Geometric PDE Workshop Titles + Abstracts

Lucas Ambrozio

Title: Some new results for free boundary minimal hypersurfaces

Abstract: Free boundary minimal submanifolds are critical points of the volume functional on the class of submanifolds whose boundaries are free to vary inside the boundary of the given ambient domain. We will show how the Morse index controls the first Betti number of compact free boundary minimal hypersurfaces in an ambient domain whose boundary satisfies weak convexity assumptions (joint with A. Carlotto and B. Sharp). A few applications in the case where the ambient domain is the unit ball will be discussed. Finally, we will also explain a new geometric characterisation of the critical catenoid in terms of the second fundamental form (joint with I. Nunes).

Costante Bellettini

Title: Stable CMC varifolds of codimension 1

Abstract: In a joint work with N. Wickramasekera (Cambridge) we develop a regularity and compactness theory for a class of codimension-1 integral n-varifolds with generalised mean curvature in L^{p}_{loc} for some p > n. Subject to suitable variational hypotheses on the regular part (namely criticality and stability for the area functional with respect to variations that preserve the "enclosed volume") and two necessary structural assumptions, we show that the varifolds under consideration are "smooth" (and have constant mean curvature in the classical sense) away from a closed singular set of codimension 7. In the case that the mean curvature is non-zero, the smoothness is to be understood in a generalised sense, i.e. also allowing the tangential touching of two smooth CMC hypersurfaces (e.g. two spheres touching).

Pieter Blue

Title: Hidden symmetries and decay of fields outside black holes

Abstract: I will discuss energy and Morawetz (or integrated local decay) estimates for fields outside black holes, in particular the Vlasov equation. This builds on earlier work for the wave and Maxwell equation. Much of the work on these problems in the last decade has used the vector-field method and its generalisations. One generalisation has focused on using symmetries, differential operators that take solutions of a PDE to solutions. In this context, a hidden symmetry is a symmetry that does not decompose into first-order symmetries coming from a smooth family of isometries of the underlying manifold. In this talk, I will build on applications of the vector-field method to the Vlasov equation to prove an integrated energy decay for the Vlasov equation outside a very slowly rotating Kerr black hole, and I will discuss some new features of the symmetry algebra for the Vlasov equation, which illustrate the

difficulties in passing to pointwise-decay estimates for the Vlasov equation in this context. This is joint work with L. Andersson and J. Joudioux.

Paul Bryan

Title: Harnack Inequalities and Ancient Solutions of Hypersurface Flows

Abstract: Harnack inequalities are an important tool in the analysis of elliptic and parabolic PDE. For geometric flows such at the Ricci flow and the Mean Curvature Flow, the so-called Li-Yau-Hamilton Harnack inequality plays a central role in a number of places, in particular in the study of singularity formation. Ancient solutions of a parabolic PDE are those solutions existing on an interval $(-\infty,T)$, and are closely related to the Harnack inequality. Ancient solutions include, in particular, certain solitons which arise as the blow up of singularities. To date, most work on hypersurface flows has focused on Euclidean space, where a Harnack inequality for general hypersurface flows was obtained by Ben Andrews via a simple computation using the support function and Gauss map parametrisation. I will a describe a variation of this approach that works in arbitrary Riemannian backgrounds, tying together several different aspects of the Harnack inequality. I will also describe how this relates to ancient solutions, and give a survey of several classification techniques.

Reto Buzano

Title: The Moduli Space of (2-Convex) Embedded Spheres

Abstract: We investigate the topology of the space of smoothly embedded n-spheres in R^{n+1}. By Smale's theorem, this space is contractible for n=1 and by Hatcher's proof of the Smale conjecture, it is also contractible for n=2. These results are of great importance, generalising in particular the Schoenflies theorem and Cerf's theorem. In this talk, I will explain how geometric analysis can be used to study a higher-dimensional variant of these results. The main result (joint with Robert Haslhofer and Or Hershkovits) is that the space of 2-convex embedded spheres is path-connected in every dimension n. The proof uses mean curvature flow with surgery.

Filippo Cagnetti

Title: The rigidity problem for symmetrization inequalities

Abstract: We will discuss several symmetrizations (Steiner, Ehrhard, and spherical symmetrization) that are known not to increase the perimeter. We will show how it is possible to characterize those sets whose perimeter remains unchanged under symmetrization. We will also characterize rigidity of equality cases. By rigidity, we mean the situation when those sets whose perimeter remains unchanged under symmetrization, are trivially obtained through a rigid motion of the (Steiner, Ehrhard or spherical) symmetral. We will achieve this through the introduction of a suitable measure-theoretic notion of connectedness, and through a fine analysis of the barycenter function for a special class of sets. These results are obtained

together with several collaborators (Maria Colombo, Guido De Philippis, Francesco Maggi, Matteo Perugini, Dominik Stöger).

Alessandro Carlotto

Title: Min-max embedded geodesic lines and a question of Bangert

Abstract: The quest for (properly) embedded geodesic lines is one of the most fundamental and natural global problems in Differential Geometry and yet even in the simplest case of complete Riemannian metrics on the plane a general existence result was only obtained in 1981 by V. Bangert. But what can we say about multiplicity, namely are there always infinitely many such lines or is there a finite, effective, upper bound? Surprisingly enough, this problem is still completely open in its full generality. In this talk, I shall present some recent joint work with Camillo De Lellis where we employ min-max methods to construct uncountably many, geometrically distinct, properly embedded geodesic lines in any asymptotically conical surfaces of non-negative scalar curvature, where minimization schemes are inevitably doomed to fail, with full control on the Morse index and the asymptotic behaviour of the lines themselves.

Guido De Philippis

Title: Allard's rectifiability theorem for anisotropic energies

Abstract: Allard's rectifiability theorem asserts that every d-varifold with bounded first variation and positive d-dimensional density is rectifiable. In this talk I will show that there exists a necessary and sufficient condition on the integrand in order to extend this result to varifolds with bounded first variation with respect to anisotropic integrands. Joint work with A. De Rosa and F. Ghiraldin.

Grigorios Fournodavlos

Title: Dynamics of the Einstein equations near a Schwarzschild singularity

Abstract: We will discuss dynamical properties of the Schwarzschild interior, backwards and forwards (in time) with respect to the initial value problem for the Einstein vacuum equations.

Dejan Gajic

Title: Late-time tails for the wave equation on stationary, asymptotically flat spacetimes

Abstract: I will discuss the precise late-time asymptotics (“tails”) of solutions to the wave equation on stationary, asymptotically flat spacetime backgrounds. These were first studied heuristically by Price on Schwarzschild and are also known as “Price's law”. In particular, I will introduce energy decay estimates that go beyond those obtained by the traditional vector field method. This is based on joint work with Yannis Angelopoulos and Stefanos Aretakis.

Gustav Holzegel

Title: Linear Stability of the Schwarzschild Solution

Abstract: I will discuss our recent proof (with M. Dafermos and I. Rodnianski) of the Linear Stability of the Schwarzschild Solution in General Relativity. The proof relies on progress in the study of linear wave equations on black hole backgrounds combined with some classical insights into the structure of the system of gravitational perturbations, which I will review. An important role is played by the pure gauge solutions of the system. Time permitting I will also discuss conservations laws for the system of gravitational perturbations on Schwarzschild and how to exploit them to control the dynamics.

Wenshuai Jiang

Title: Regularity and compactness of Harmonic-Einstein equations

Abstract: We consider Harmonic-Einstein equations which is a generalization of Einstein equation. We first show an Anderson type ϵ-regularity theorem. As an application, we prove an orbifold type compactness theorem by proving some decay curvature estimates. This is a joint work with Huabin Ge.

Andrea Mondino

Title: Isoperimetric inequalities in non-smooth spaces with Ricci curvature bounded below.

Abstract: In the first part of the talk I will recall the notion of Ricci curvature lower bounds in non-smooth spaces via optimal transport introduced by Lott-Sturm-Villani, then I will discuss some basic properties of such spaces and finally I will discuss the non-smooth extension of the celebrated Levy-Gromov isoperimetric inequality.

Rafael Montezuma

Title: Metrics of positive scalar curvature and unbounded min-max widths

Abstract: In this talk, I will construct a sequence of Riemannian metrics on the three-dimensional sphere with scalar curvature greater than or equal to 6 and arbitrarily large min-max widths. The search for such metrics is motivated by a rigidity result of min-max minimal spheres in three-manifolds obtained by Marques and Neves.

Luc Nguyen

Title: Lipschitz regularity for nonlinear degenerate elliptic equations arising in conformal geometry

Abstract: It is known that the strong maximum principle does not hold for nonlinear degenerate elliptic equations. We however show that a mild form of the strong maximum principle holds for

a class of equations arising in conformal geometry and hence establish as an application that continuous viscosity solutions of such equations are in fact Lipschitz.

Filip Rindler

Title: Rademacher's Theorem, Cheeger's Conjecture and PDEs for measures

Abstract: The classical Rademacher Theorem asserts that every Lipschitz function is differentiable almost everywhere with respect to Lebesgue measure. On the other hand, Preiss ('90) gave a surprising example of a null set in the plane such that every Lipschitz function is differentiable at at least one point of this set. Thus, it is a natural question to ask whether there exists a singular measure such that all Lipschitz functions are differentiable with respect to this singular measure. It turns out that this question has an intricate connection to the geometric structure of normal one-currents. In this talk I will present a converse to Rademacher's Theorem, which settles the question in the negative in all dimensions: if a positive measure μ has the property that all Lipschitz functions are μ-a.e. differentiable, then μ is absolutely continuous with respect to Lebesgue measure (in the plane, this question was already solved by Alberti, Csornyei and Preiss in '05).

In a geometric context, Cheeger conjectured in '99 that in all Lipschitz differentiability spaces (which are essentially Lipschitz manifolds in which Rademacher's Theorem holds) likewise there is a "functional converse" to Rademacher's Theorem. As the second main result, I will present a recent solution to this conjecture.

Technically, the proofs of both of these theorems are based on a recent structure result for the singular parts of PDE-constrained measures, its corollary on the structure of normal one-currents, and the powerful theory of Alberti representations.

This is joint work with A. Marchese and G. De Philippis.

Melanie Rupflin

Title: Fine properties of degenerating hyperbolic surfaces and applications

Abstract: The tangent space of the set of hyperbolic surfaces splits into directions generated by pull-back with diffeomorphisms and a finite dimensional horizontal space, which can be characterized as real part of the space of holomorphic quadratic differentials. In this talk we will discuss the fine properties of this horizontal space in situations where one or more geodesics in the domain collapse and applications to the study of eigenvalues on degenerating surfaces and geometric flows.

Felix Schulze

Title: Ricci flow from spaces with isolated conical singularities

Abstract: Let (M,g_0) be a compact n-dimensional Riemannian manifold with a finite number of singular points, where at each singular point the metric is asymptotic to a cone over a compact (n-1)-dimensional manifold with curvature operator greater or equal to one. We show that there exists a smooth Ricci flow, possibly with isolated orbifold points, starting from such a metric with curvature decaying like C/t. The initial metric is attained in Gromov-Hausdorff distance and smoothly away from the singular points. To construct this solution, we desingularize the initial metric by glueing in expanding solitons with positive curvature operator, each asymptotic to the cone at the singular point, at a small scale s. Localizing a recent stability result of Deruelle-Lamm for such expanding solutions, we show that there exists a solution from the desingularized initial metric for a uniform time T>0, independent of the glueing scale s. The solution is then obtained by letting s->0. We also show that the so obtained limiting solution has the corresponding expanding soliton as a forward tangent flow.

Arick Shao

Title: Uniqueness Theorems on Asymptotically Anti-de Sitter Spacetimes

Abstract: In theoretical physics, it is often conjectured that a correspondence exists between the gravitational dynamics of asymptotically Anti-de Sitter (AdS) spacetimes and a conformal field theory of their boundaries. In the context of classical relativity, one can attempt to rigorously formulate a correspondence statement as a unique continuation problem for PDEs: Is an asymptotically AdS solution of the Einstein equations uniquely determined by its data on its conformal boundary at infinity? In this presentation, we establish a key step in toward a positive result; we prove an analogous unique continuation result for linear and nonlinear wave equations on fixed asymptotically AdS spacetimes satisfying a positivity condition at infinity. We show, roughly, that if a wave ϕ on this spacetime vanishes on a sufficiently large but finite portion of its conformal boundary, then ϕ must also vanish in a neighbourhood of the boundary. In particular, we highlight the analytic and geometric features of AdS spacetimes which enable this uniqueness result, as well as obstacles preventing such a result from holding in other cases. This is joint work with Gustav Holzegel.

Ben Sharp

Title: Minimal hypersurfaces with bounded index

Abstract: An embedded hypersurface in a Riemannian manifold is said to be minimal if it is a critical point with respect to the induced area. The index of a minimal hypersurface (roughly speaking) tells us how many ways one can locally deform the surface to decrease area (so that strict local area-minimisers have index zero). We will give an overview of recent works linking the index, topology and geometry of closed and embedded minimal hypersurfaces.

Giuseppe Tinaglia

Title: The geometry of constant mean curvature surfaces in Euclidean space.

Abstract: In this talk I will begin by reviewing classical geometric properties of constant mean curvature surfaces, H>0, in R^3. I will then talk about several more recent results for surfaces embedded in R^3 with constant mean curvature, such as curvature and radius estimates for simply-connected surfaces embedded in R^3 with constant mean curvature. Finally I will show applications of such estimates including a characterisation of the round sphere as the only simply-connected surface embedded in R^3 with constant mean curvature and area estimates for compact surfaces embedded in a flat torus with constant mean curvature and finite genus. This is joint work with Meeks.

Daniele Valtorta

Title: Sharp estimates for the singular sets of harmonic maps

Abstract: In this talk we present the new regularity results proved for the singular sets of minimizing and stationary harmonic maps in collaboration with Aaron Naber (see arXiv:1504.02043).

We prove that the singular set of a minimizing harmonic map is rectifiable with effective n-3 volume estimates. The results are based on an improved quantitative stratification technique, which consists in a detailed analysis of the symmetries and almost symmetries of the map u and its blow-ups at different scales, and rely on a new W^{1,p} version of Reifenberg's topological disk theorem. The application of this theorem in the situation of harmonic maps hinges on the monotonicity formula for the normalized energy.

This technique can be applied to other problems in geometrical analysis. For example, similar results are available for minimizing and stationary currents (see arXiv:1505.03428).