longrightarrow s_{\rm RK} = 0$$

Classical **East** glass model $(T=\infty)$: **11** $\rightleftarrows$ **01** **10** ~~$\rightleftarrows$~~ **00**

{Jackle+, Sollich-Evans, Aldous-Diaconis, JPG-Chandler, Chleboun-Faggionato-Martinelli, Blondel-Toninelli, many others}

s<0 active to s>0 inactive transition ≡ quantum phase transition in g.s. of H





$N \longrightarrow \infty$

{JPG-Jack-Lecomte-Pitard-van Duijvendijk-van Wijland **PRL 2007**}

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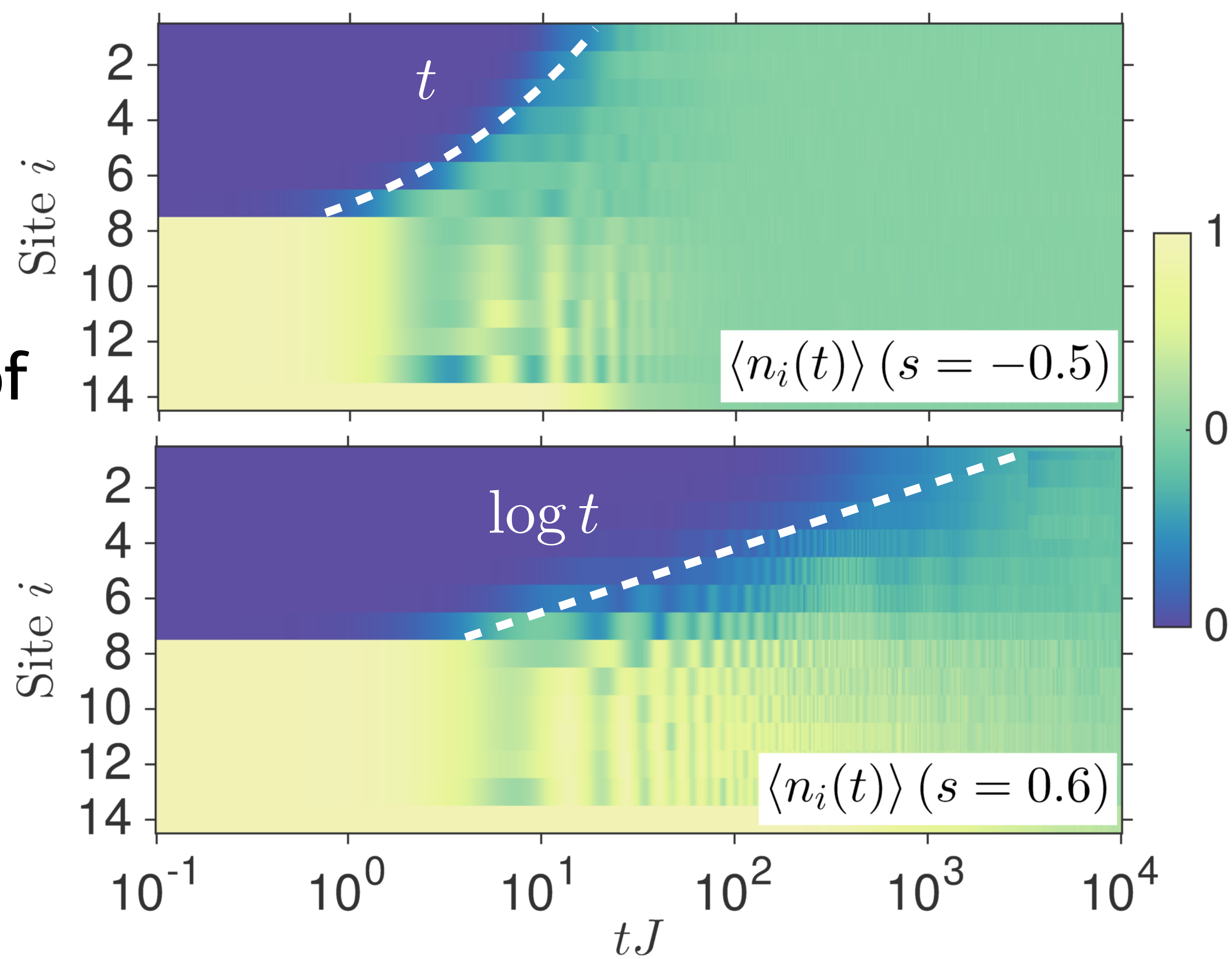
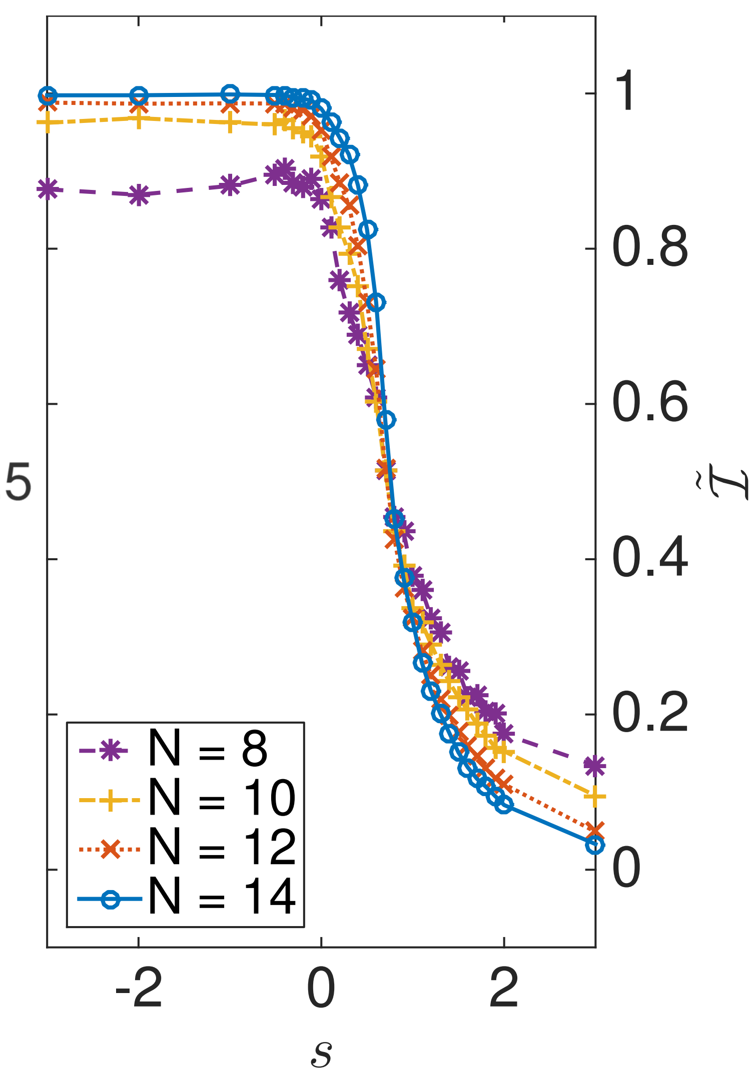
Signatures of MBL dynamics for s>0 (inactive):

Propagation of excitations





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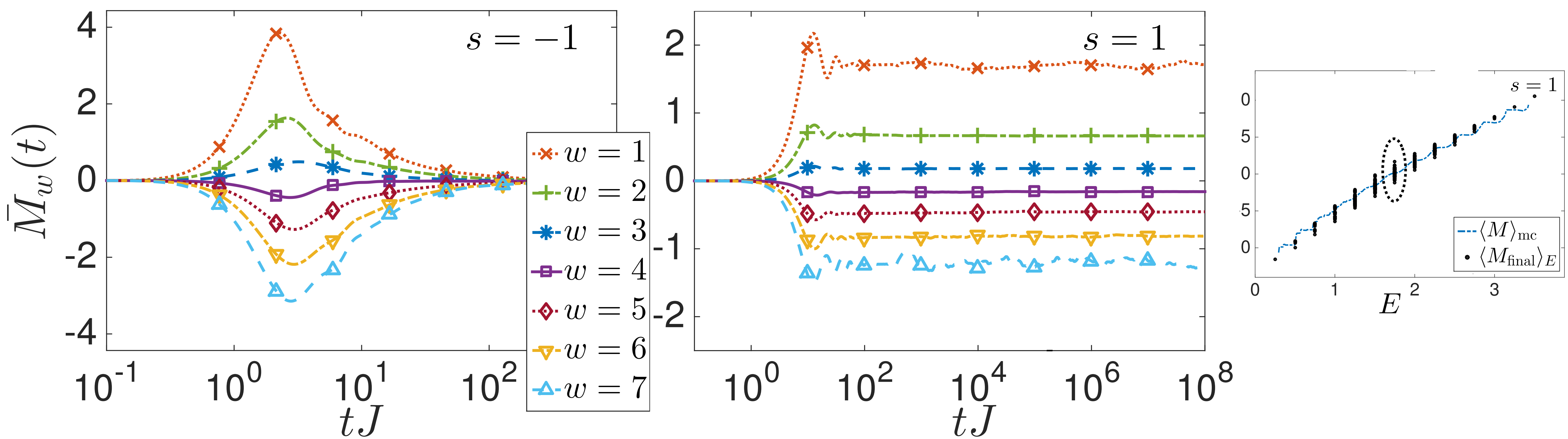
Signatures of MBL dynamics for s>0 (inactive):

time averaged magnetisation

dependence on initial conditions - ETH (active) v. no-ETH (inactive)



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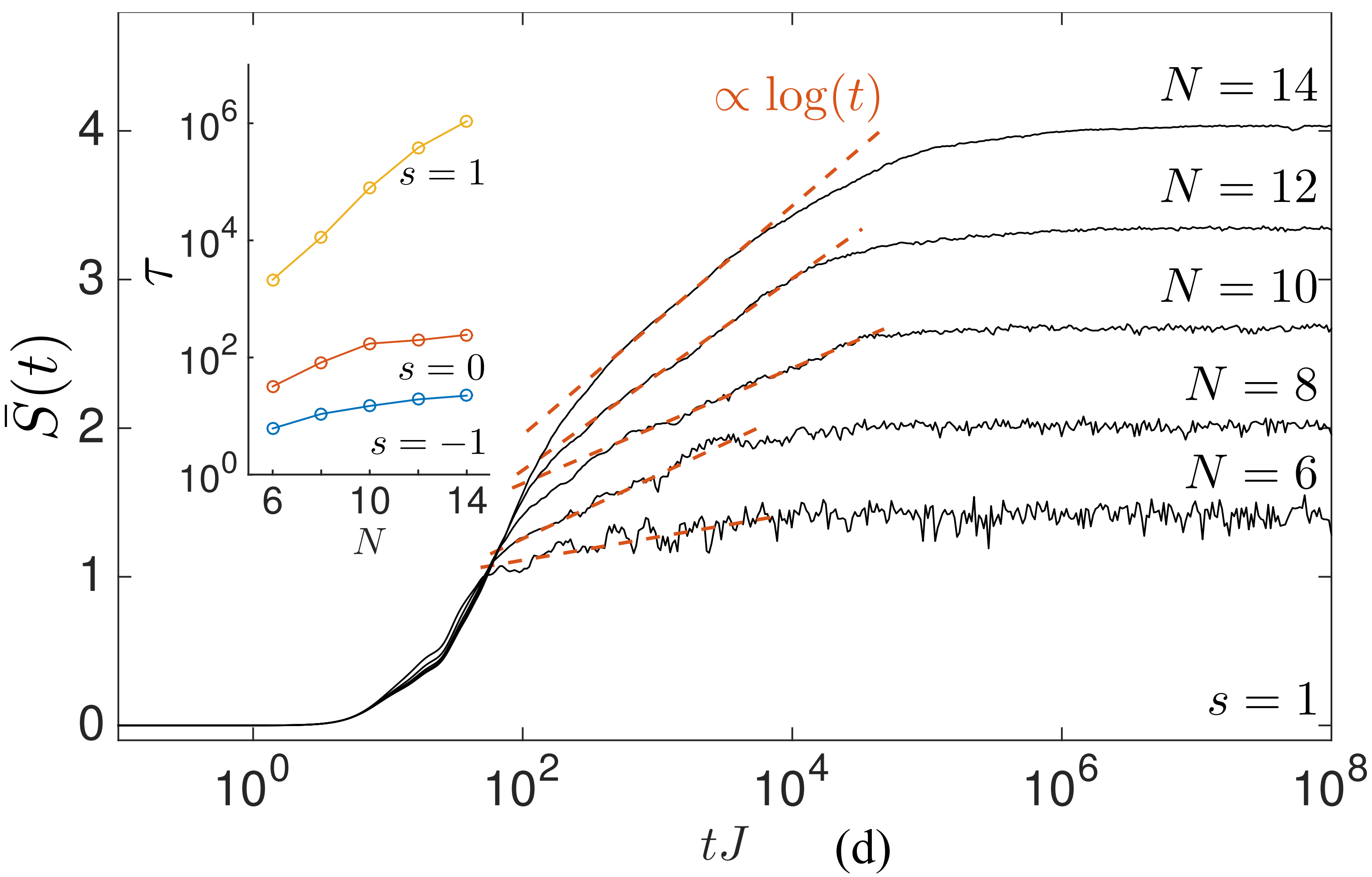
Signatures of MBL dynamics for s>0 (inactive):

log t
growth of
entanglement
entropy

{Serbyn-Papic-Abanin,
Moore+, others}



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Signatures of MBL dynamics for s>0 (inactive):

relaxation of correlations

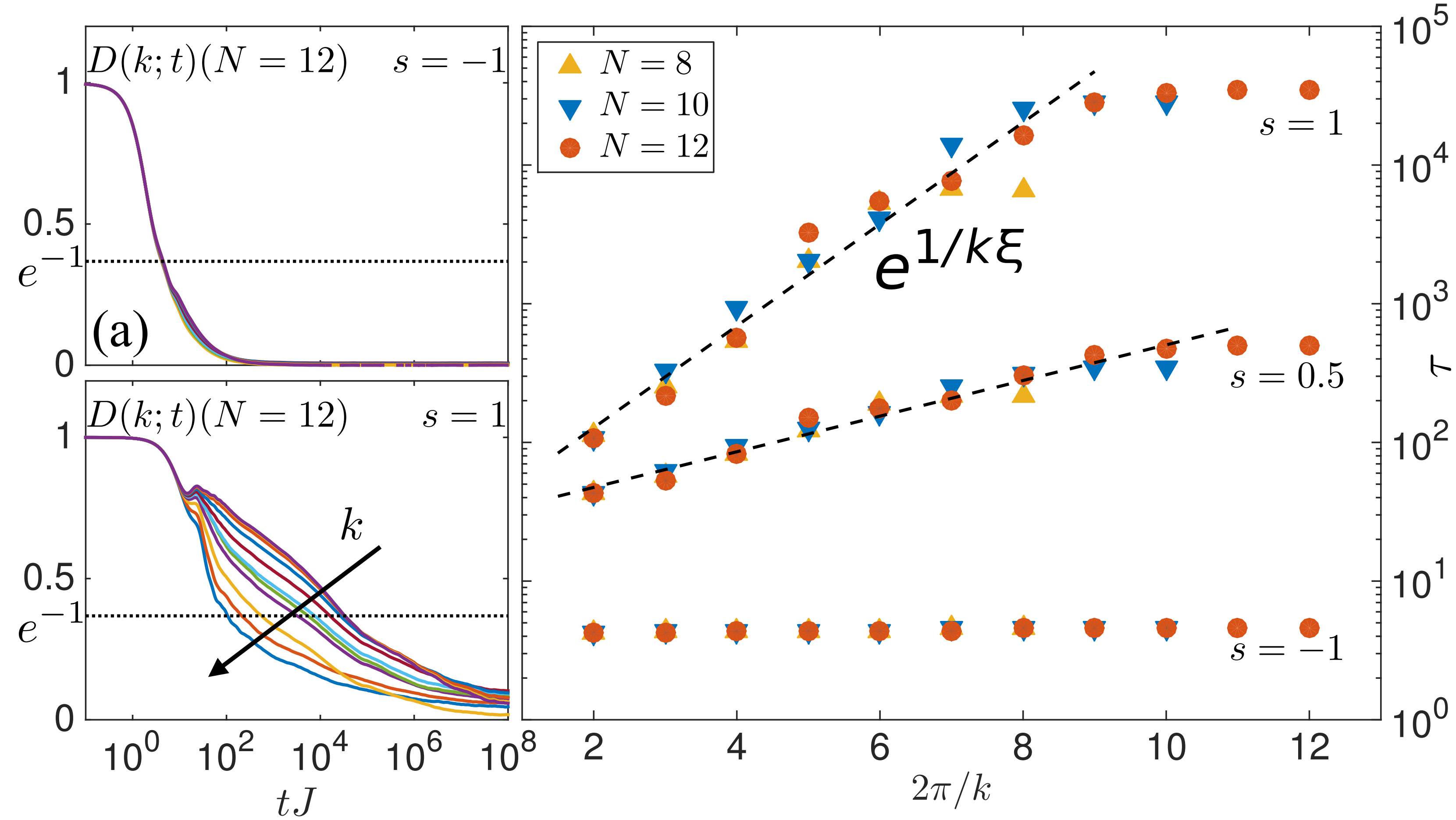
$$D(k,t) = \mathrm{Tr}\, F_k(t) F_k(0)$$

$$F(k) = \sum_j \sigma_j^z e^{ikj}$$

$$w \sim \xi \log t \;\rightarrow\; \tau(k) \sim e^{1/k\xi}$$

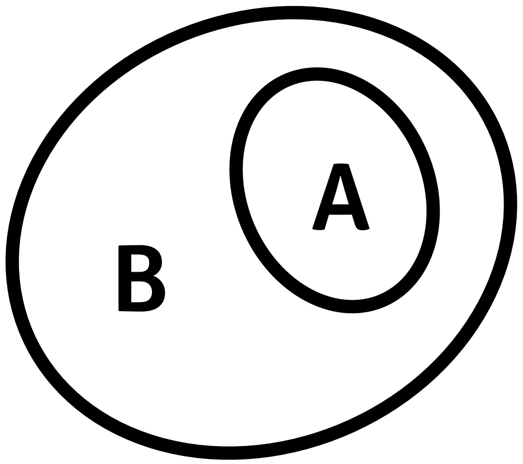
MBL or quasi-MBL

{cf. Yao-Laumann-Cirac-Lukin-Moore, De Roeck-Huveneers}



3. Towards a theory of quantum metastability

{Macieszczak-Guta-Lesanovsky-JPG, **arXiv:1512.05801**}



→ in A+B (closed) = **difficult**

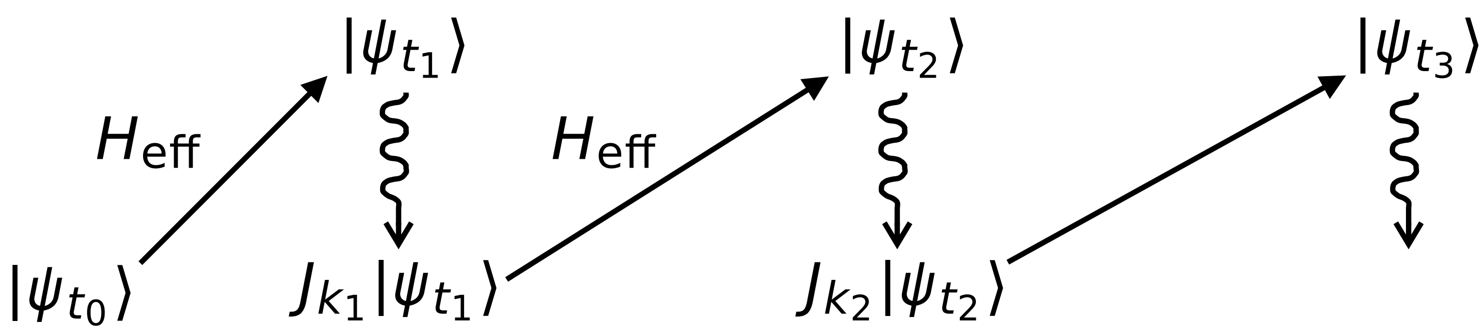
→ in A only (open) = **easier** by analogy with classical {Gaveau-Shulman, Bovier et al.}

quantum master equation: {Lindblad, Gorini-et-al}

$$\partial_t \rho = -i[H,\rho] + \sum_\mu \left[J_\mu \rho J_\mu{}^\dagger - \tfrac{1}{2}\left\{ J_\mu{}^\dagger J_\mu, \rho \right\} \right] \equiv \mathcal{L}(\rho)$$

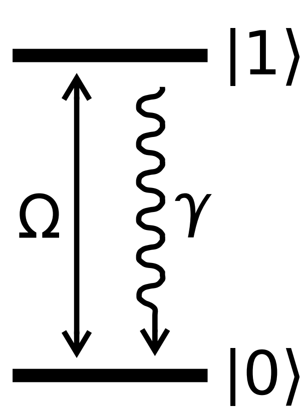
stochastic wave function:

{Dalibard-et-al, Belavkin}



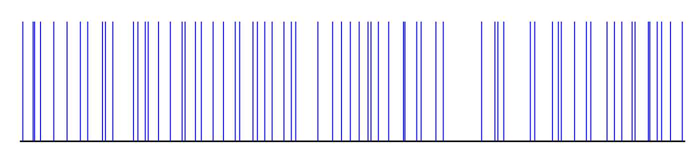
$$\left(H_{\text{eff}} = H - \frac{i}{2} \sum_\mu J_\mu{}^\dagger J_\mu \right)$$

E.g. 2-level system at T = 0:



$$H = \Omega\, \sigma_x$$

$$J = \sqrt{\gamma}\, \sigma_-$$



quantum jump trajectory

Note: if ρ = diagonal, H = 0 and J_μ = rank 1 (e.g. $J_\mu = |C'\rangle\langle C|$) **then QME → classical ME**

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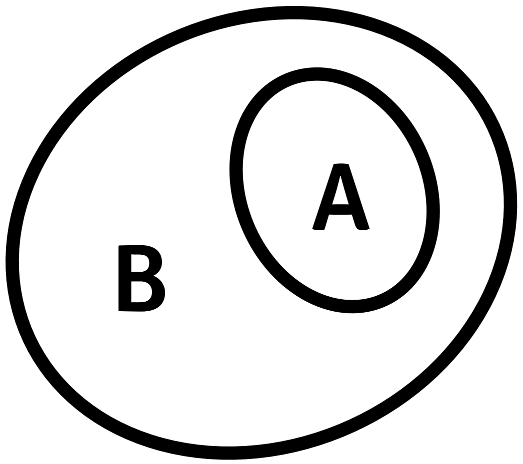

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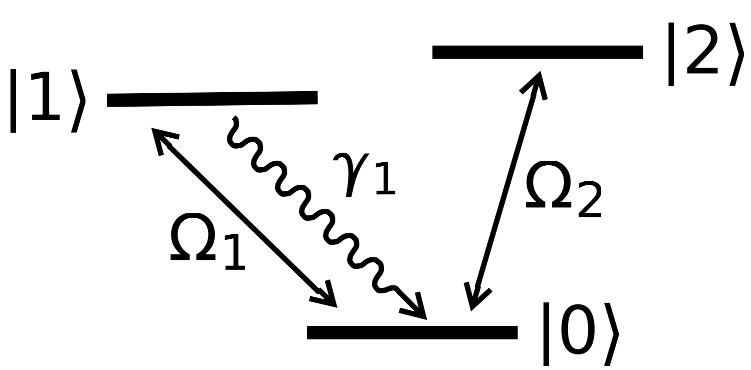
Examples of metastability in open quantum dynamics

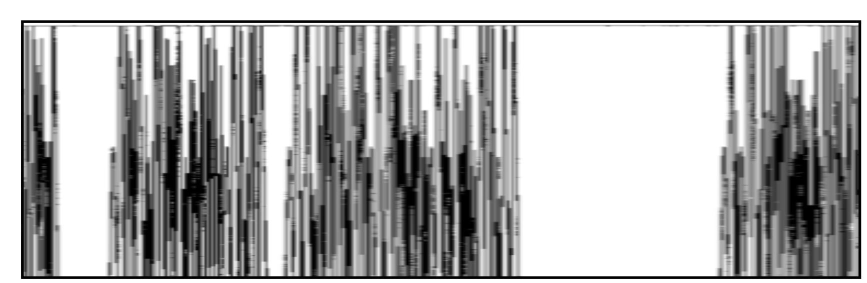
(i) 3-level system (electron shelving, blinking q. dot …)



$$H = \Omega_1 |0\rangle\langle 1| + \Omega_2 |0\rangle\langle 2| + \text{c.c.}$$

$$J = \sqrt{\gamma_1}\, |0\rangle\langle 1|$$

$\Omega_2 \ll \Omega_1,\ \gamma_1$



intermittency

{Hegerfeldt-Plenio}

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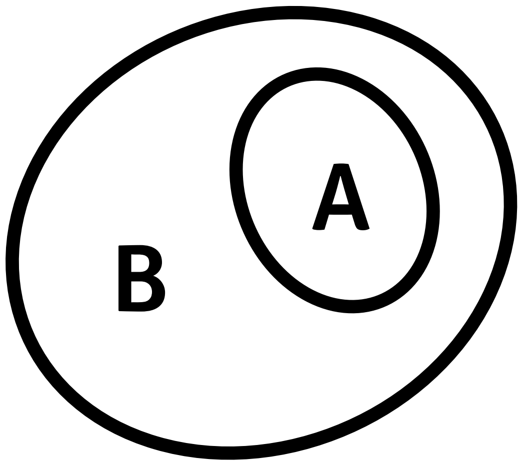

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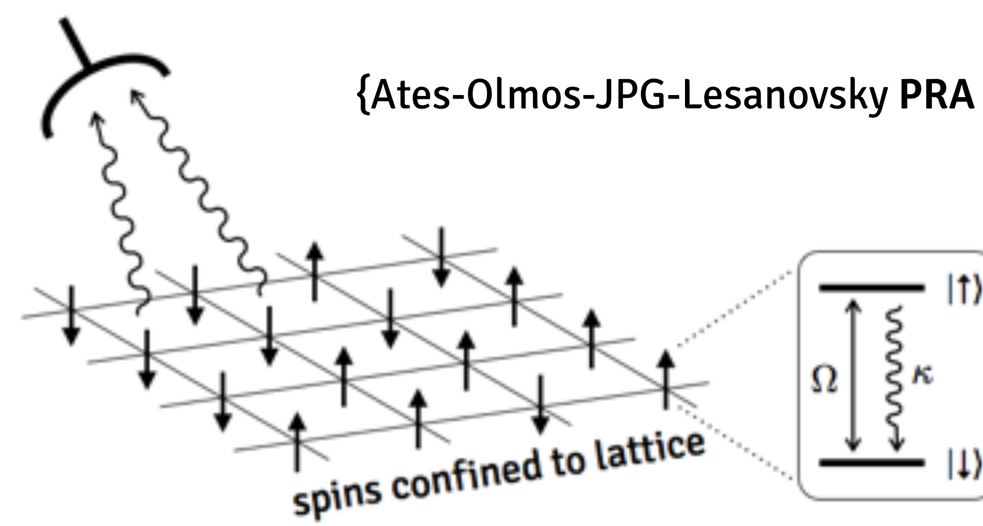
Examples of metastability in open quantum dynamics

(ii) dissipative transverse field Ising model

{Ates-Olmos-JPG-Lesanovsky **PRA 2012**}

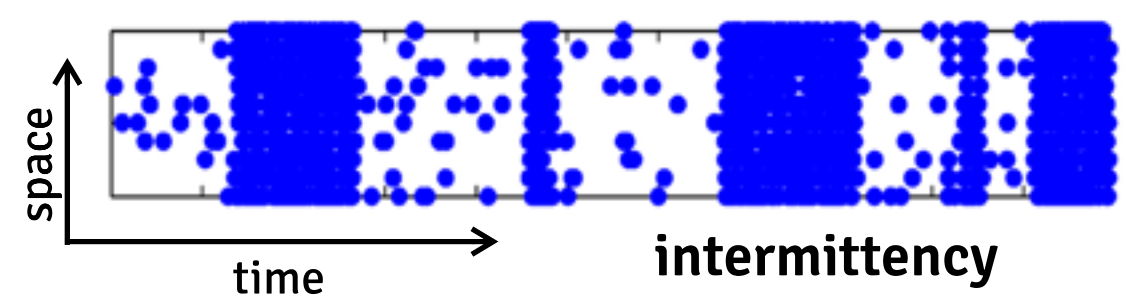


$$H = \Omega \sum_i \sigma_i^x + V \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z$$

$$J_i = \sqrt{\kappa}\, \sigma_i^-$$

stationary state transition

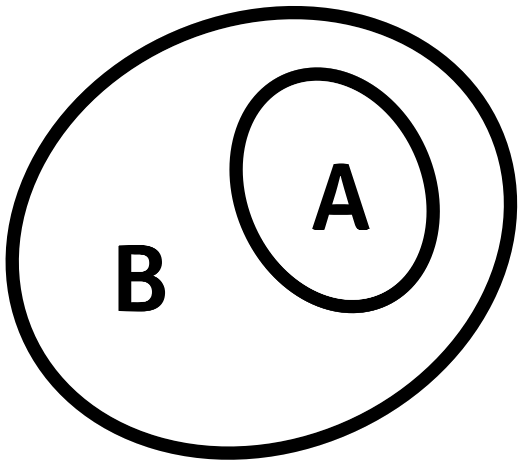
ferro $(\downarrow\downarrow\cdots\downarrow)$ → **para** $(\rightarrow\rightarrow\cdots\rightarrow)$



intermittency

3. Towards a theory of quantum metastability

{Macieszczak-Guta-Lesanovsky-JPG, **arXiv:1512.05801**}



→ in A+B (closed) = **difficult**

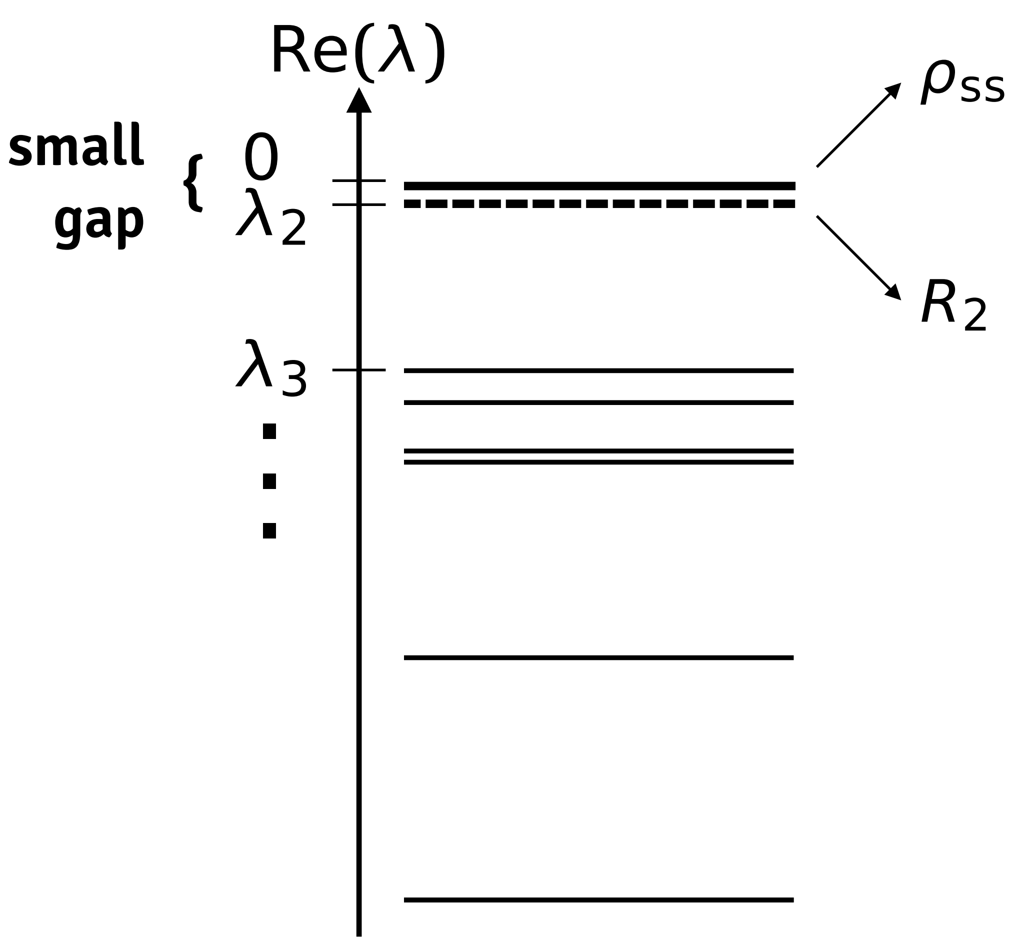
→ in A only (open) = **easier** by analogy with classical {Gaveau-Shulman, Bovier et al.}

quantum master equation: {Lindblad, Gorini-et-al}

$$\partial_t \rho = -i[H,\rho] + \sum_\mu \left[J_\mu \rho J_\mu{}^\dagger - \tfrac{1}{2}\left\{ J_\mu{}^\dagger J_\mu, \rho \right\} \right] \equiv \mathcal{L}(\rho)$$

metastable states from spectrum : $\mathcal{L}(R_k) = \lambda_k R_k\,,\ \ \mathcal{L}^\dagger(L_k) = \lambda_k L_k\,,\ \ \mathrm{Tr}(L_k R_l) = \delta_{kl}$

$\lambda_1 = 0\,,\ \ L_1 = \mathbb{1}\,,\ \ R_1 = \rho_{ss}$ but $R_{k>1} < 0$



$$\rho(t) = e^{t\mathcal{L}}\rho_0$$

$$= \rho_{ss} + \sum_{k=2}^{m} e^{t\lambda_k} R_k \mathrm{Tr}(L_k \rho_0) + \cdots$$

$$\approx \rho_{ss} + \sum_{k=2}^{m} R_k \mathrm{Tr}(L_k \rho_0) \qquad \mathcal{O}(t\lambda_m)$$

metastable state $\in$ manifold dim=m-1

dimensional reduction $(\dim \mathcal{H})^2 \to (m-1)$

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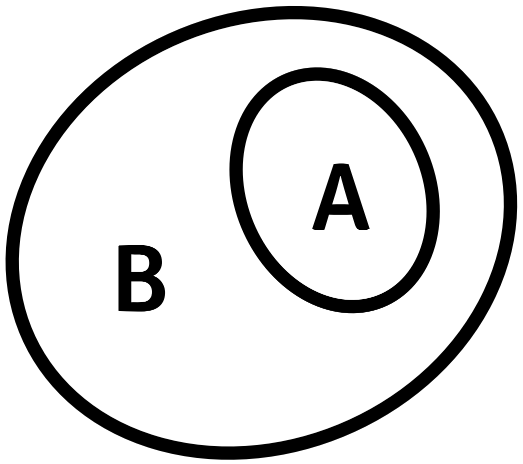

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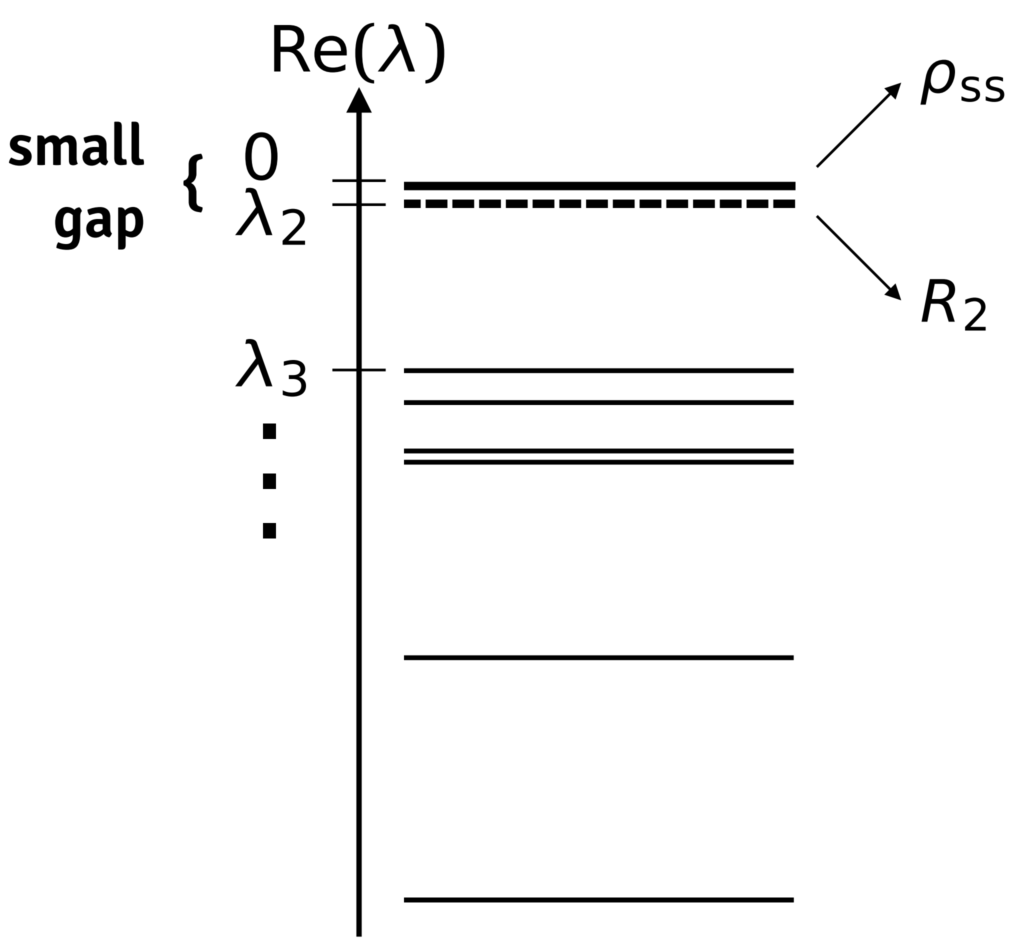

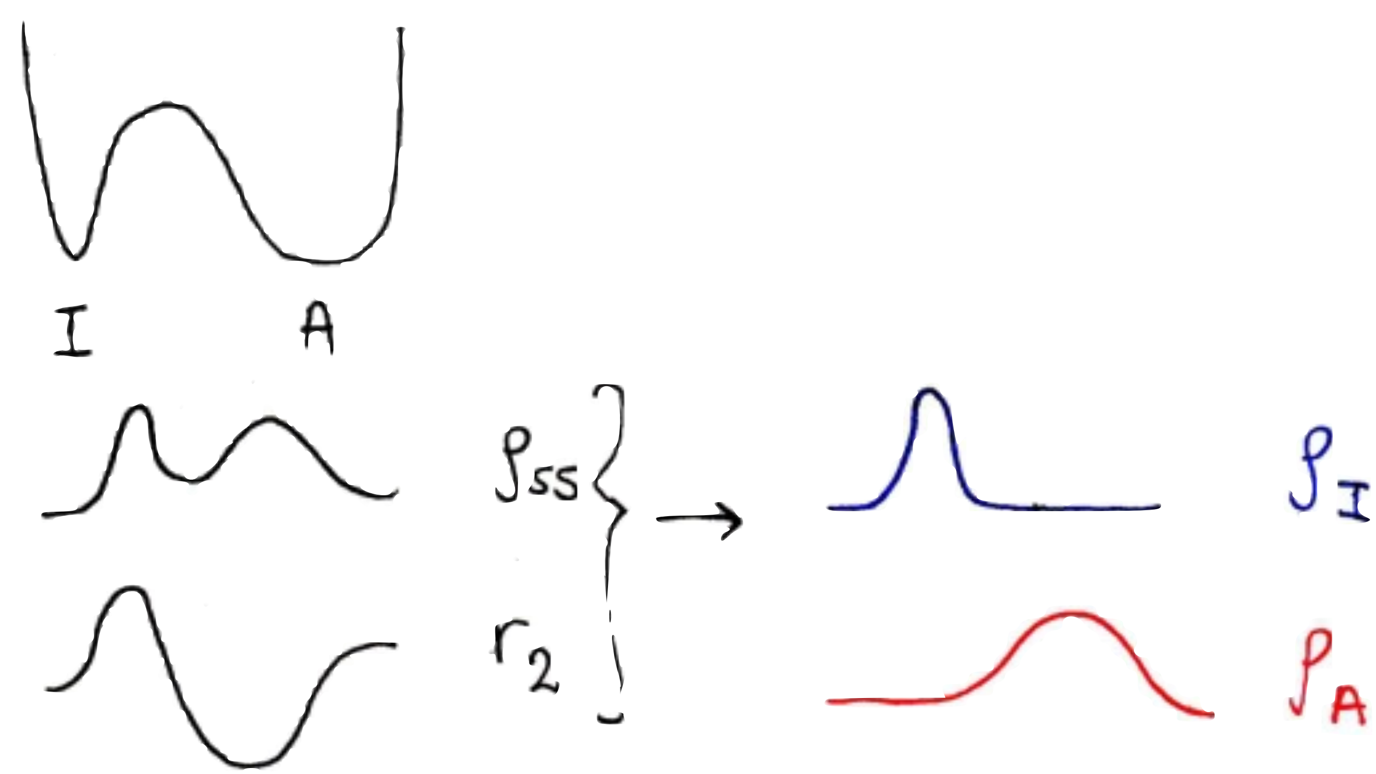

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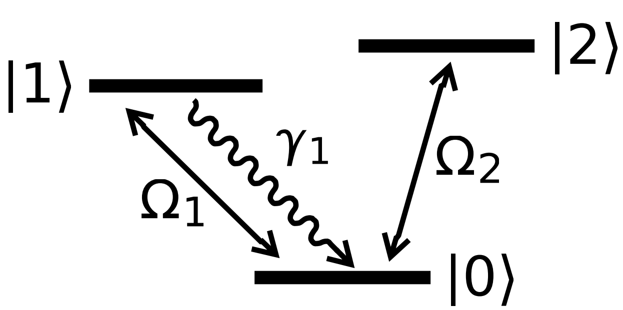
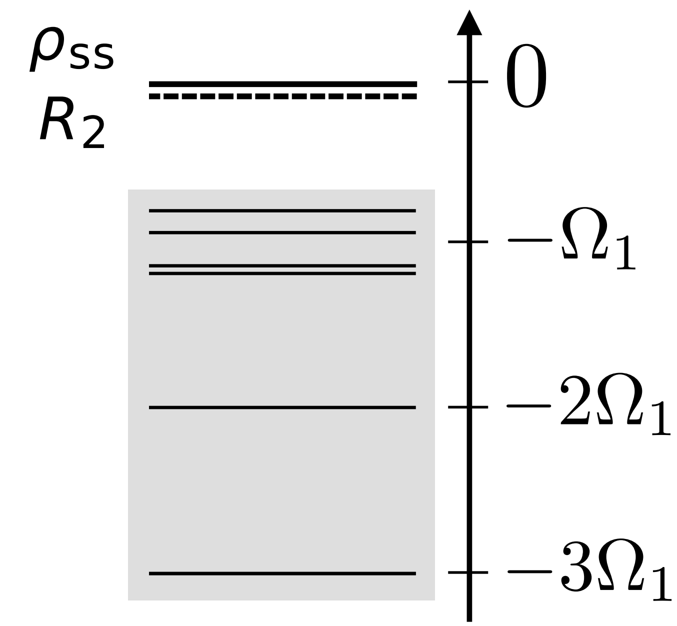


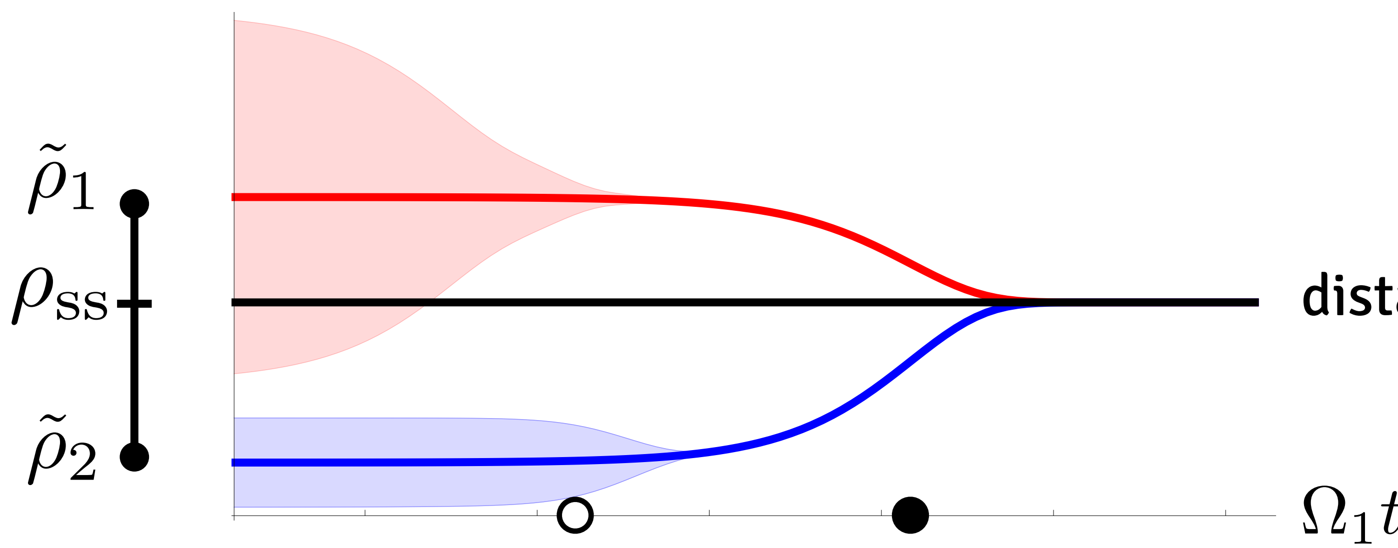
$$\tilde{\rho}_1 = \rho_{ss} + c_2^{max} R_2$$

$$\tilde{\rho}_2 = \rho_{ss} + c_2^{min} R_2$$

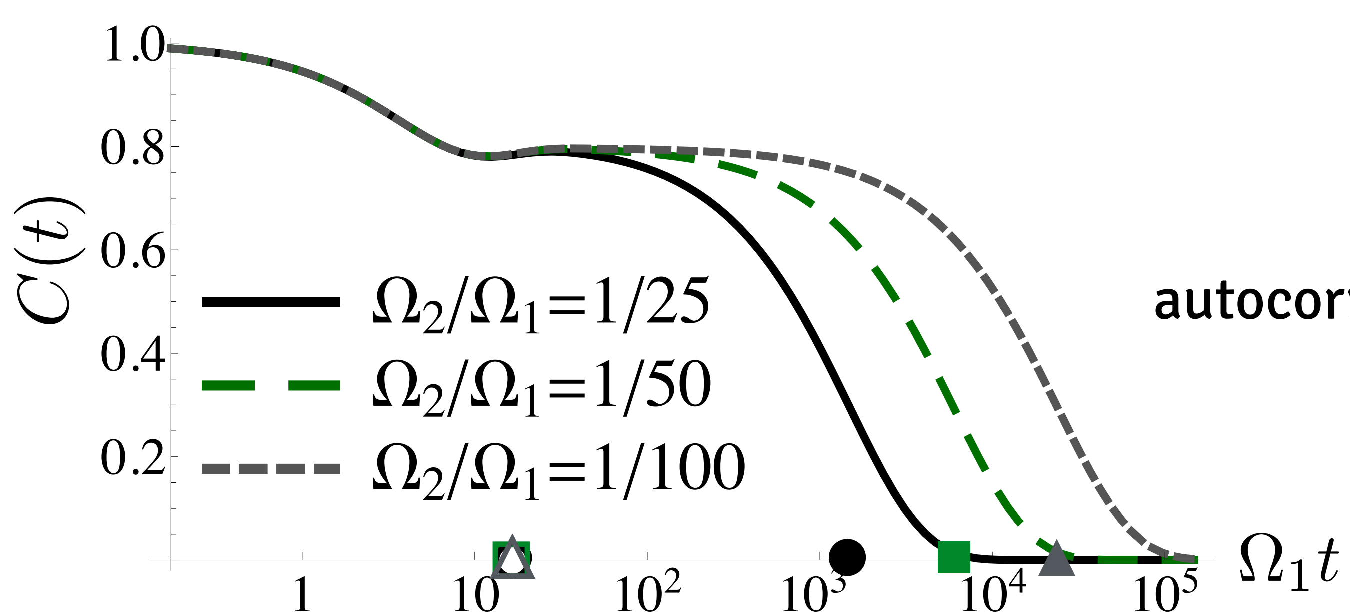
$c_2^{max,min}$ = max/min evals of L_2



metastable manifold is 1d simplex



distance to metastable manifold



autocorrelation of $|1\rangle\langle 1| - |2\rangle\langle 2|$

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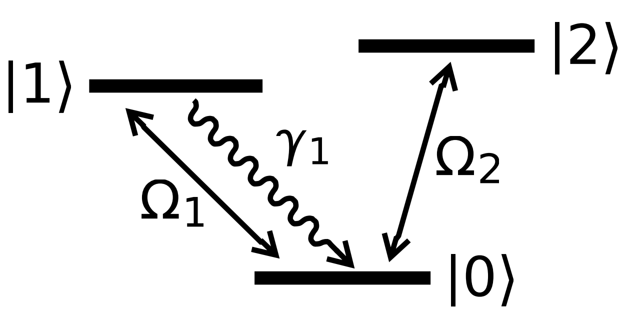

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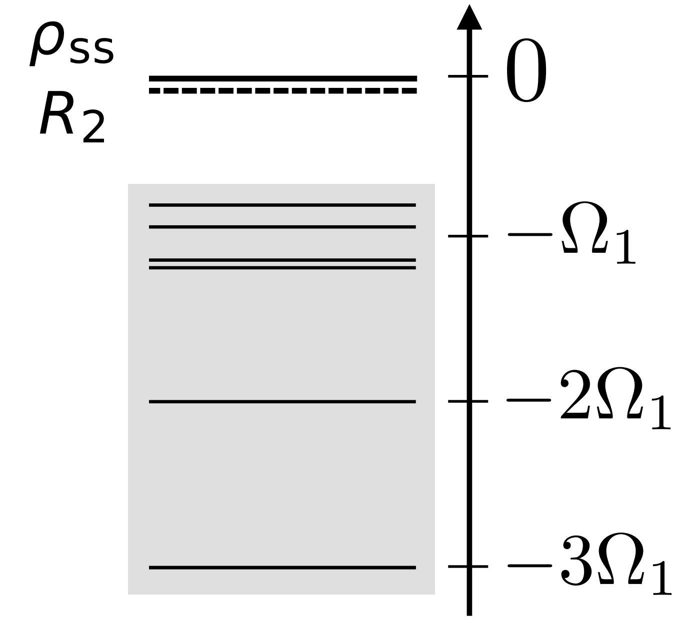
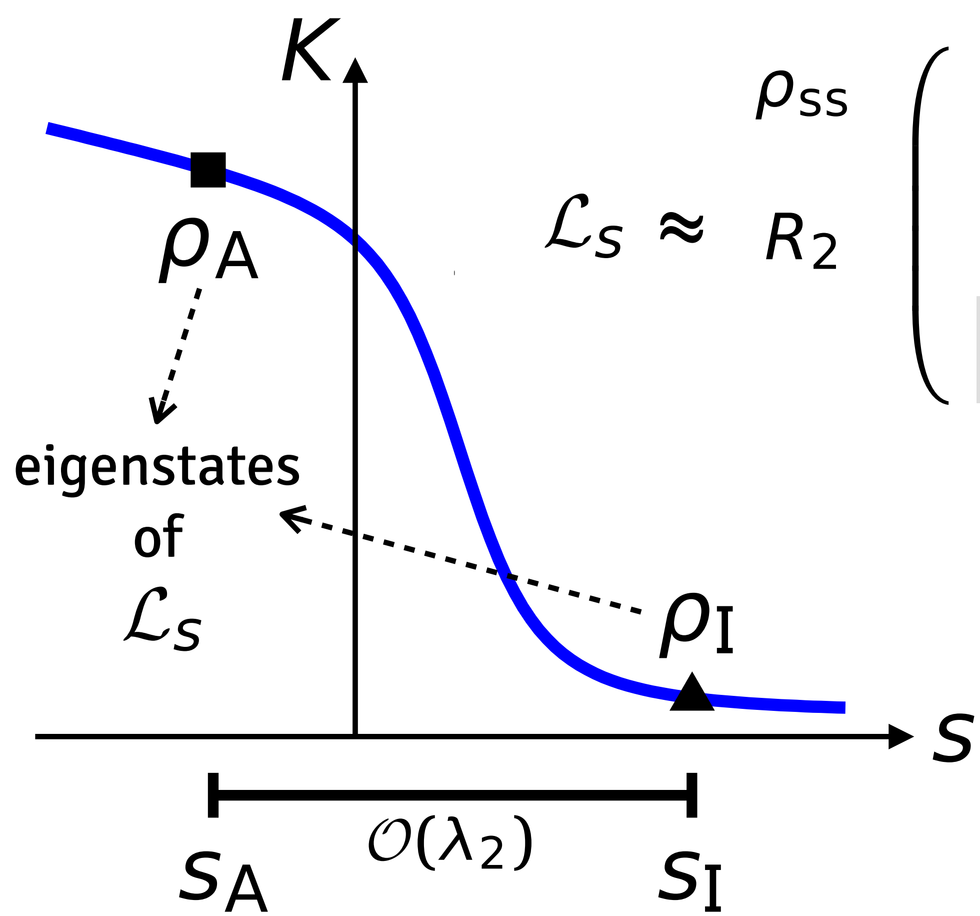


Connection to s-ensemble?



$$\mathcal{L}_s \approx \begin{array}{c} \\ \rho_{ss} \\ R_2 \\ \end{array} \begin{array}{c} \begin{array}{ccc} \mathbb{1} & L_2 & \end{array} \\ \begin{pmatrix} 0 & -sJ^\dagger J & \cdots \\ -sJ^\dagger J & \lambda_2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \end{array}$$

→ **almost degenerate P.T.**

ρ_A , ρ_I

$$\rho_A = \tilde{\rho}_1 + \mathcal{O}(t\lambda_2)$$

$$\rho_I = \tilde{\rho}_2 + \mathcal{O}(t\lambda_2)$$

3. Towards a theory of quantum metastability

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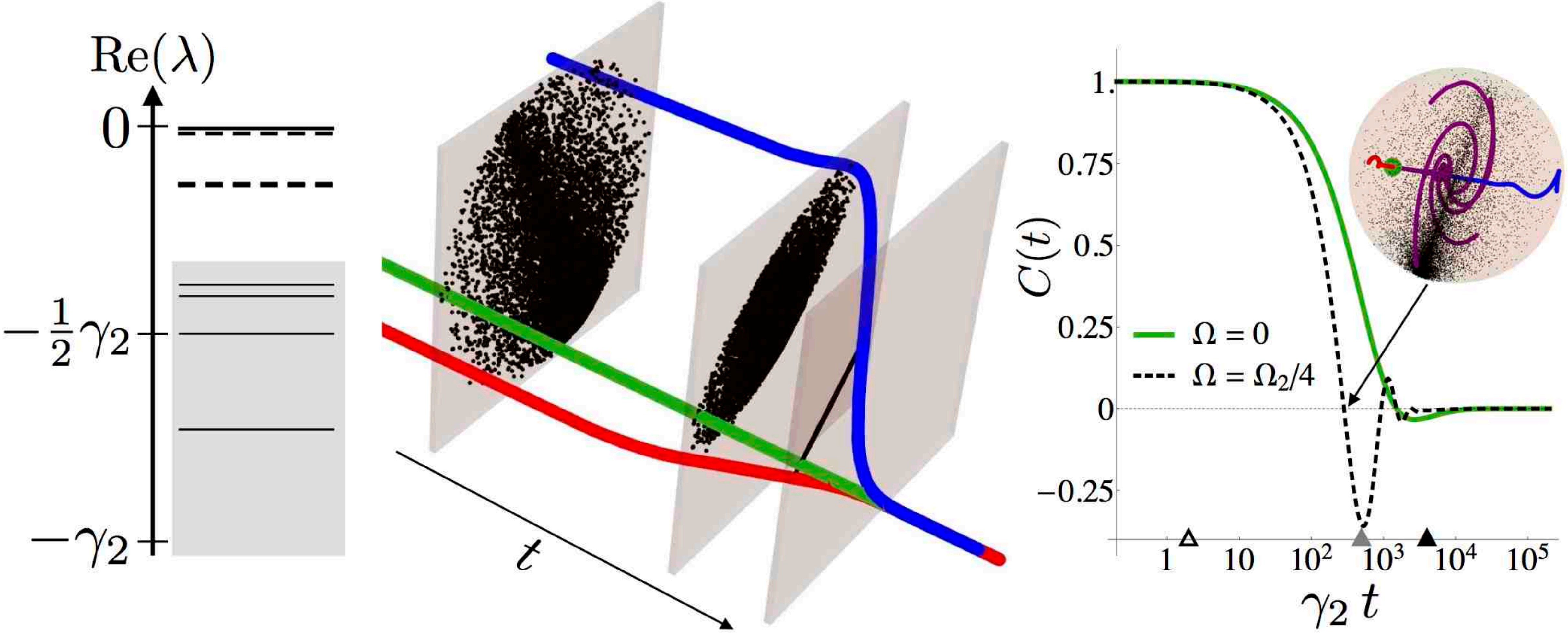
Metastable Manifold = disjoint extreme metastable states (cf. classical {Gaveau-Shulman})
+ Decoherence free subspaces + Noiseless subsystems

$\mathcal{L} \longrightarrow \mathcal{L}_{\text{eff}}$ **in MM**

e.g. Metastable M/fold = **qubit** $H = \Omega_1 \sigma_1^x + \Omega_2 \sigma_2^x$, $L = \gamma_1 n_1 \sigma_2^- + \gamma_2 \sigma_1^+ (1 - n_2)$



SUMMARY

1. Slow relaxation through dynamical constraints

Constrained hopping, fast-slow crossover at RK point

2. Signatures of MBL dynamics in absence of disorder

Quantum East model, thermal-MBL (quasi-MBL) transition at RK point

3. Theory of metastability of open quantum systems

Metastable states and effective dynamics from spectrum of dynamical generator

Reduction to low-dimensional metastable manifold