GEOMETRY

IN HONOUR OF MARIO MICALLEF'S 60TH BIRTHDAY

8-10 MAY 2017

Claudio Arezzo. (ICTP)

On the existence of constant scalar curvature Kahler metrics

I will present some old and new results about existence of Kahler constant scalar curvature metrics on compact and non-compact complete manifolds.

Ailana Fraser. (UBC)

Existence of harmonic maps into CAT(1) spaces

The celebrated work of Eells and Sampson initiated a wide interest in the study of harmonic maps between Riemannian manifolds, and harmonic maps have proven to be a useful tool in geometry. A more recent development is the harmonic map theory for non-smooth spaces. The seminal works of Gromov-Schoen and Korevaar-Schoen consider harmonic maps from a Riemannian domain into a non-Riemannian target. Further exploration of harmonic map theory to the singular setting includes works of Jost, J. Chen, Eells-Fuglede and Daskalopoulos-Mese. The above mentioned works all assume non-positivity of curvature. In this talk I will discuss joint work with Breiner, Huang, Mese, Sargent, Zhang on existence and regularity results for harmonic maps when the target curvature is bounded above by a constant that is not necessarily 0, extending earlier works of Serbinowski, Mese, and Fuglede.

Tobias Lamm. (KIT)

Limits of α-harmonic maps

Yng-Ing Lee. (NTU/Oxford)

On the existence of Lagrangians with special properties

Date: 5 May 2017.

It is interesting to combine Riemannian structure and symplectic structure, and investigate objects with special properties with respect to both structures. Examples include special Lagrangians, minimal Lagrangians, Hamiltonian stationary Lagrangians, and Lagrangian soliton solutions of mean curvature flow. These submanifolds have nicer properties and can serve as model examples or canonical representatives for related classes for submanifolds in middle dimension. However, existence theory for these objects is still wildly open. In this talk, I will survey results and different techniques on studying the problem. The survey is biased by the speakers own interests and research, and does not mean to be complete.

Martin Li. (CUHK)

Chord shortening flow and a theorem of Lusternik and Schnirelmann

We introduce a new geometric flow called the chord shortening flow which is the negative gradient flow for the length functional on the space of chords with end points lying on a fixed submanifold in Euclidean space. As an application, we give a simplified proof of a classical theorem of Lusternik and Schnirelmann (and a generalisation by Riede and Hayashi) on the existence of multiple orthogonal geodesic chords. For compact convex planar domain, we show that any convex chord which is not orthogonal to the boundary would shrink to a point in finite time under the flow. These works are partially supported by an RGC grant from the Hong Kong Government.

Davi Maximo. (U Penn)

On Morse index estimates for minimal surfaces

In this talk we will survey some recent estimates involving the Morse index and the topology of minimal surfaces

Andrea Mondino. (Warwick)

Some smooth applications of non-smooth Ricci curvature lower bounds

The idea of compactifying the space of Riemannian manifolds satisfying Ricci curvature lower bounds goes back to Gromov in the 80s and was pushed by Cheeger and Colding in the 90s who investigated the fine structure of possibly non-smooth limit spaces. A completely new approach via optimal transportation was proposed by Lott-Villani and Sturm almost ten years ago. Via such an approach one can give a precise definition of what it means for a non-smooth space to have Ricci curvature bounded below. Such an approach has been refined in the last years giving new insights to the theory

and yielding applications which seem to be new even for smooth Riemannian manifolds.

Georgios Moschidis. (Princeton/Cambridge)

A proof of the instability of AdS spacetime for the Einstein–null dust system

The AdS instability conjecture is a conjecture related to the initial value problem for the vacuum Einstein equations with a negative cosmological constant in general relativity. It states that generic, arbitrarily small perturbations to the initial data of the AdS spacetime, under evolution by the vacuum Einstein equations with reflecting boundary conditions on conformal infinity, lead to the formation of black holes. Formulated in 2006 by Dafermos and Holzegel, this conjecture has attracted a vast amount of numerical and heuristic works by several authors, focused mainly on the simpler setting of the spherically symmetric Einstein–scalar field system. In this talk, we will provide the first rigorous proof of the AdS instability conjecture in the simplest possible setting, namely for the spherically symmetric Einstein–massless Vlasov system, in the case when the Vlasov field is moreover supported only on radial geodesics. This system is equivalent to the Einstein–null dust system, allowing for both ingoing and outgoing dust. In order to overcome the "trivial" break down occuring once the null dust reaches the centre $r = 0$, we will study the evolution of the system in the exterior of an inner mirror with positive radius r_0 and prove the conjecture in this setting. After presenting our proof, we will briefly explain how the main ideas can be extended to more general matter fields.

Andre Neves. (Imperial/Chicago)

The Weyl Law for the volume spectrum

Melanie Rupflin. (Oxford)

Eigenvalues of the Laplacian on degenerating surfaces

We consider the first non-zero eigenvalue λ_1 of the Laplacian on hyperbolic surfaces for which one disconnecting collar degenerates and show that the gradient of λ_1 is given essentially explicitly in terms of the dual of the differential of the degenerating length coordinate. As a corollary we obtain estimates which establish that λ_1 essentially only depends on this length coordinates with error estimates that are sharp for surfaces of genus at least 3.

Rick Schoen. (Irvine)

Manifolds with positive isotropic curvature

In 1988 Micallef and Moore introduced a new curvature notion which has become fundamental in Riemannian geometry. It is called positive curvature on isotropic two planes (PIC). The notion grew out of considerations of the second variation of area for surfaces in Riemannian manifolds whose study was pioneered by Mario Micallef in the early 1980s. The 1988 paper proves the vanishing of certain homotopy groups for PIC manifolds which implies a far reaching generalization of the topological sphere theorem for simply connected manifolds. The preservation of the PIC condition under Ricci flow in four dimensions was shown by R. Hamilton in the 1990s, and he constructed the Ricci flow with surgeries in order to classify four dimensional PIC manifolds. In higher dimensions S. Brendle and the speaker showed that PIC is preserved under Ricci flow, and stronger conditions related to PIC were used to show that strictly quarter pinched manifolds are diffeomorpic to spherical space forms. For manifolds with infinite fundamental group the structure of PIC manifolds is only conjectured. In this talk we will survey the PIC theory and discuss these conjectures and progress which has been made on them.

Ben Sharp. (Warwick)

The spectrum of the Jacobi operator on minimal hypersurfaces

We will discuss some recent results which relate the spectrum of a minimal hypersurfaces to their geometry and topology - most of the talk will deal with the case of closed and smooth n-dimensional embedded minimal hypersurfaces in a closed Riemannian manifold with $1 < n < 7$. In particular we will show that within the class of bounded-volume, closed, embedded minimal hypersurfaces there is an equivalence between total curvature, Morse index and bounding one of the eigenvalues (of the Jacobi operator) away from minus infinity. The talk will consist of (separate) joint works with L. Ambrozio - A. Carlotto and R. Buzano.

Emanuele Spadaro. (Leipzig)

The structure of the singular set of Dirichlet minimizing multiple valued functions

Dirichlet minimizing multiple valued functions are the blowup profiles of mass minimizing integer rectifiable currents in higher co-dimension, and their regularity properties resemble those of minimal surfaces. In this talk I will present some results on the structure of the singular set of such functions, in particular showing its rectifiability. This is a joint work with De Lellis, Marchese and Valtorta.