RECENT DEVEOPMENTS IN THE STDUY OF GROWTH PROCESSES

WORKSHOP ABSTRACTS (Sept 17-19, 2018, Warwick)

Emre Esenturk: **Mathematical Theory of Exchange-driven Growth (Mon 10:30)**

Warwick Mathematics Institute

Exchange-driven process is a mechanism where clusters of (constituent) elements of a system interact with each other by exchanging single unit-element at a time. Such models have recently become popular in describing a variety of physical and social phenomena (population dynamics, wealth exchange etc.).

In this talk we present first rigorous results for the fundamental mathematical properties of the mean field rate equations of this process. We show two different types of behavior depending on whether K(j, k) is symmetric or not. For the non-symmetric case, we show global existence and uniqueness of solutions for kernels satisfying $K(j, k) \leq C\ j\ k$. This result is optimal in the sense that we show for a large class of initial conditions and kernels satisfying $K(j, k) \geq C\ j^{\beta}$ ($\beta > 1$) the solutions cannot exist. On the other hand, for symmetric kernels solutions exist globally for $K(j, k) < C((j^{\mu}) (k^{\nu}) + (j^{\nu}) (k^{\mu}))$ ($\mu, \nu \leq 2$, $\mu + \nu \leq 3$), while existence is lost for $K(j, k) \geq C\ j^{\beta}$ ($\beta > 2$). In the intermediate regime $3 < \mu + \nu < 4$, we can only show local existence.

Furthermore, the classical exchange-driven models allow the exchange of elements to happen among non-zero clusters which leads to uni-directional growth. In this talk, we also present results on exchange-driven process with the possibility of breakages (chipping) from massive clusters to form one-clusters (a cluster of single unit element). We show that for systems which allow chipping, the classical indefinite growth trend is broken and the system approaches to an equilibrium even without the inclusion of a sink.

We also demonstrate these phenomena numerically, that is, without the chipping assumption the system eventually approaches to self-similar form. If chipping is assumed, then the system is driven towards equilibrium. In systems with source, the system again behaves very differently depending on whether chipping (to available volume) is allowed or not.

Johannes Zimmer: **From fluctuations to evolution operators (Mon. 11:30)**

Bath University, Mathematics Department

Fluctuations of particle processes can often provide useful information on the processes, for example in form of fluctuation-dissipation relations. We present some results, old and new. In particular, we consider processes out of equilibrium and aim to gain an understanding of the associated macroscopic evolution equation (the hydrodynamic limit). We present a method to determine transport coefficients, and an extension to determine the entire associated evolution operator. Zero-range processes will be one of the test cases. This is joint work with X. Li, N. Dirr, P. Embacher and C. Reina.

John King: Mathematical modelling of tissue-engineered growth (Mon 14:00)

Nottingham University, Mathematics Department

Some macroscale (PDE) models for the growth of biological tissue will be described and analysed.

Nikolai Brilliantov: Vectorial Smoluchowski equations, steady oscillations and phase transitions in aggregation and fragmentation kinetics (Mon 15:30)

Skoltech, Moscow, Russia & University of Leicester, Leicester, UK

We analyze a wide class of systems with aggregation and fragmentation. First, we consider physical models with the ballistic motion of particles and perform microscopic derivation of coupled systems of equation – for the concentration of aggregates of different size and for the average kinetic energy of such aggregates. These coupled systems of equations have the form of Smoluchowski equations, with microscopic expressions for the reaction rates; hence we call them "vectorial Smoluchowski equations" [1]. We analyze these equations analytically and numerically. Next, we consider aggregation-fragmentation kinetics for model reaction rates, which mimic the most prominent features of microscopic reaction kernels. We demonstrate the existence of steady oscillations in the time dependence of aggregate concentrations [2,3]. For a wide class of aggregation-fragmentation model of an additive type we observe a rich kinetic behavior with a first and second-order phase transition and formation of equilibrium and jammed final states [4]. Analytical findings agree well with the numerical results.

References:

1. N. Brilliantov, T. Poeschel, and A. Formella, Nature Communications, 9 (2018) 797.
2. S. A. Matveev, P. L. Krapivsky, A. P. Smirnov, E. E. Tyrtyshnikov, and N. V. Brilliantov, Phys. Rev. Lett., 119 (2017) 260601.
3. N. V. Brilliantov, W. Otieno, S. A. Matveev, A. P. Smirnov, E. E. Tyrtyshnikov, and P. L. Krapivsky, Phys. Rev. E 98 (2018) 012109.
4. P. L. Krapivsky, W. Otieno and N. V. Brilliantov, Phys. Rev. E, 96 (2017) 042138.

Paul Chleboun: Glassy dynamics and growth processes in kinetically constrained models (Mon 16:30)

Oxford University, Department Statistics

We will discuss the relaxation and out-of-equilibrium dynamics of certain kinetically constrained models (KCMs). KCMs are particle systems on integer lattices, where each vertex is labelled either 0 or 1, which evolve according to very simple rules: i) with rate one and independently for each vertex, a new value 1/0 is proposed with probability 1-q and q respectively; ii) the proposed value is accepted if and only if the current state in a neighbourhood of this vertex satisfy a certain constraint. The out-of-equilibrium dynamics of KCMs are extremely rich and display many of the key features of real glasses. We discuss recent advances on the out-of-equilibrium dynamics of these models, including the relaxation and mixing properties. A particularly interesting feature of the out-of-equilibrium behaviour are the associated growth processes. Rigorous results here are so far limited to one dimension, we will discuss some simulation results which motivate interesting limit shape conjectures in higher dimensions.

This will include joint work with Alessandra Faggionato and Fabio Martinelli.
Juan Velazquez: TBA (Tue 09:00)

University of Bonn, Hausdorff Centre for Mathematics

Michael Herzog: Modelling of Volcanic Ash Aggregation: when theory meets application (Tue 10:30)

Cambridge Centre for Climate Science, University of Cambridge

Aggregation of volcanic ash particles is ubiquitous in volcanic plumes and strongly affects the deposition and long range transport of volcanic ash. Despite this, ash aggregation is commonly ignored in transport models used for predicting and assessing aviation hazards due to volcanic ash. In this talk, we will present and compare two approaches to represent volcanic ash aggregation in tracer transport models for the atmosphere. Practical and theoretical problems in formulating and solving a coagulation kernel for volcanic ash aggregation will be discussed. Simplifications, uncertainties and some of the missing physics will be described.

Michael Wilkinson: Large deviation theory for the collisional growth of rain drops (Tue 11:30)

Department of Mathematics and Statistics,
The Open University, Walton Hall, Milton Keynes, MK7 6AA, England

Rainfall from ice-free cumulus clouds requires collisions of very large numbers of microscopic droplets to create every raindrop, and the collision rate for the first few droplet coalescences is typically less than one per hour. The onset of rain showers can be surprisingly rapid, much faster than the mean time required for a single collision. Large-deviation theory is used to explain this fact.

Miguel Escobedo: An inverse problem for fragmentation equation in the study of amyloid fibril breakage (Tue 14:00)

University of Pais Vasco, Department of Mathematics

We aim at the determination of the total fragmentation rate and the fragmentation kernel of a binary fragmentation equation that is currently used to describe the breakage of amyloid fibrils. Under the assumption of a polynomial division rate $B(x) = \alpha x^\gamma$ and a self-similar fragmentation kernel k_0 we firs prove the existence and uniqueness of a triplet (\alpha , \gamma , k_0$ and a representation formula for k_0 . Some considerations on how to actually obtain a reasonable approximation will also be presented.

Robert Jack: An effective model for a first-order dynamical phase transition in a kinetically constrained model (Tue 15:30)
University of Cambridge, Department of Chemistry

By considering large deviations of time-averaged quantities, one may observe dynamical phase

transitions in many model systems, including simple spin models. We consider the Fredrickson-Andersen model, which is known to have such a first-order transition[1]. We analyse the behaviour of finite systems, close to the phase transition, and we show how their properties can be captured by a simple effective model[2], similar to that proposed by Bodineau, Lecomte, and Toninelli[3]. We will discuss how these theoretical results can rationalise the results of numerical computations, and how they can be used to improve the convergence of numerical methods.

[1] JP Garrahan et al, Phys. Rev. Lett. 98, 195702 (2007)
[2] T Nemoto, RL Jack, and V Lecomte, Phys. Rev. Lett. 118, 115702 (2017)
[3] T Bodineau, V Lecomte and C Toninelli, J. Stat. Phys. 147, 1 (2012)

Bartlomiej Waclaw: Phase transitions in stochastic mass-transport models (Tue 16:30)

University of Edinburgh, School of Physics

Driven diffusive systems, in which diffusive motion of particles on a lattice is biased in a certain direction, are often used as prototypes of non-equilibrium mass transport models. Two such minimal models, the zero-range process (ZRP) and totally asymmetric simple exclusion process (TASEP) are well known for undergoing phase transitions even in one dimension. ZRP exhibits a transition from a liquid to condensed state, whereas open-boundary TASEP is known for its phase diagram with low-, high-, and maximum-current phases and the phenomenon of congestion. Both models are exactly solvable thanks to ultra-local interactions which leads to the factorization of their steady states

In this talk I will discuss some extensions of these models to finite-range interactions and annealed (dynamic) disorder. I will show that although the nature of the steady state changes in these extended models, the basic physics and phase diagrams remain the same as in the archetypal ZRP and TASEP. I will also discuss some interesting connections of these models to fragmentation and aggregation processes.

Philippe Laurencot: Self-similar solutions to coagulation-fragmentation (Wed 09:00)

University of Toulouse, Department of Mathematics

When the coagulation kernel and the overall fragmentation rate are homogeneous, there is a specific value of both homogeneity orders which separates two different behaviours: all solutions are expected to be mass-conserving in one case while gelation is expected to take place in the other, provided the mass of the initial condition is large enough. The focus of this talk is the critical case for which we establish the existence of mass-conserving self-similar solutions, possibly for small masses. This is partly a joint work with Henry van Roessel (Edmonton).

Tibor Antal: The growth process of cancer (Wed 10:30)

University of Edinburgh, School of Mathematics

We consider a population of cells where cells divide, die and mutate into different types. The simplest framework is a multi-type branching process, but other extension will also be discussed. A central question is the time required for the appearance of a certain cell type, for example a therapy resistant type in a cancer model context. Apart from the time it takes, one would also like to know the probable order of elementary steps which led to this mutation. Applications range from the effects of imperfect drug penetration, the use of multiple drugs in therapy, or the relapse time of disease after treatment.

Yanghong Huang: **Stationary States and Asymptotic Behaviours of Aggregation Models with Nonlinear Local Repulsion (Wed 11:30)**

University of Manchester, Department of Mathematics

We consider a continuum aggregation model with nonlinear local repulsion given by a degenerate power-law diffusion with general exponent. The steady states and their properties in one dimension are studied both analytically and numerically, suggesting that the quadratic diffusion is a critical case. The focus is on finite-size, monotone and compactly supported equilibria. We also investigate numerically the long time asymptotics of the model by simulations of the evolution equation. Issues such as metastability and local/ global stability are studied in connection to the gradient flow formulation of the model.

Andre Schlichting: **Convergence to equilibrium for exchange-driven growth (Wed 14:00)**
University of Bonn, Hausdorf Centre for Mathematics

The exchange-driven growth dynamic was recently proposed as a generalization of the classical Becker-Döring model. In exchange-driven growth, pairs of clusters consisting of individual monomers interact by the exchange of a single monomer. In contrast to this, the classic Becker-Döring mode, where a single cluster exchanges monomers with an environment.

We review the derivation of the exchange-driven growth dynamic as the mean-field limit of stochastic particle systems and provide an analysis for its well-posedness. Moreover, under suitable assumptions on the rates, we analyze the equilibrium states and give the convergence to equilibrium.

Besides, we provide a first step and numerical illustration towards the description of more detailed behavior. On the one hand, this is the construction of possible metastable states and on the other hand the explanation of the coarsening phenomena within this model.