



BANK OF ENGLAND

The Dynamics of Money
University of Warwick
4th February 2019

Contagion in Financial Networks

Marco Bardoscia

Any views expressed are solely those of the speaker and so cannot be taken to represent those of the Bank of England or to state Bank of England policy.

Clearing & Conservation of Money



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Locally conserved quantity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0$$

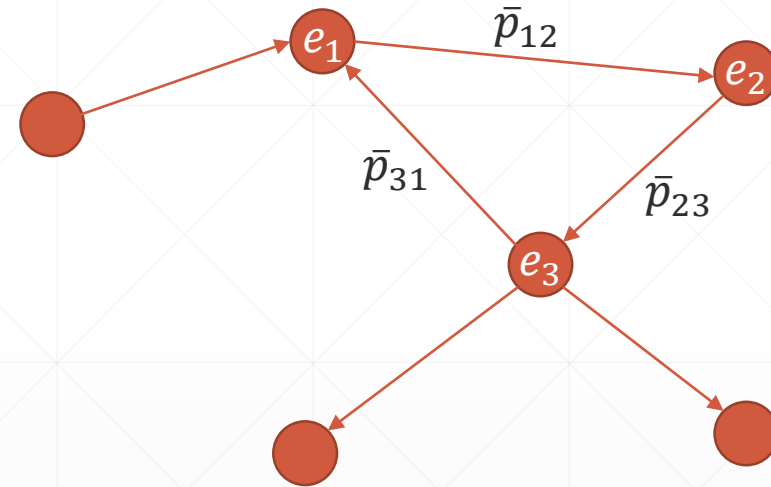
- Fluid dynamics
- Heat
- Electromagnetism
- Quantum mechanics
- General relativity



Clearing payments

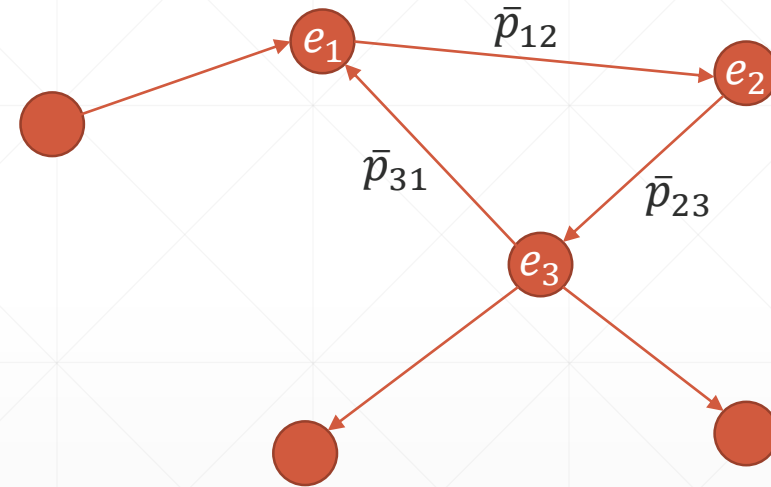
- Financial institutions with some cash buffers: e_i
- And with bilateral obligations between them: \bar{p}_{ij}

- How do we compute payments between institutions?



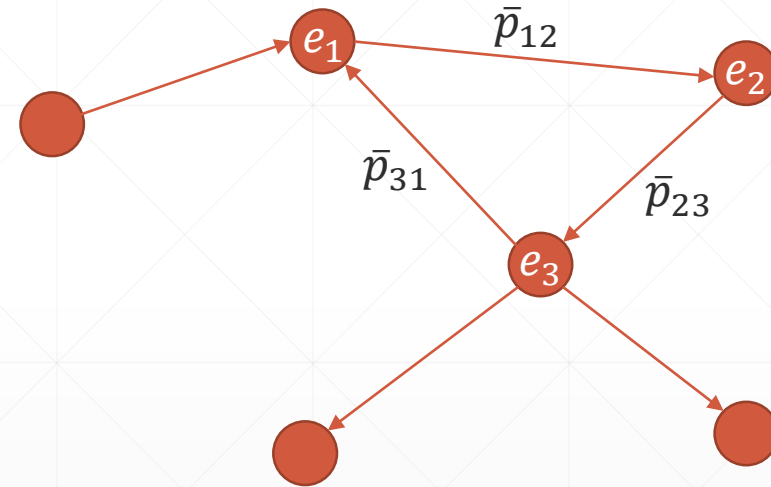
Eisenberg and Noe (2001)

- Main assumptions:
 - Limited liability
 - Absolute priority
- Institutions pay:
 - All their obligations, if they can
 - As much as they can pro rata, otherwise
- Key results:
 - There exist greatest and least clearing payments
 - In many cases clearing payments are unique
 - Clearing payments can be easily computed



Local conservation of money

- Visentin et al (2016): Money is locally conserved in Eisenberg and Noe
- Local conservation implies global conservation
- No amplification!
- Can we break it?
 - Rogers and Veraart (2013): Bankruptcy costs destroy money
 - Barucca et al (2016): Ex-ante valuation



Valuation framework

P Barucca, MB, F Caccioli, M D'Errico, G Visentin, S Battiston, G Caldarelli



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Balance sheet

- Asset side:
 - **External assets** (e.g. loans)
 - **Interbank assets**
- Liability side:
 - **External liabilities** (e.g. deposits)
 - **Interbank liabilities**
 - **Equity**
- Balance sheet identity:

Bank A	
External Assets	External Liabilities
	B → A
A → ...	Equity

Bank B	
External Assets	External Liabilities
B → A	
B → ...	Equity

$$E_i(t) = A_i^e(t) + \sum_j A_{ij}(t) - L_i^e(t) - \sum_j L_{ij}(t)$$



Intuition

Bank A	
External Assets	B → A
	Equity



Intuition

Bank A	
External Assets	B → A
Shock	
	Equity

1. Exogenous shock



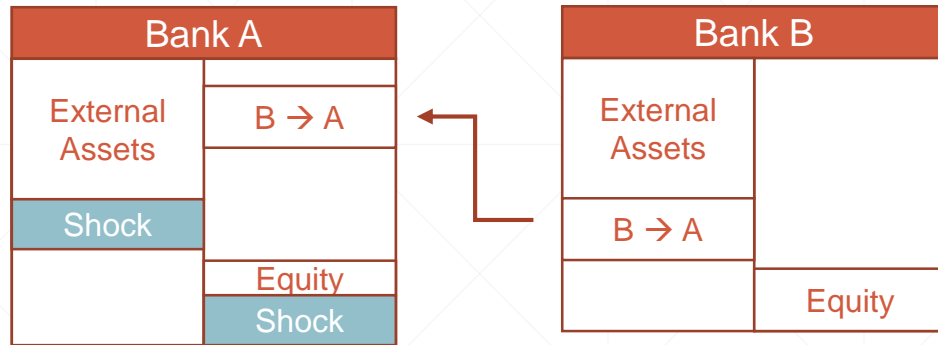
Intuition

Bank A	
External Assets	B → A
Shock	
	Equity
	Shock

1. Exogenous shock
2. First round: impact on equity



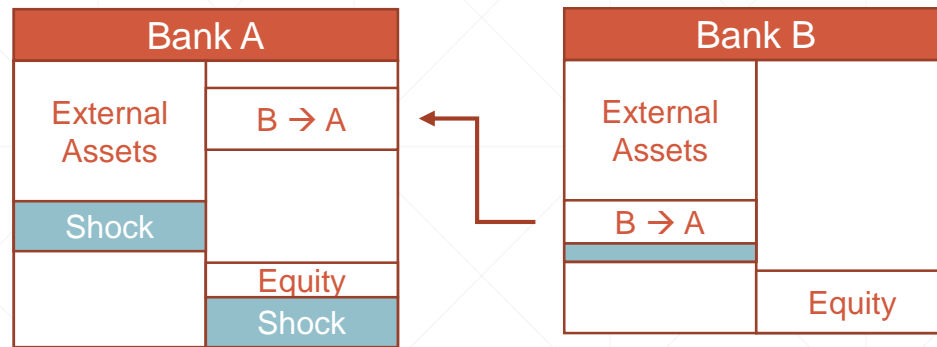
Intuition



1. Exogenous shock
2. First round: impact on equity



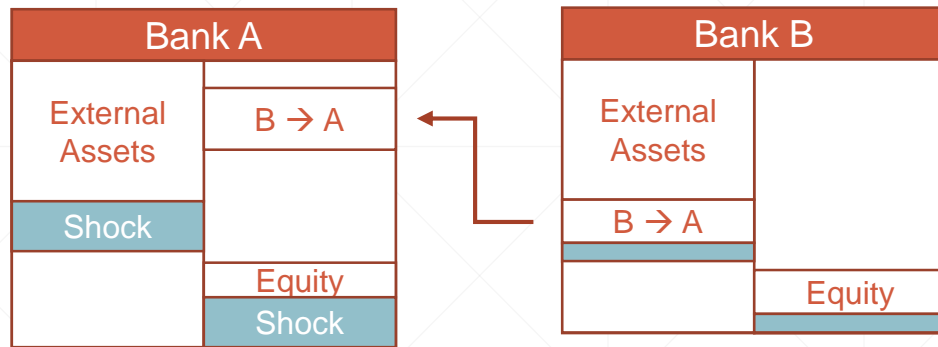
Intuition



1. Exogenous shock
2. First round: impact on equity
3. Counterparties reevaluate interbank assets



Intuition



1. Exogenous shock
2. First round: impact on equity
3. Counterparties reevaluate interbank assets
4. Second round: impact on counterparties' equities
5. Contagion spreads to Bank B's counterparties



Valuation functions

$$E_i(t) = A_i^e(t) + \sum_j A_{ij}(t) \mathbb{V}_{ij}(\mathbf{E}(t) | \dots) - L_i^e(t) - \sum_j L_{ij}(t)$$

- Between 0 and 1
 - When equal to one, interbank assets are worth the book value
 - When equal to zero, interbank assets are worth nothing
- Non decreasing in the equity vector
- Continuous from above
- Greatest and least solutions computed iteratively



Valuation functions: Zoology

By using specific valuation functions we can recover several contagion algorithms:

- Eisenberg and Noe (2001)
- Furfine (2003)
- Rogers and Veraart (2013)
- DebtRank (2015)



Forward-looking solvency contagion

MB, P Barucca, A Brinley Codd, J Hill



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Valuation functions: At maturity

- Valuation that accounts for uncertainty on solvency of counterparties:

$$\tilde{E}_i(T) = A_i^e(T) + \sum_{j=1}^n A_{ij}(T) \mathbb{V}_j(\tilde{E}_j(T); \dots) - L_i^e(T) - \sum_{j=1}^n L_{ij}(T)$$

- The valuation of interbank assets is performed via a discount factor:

$$\mathbb{V}_j(\tilde{E}_j(T); \dots) = \begin{cases} 1 & \text{for } \tilde{E}_j(T) > 0 \\ r_j(\tilde{E}_j(T); \dots) & \text{for } \tilde{E}_j(T) \leq 0 \end{cases}$$

If the borrower has not defaulted, then the discount factor is equal to one and the interbank asset is worth its book value; otherwise it will be worth less.



Valuation functions: Forward-looking

- We now perform a forward-looking valuation at time $t < T$: Average over the risk-neutral measure:

$$\tilde{E}_i(t) = A_i^e(t) + \sum_{j=1}^n A_{ij}(t) \mathbb{E}^{\mathbb{Q}} [\mathbb{V}_j(\tilde{E}_j(T); \dots) | \mathbf{A}^e(t)] - L_i^e(t) - \sum_{j=1}^n L_{ij}(t)$$

- We also account for the possibility that banks can default at any point in time

$$\mathbb{V}_j(\tilde{E}_j(T); \dots) = \begin{cases} 1 & \text{for } \tilde{E}_j(s) > 0, \forall s < T \\ \rho_j & \text{otherwise} \end{cases}$$

- It turns out that:

$$\mathbb{E}^{\mathbb{Q}} [\mathbb{V}_j(\tilde{E}_j(T); \dots) | \mathbf{A}^e(t)] = 1 - p_j^D(\tilde{E}_j(t)) + \rho_j p_j^D(\tilde{E}_j(t))$$

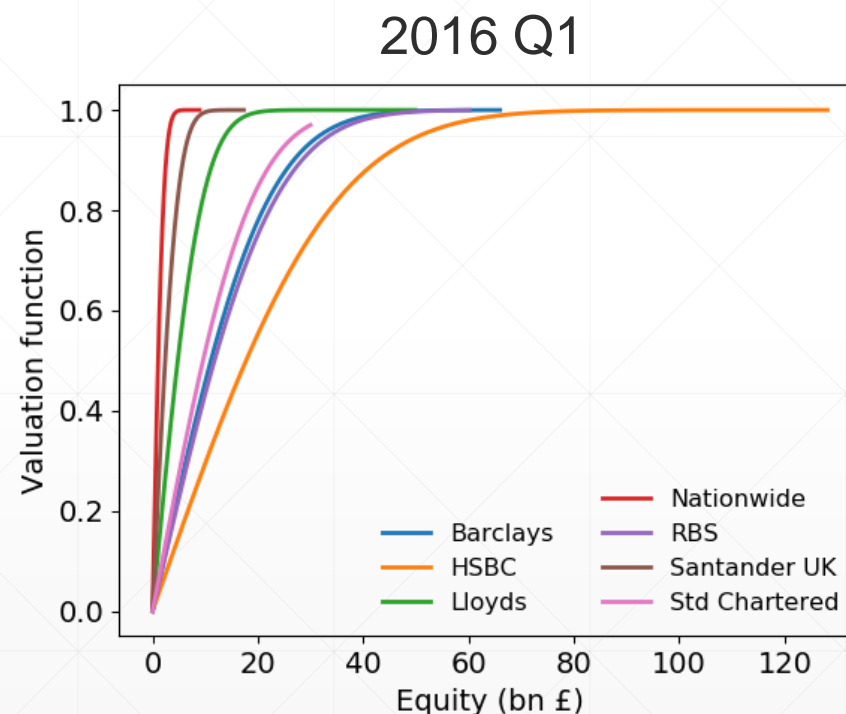
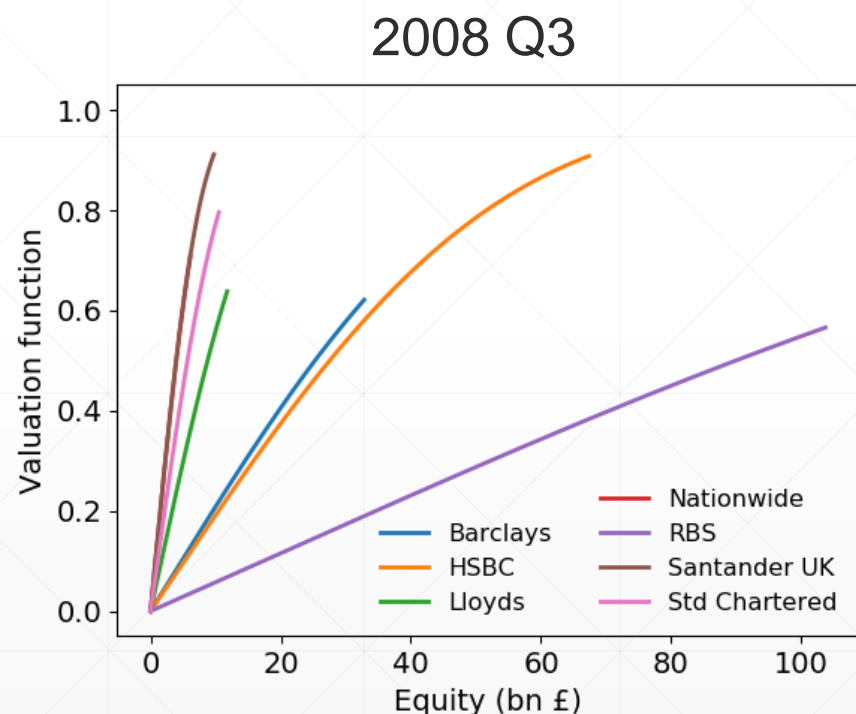


Data

- We use real interbank exposures between banks part of the Bank of England's annual concurrent stress test:
 - 7 banks, which account for 80% of the regulated UK lending
 - 2008 – 2013: exposures larger than 10% of capital
 - 2014 – 2015: no threshold, more granular data (subordinated and unsecured lending)
- When possible (2014 – 2016) we interpret the equity of our model as the CET1 buffer, otherwise we use shareholders' equity for consistency.
- Valuation functions:
 - Volatilities estimated from share prices
 - Forward-looking horizon: 1 year

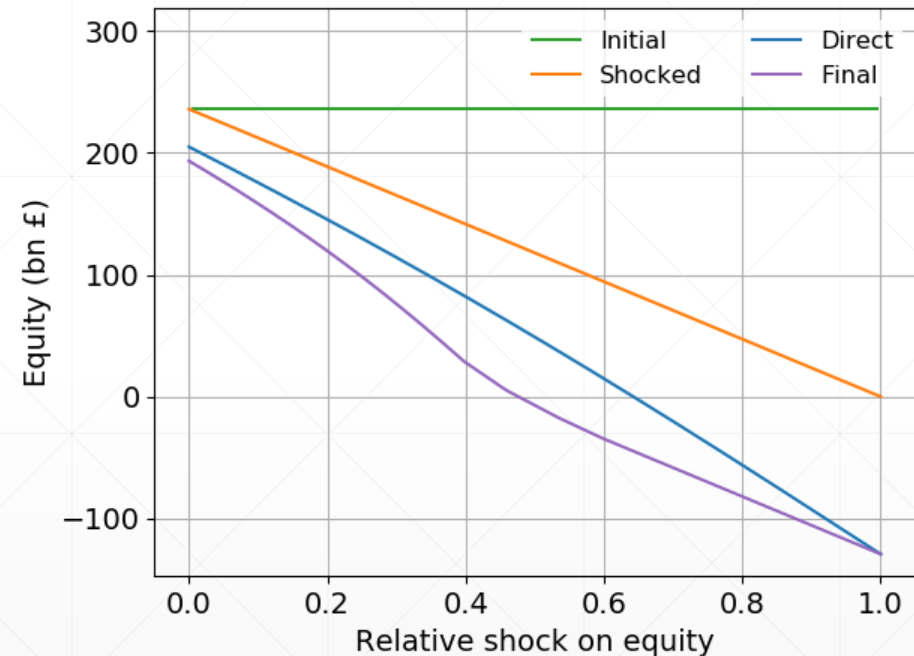


Valuation functions: Calibration



Simplified stress tests

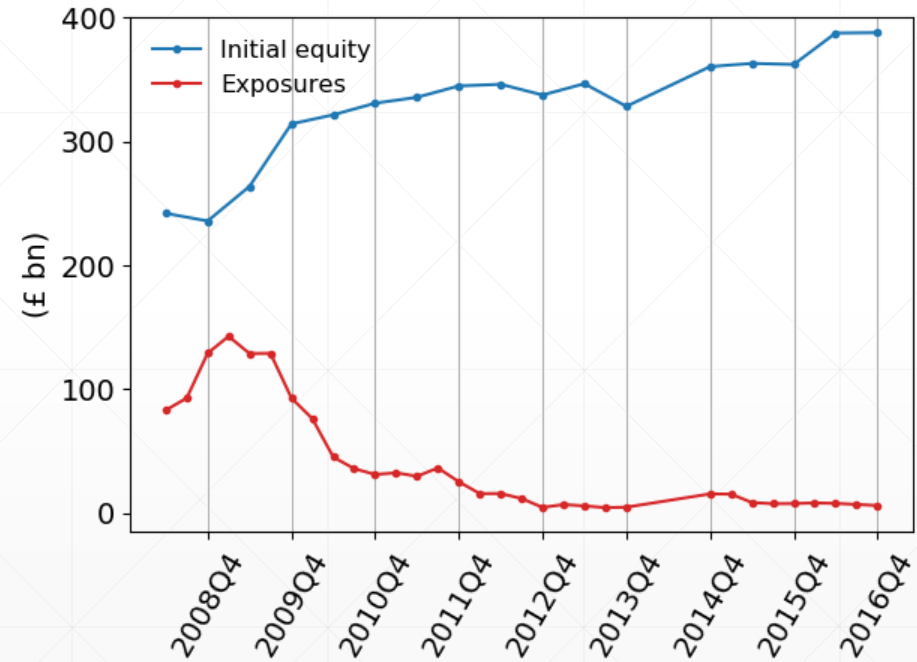
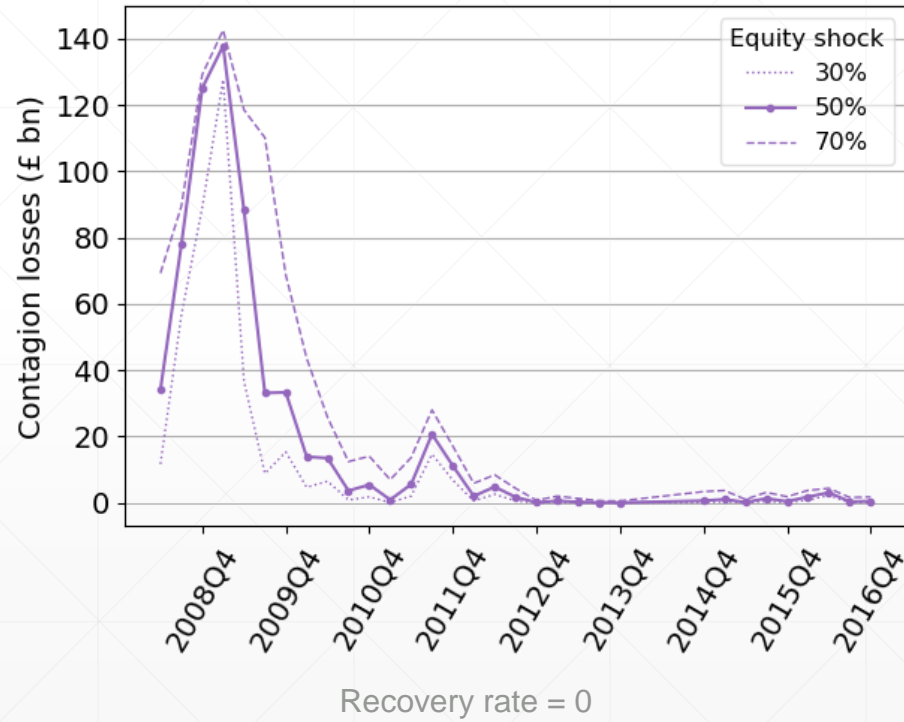
- We run simplified stress tests. In the first “scenario” all banks suffer a homogeneous (relative) shock to their equity.
- Losses due to contagion (orange to purple) can be as large as the exogenous shock.
- Losses caused by direct exposures (orange to blue) can be as large as those caused by indirect exposures (blue to purple).



2008, recovery rate = 0

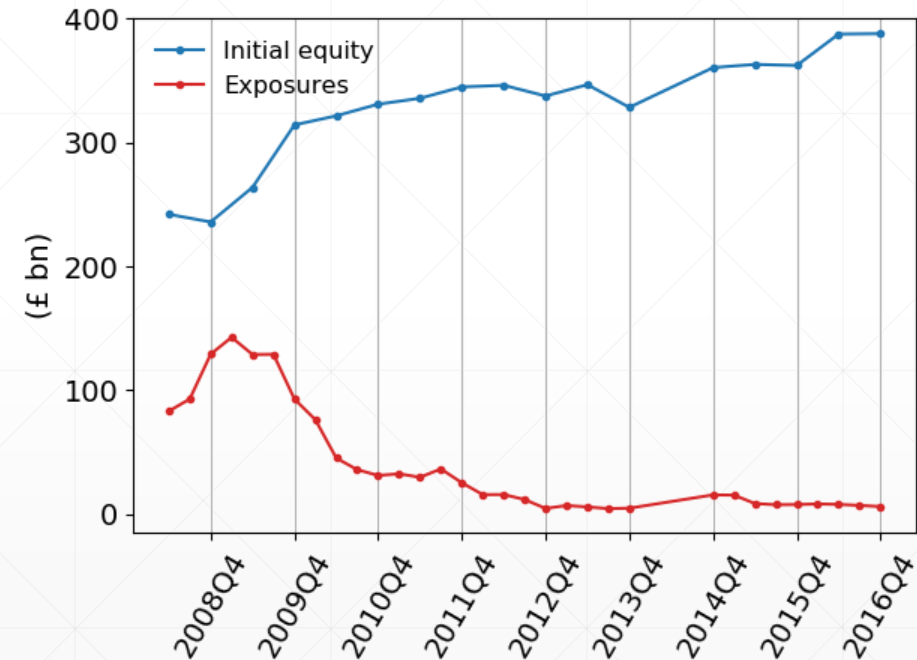


Contagion losses decline

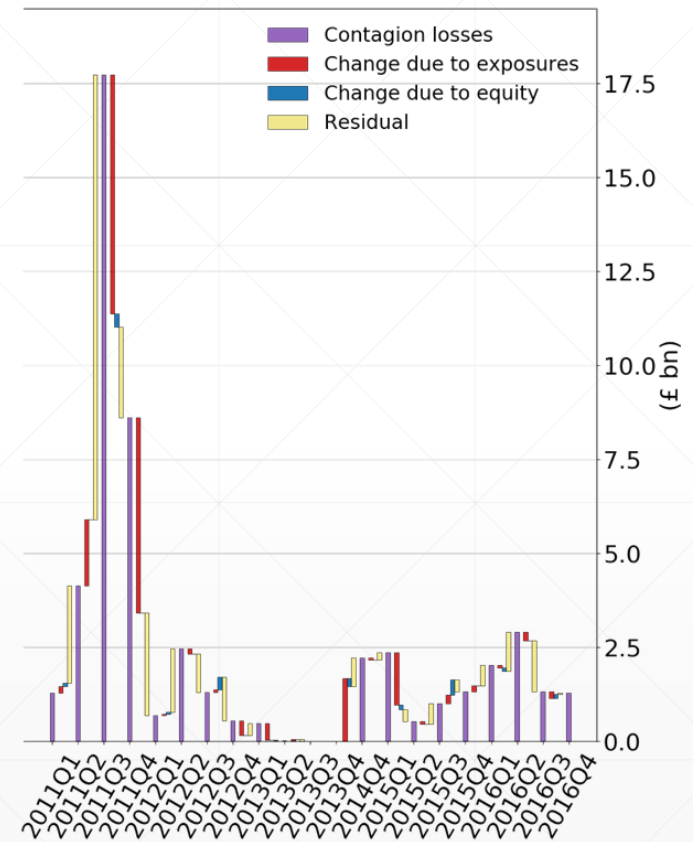
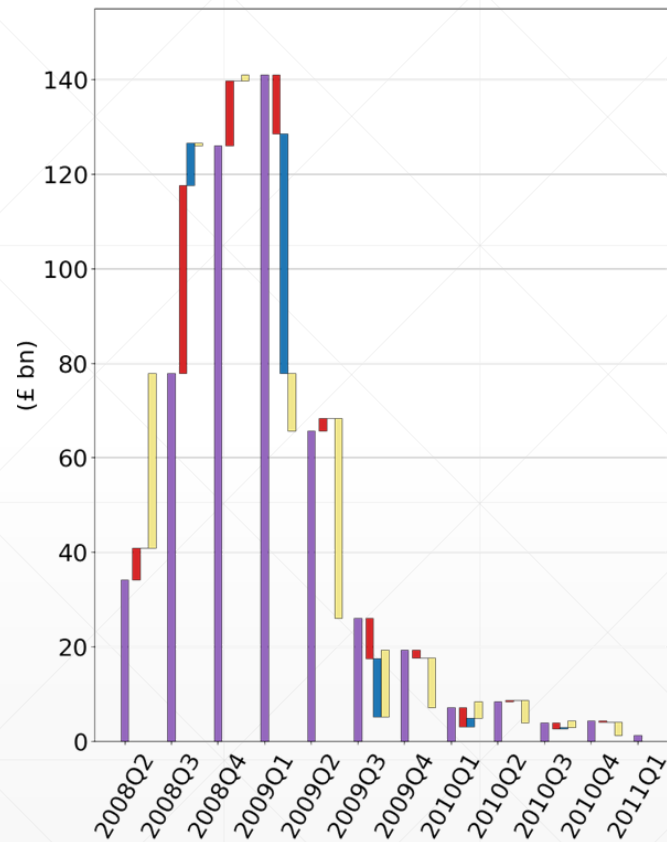


Contagion losses decline

- In order to isolate the effect in changes of equity and exposures we build synthetic balance sheets:
 1. Balance sheets of period 1 with exposures from period 2,
 2. Balance sheets of point 1 with initial equity from period 2
- As a robustness check we also do vice versa.



Decomposing the fall

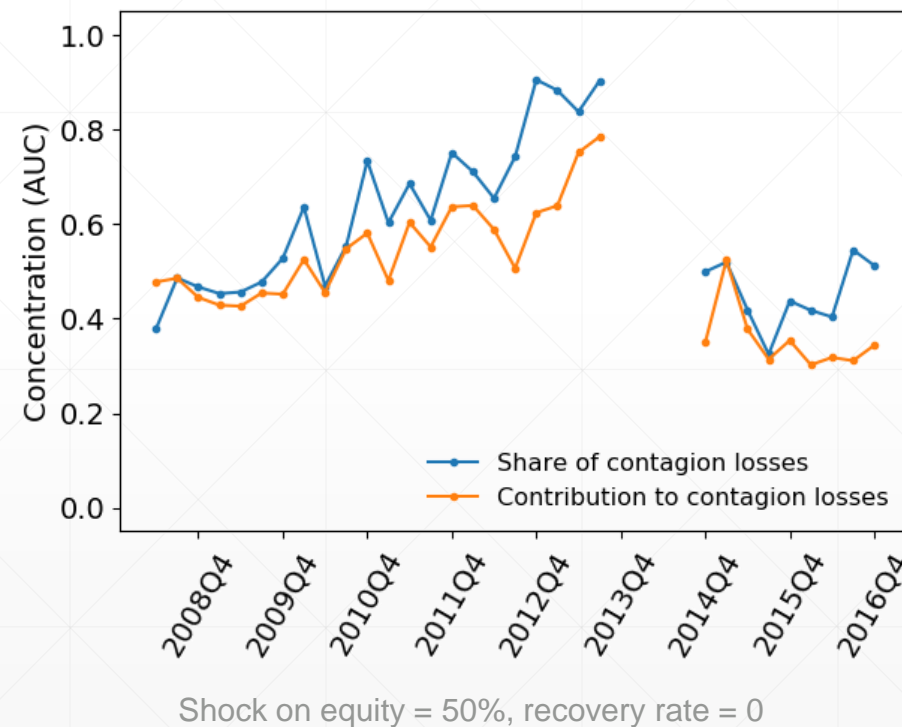


Shock on equity = 50%, recovery rate = 0



Measures of concentration

- **Relative vulnerability**: fraction of contagion losses experienced by a bank
- **Systemic importance**: fraction of contagion losses caused by a bank
 - Selective bail-outs
 - Shapley decomposition of aggregate contagion losses



Conclusions

- When clearing payments money is locally conserved.
- Bankruptcy costs or ex-ante valuation can break conservation.
- There is a valuation framework that includes all those cases.
- We apply the framework to stress testing: The risk related to solvency contagion has shapely decreased from the peak of the crisis to today.



Info

- Neva Working Paper:
<https://ssrn.com/abstract=2795583>
- Bank of England Staff Working Paper:
<https://ssrn.com/abstract=2996689>
- Run solvency contagion on your own data!
<https://github.com/marcobardoscia/neva>



