RECENT ADVANCES IN SYMMETRIZATION TECHNIQUES FOR NONLOCAL EQUATIONS

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Symmetrization techniques are nowadays widely renowned for being very efficient tools to get sharp estimates for solutions to PDE. In the first part of this talk, excerpted from the joint work [1] with G. Di Blasio, we shall describe how symmetrization techniques allow to get a concentration comparison result for solutions of elliptic equations involving the fractional Laplacian, of the type

$$(-\Delta)^{\alpha/2}u = f$$

posed in an open bounded set Ω of $\mathbb{R}^N$, $\alpha \in (0,2)$, with homogeneous Dirichlet boundary conditions. The data f is assumed to belong to a suitable Lorentz space $L^{p,q}(\Omega)$. These results will be useful to easily obtain, as a natural consequence, some regularity results for the solutions in terms of the data f, generalizing the classical results for the Laplacian. The second part of the talk will focus on the main topics of some recent joint works with J. L. Vázquez [3], [4], [2], in which we consider the application of symmetrization techniques to obtaining concentration comparison results for linear and nonlinear parabolic equations with fractional diffusion, of the form

$$u_t + (-\Delta)^{\alpha/2}A(u) = f$$

taking into account several assumptions on the nonlinearity $A : \mathbb{R}_+ \to \mathbb{R}_+$. Finally, we will show some results from the very recent paper [5], where Neumann boundary conditions are assumed.

References

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[4] ——, *Optimal estimates for fractional fast diffusion equations*, J. Math. Pures Appl. (9), 103 (2015), pp. 535–556.

[5] B. Volzone, *Symmetrization for fractional Neumann problems*, Nonlinear Anal. 147 (2016), pp. 1–25.