

FRAMED MOTIVES IN USE

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Abstract. This is a joint work with Ivan Panin (St. Petersburg). Using the machinery of framed correspondences and framed sheaves developed by Voevodsky in the early 2000-s, a triangulated category of framed motives of smooth algebraic varieties is introduced and studied. To any smooth algebraic variety X we associate the framed motive $M_{fr}(X)$, which is an object of this category. One of the main results states that the bispectrum

$$(M_{fr}(X), M_{fr}(X)(1), M_{fr}(X)(2), \dots),$$

each term of which is a twisted framed motive of X , has motivic homotopy type of the suspension bispectrum of X (this result is an \mathbb{A}^1 -homotopy analog of a theorem of G. Segal). We also construct a triangulated category of framed bispectra $SH_{fr}(k)$ and show that it reconstructs the Morel–Voevodsky category $SH(k)$. As a topological application, it is shown that the framed motive of the point evaluated at the point yields an explicit model for the classical sphere spectrum whenever the base field is algebraically closed of characteristic zero. This machinery also allows to recover in characteristic zero the celebrated theorem of Morel stating that the stable group $\pi_{0,0}(\mathbb{S})$ of the motivic sphere spectrum equals the Grothendieck–Witt ring of the field k .